Aggregation, Reasons and Dynamics.

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Cohesive Support.

- *Sometimes*, in JA we are interested in collective judgment, regardless of the underlying distribution of reasons.
- But *sometimes*, we need a more robust notion of group judgment that is supported by cohesive reasons, or at least reasons that do not undermine each other.
- How to model this latter notion?

Undermining Reasons: an example

Here's a twist on the *discursive dilemma*:

You have invested equal amounts of money in stocks of two companies: *Cookbooks* and *Shoes*. You have no other stocks.

1. Expert #1, Bob, thinks that next week your *Cookbooks* stock will improve by 10%, while *Shoes* will break even.
2. Expert # 2, Jim, thinks that *Shoes* will improve by 10% while Cookbooks will break even.
3. Expert #3, Lara, thinks that both stocks will break even.

Should you defer to the majority on the proposition that your portfolio value will increase by 5%? Intuition vacillates...

PB to the rescue?

- *Initial Thought*: 'reason-sensitivity' can be handled by Premise-Based/Sequential Priority procedure(s). We can use these to specify what is it to be cohesive.
- But:
  1. **the model of 'reasons' incorporated in the Premise-Based procedure is too rigid.**

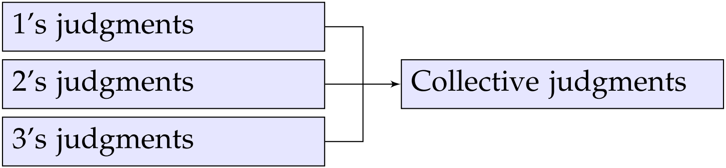
     Presupposes that the relation of X *is a reason for* Y is fixed for all judges, and flows from premises to conclusions.
  2. **PBP functions properly only under very specific circumstances.**

     It requires logically independent premises such that any combination of truth-value settles by entailment the truth value of the 'conclusion'.

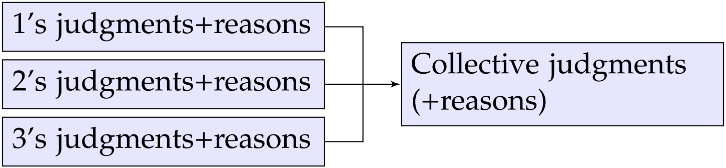
An Idea for An Alternative Framework.

Base Idea: a framework that properly handles the distinction between cohesive and non-cohesive judgments needs to operate on slightly finer inputs.

Standard Judgment Aggregation Theory



Our Framework



Aims.

1. integrate Judgment Aggregation and established formal models of reasons.
2. show how to reconstruct 'familiar' aggregation rules as special cases of more general rules expressible in our framework.
3. attempt to explain why the distinctive rules in this framework are valuable (beyond capturing the examples).

How to model reasons?

Defaults as Reasons

Import: work on *default* models of reasons by J. Horty (*via* Reiter).

- $\Sigma \hookrightarrow \phi$ means that the propositions in Σ function as reasons for ϕ (for an agent).
  - $\hookrightarrow$ is not a connective in the language.
- If $\Sigma \hookrightarrow \phi$, we require that $\Sigma \cup \{\phi\}$ be consistent.
  - But no more! **No requirement** that Σ entail ϕ.
- Given a default $\delta = \Sigma \hookrightarrow \phi$, we say:
  - Premises(δ)=Σ
  - Conclusion(δ)=ϕ

These functions are naturally lifted to sets of defaults (e.g. Conclusion($\{\delta_1, \delta_2\}$)={Conclusion(δ_1), Conclusion(δ_2)})

Expanded Agendas.

We complement an agenda $\mathcal{A}$, defined as usual, with:

- a set Δ containing all the relevant admissible defaults involving propositions in the agenda.
- an ordering $\succ$ that specifies which defaults take priority over others.

  Assume, minimally, that $\succ$ is a strict partial ordering.

All of these features are external to individual agents' epistemic states and fixed for all individual agents.

Default Epistemic States.

An agent's epistemic state is represented as a pair $\langle W, \mathcal{S} \rangle$ consisting of:

- a judgment set $W \subset \mathcal{A}$ representing the propositions that are accepted non-inferentially.
- a set $\mathcal{S} \subseteq \Delta$ of 'active' defaults.
  - The *Premises* of these defaults need not be propositions the agent accepts.
  - e.g. I can believe that it's sunny (Y), and that it being sunny is a reason to wear shorts (T), but also believe that $\sim Y$ is a reason to wear long pants.

Basic Modeling of Reasons.

R.t. a background $\langle \mathcal{A}, \Delta, \succ \rangle$, and an epistemic state $\langle W, \mathcal{S} \rangle$:

$Provable_{W,\mathcal{S}}$	$\{\phi \mid \phi$ is 'provable' from W using only rules in $\mathcal{S}\}$
$Triggered_{W,\mathcal{S}}$	$\{\delta \mid \forall\gamma \in Premise(\delta), \gamma \in Provable_{W,\mathcal{S}}\}$
$Defeated_{W,\mathcal{S}}$	$\{\delta \mid \exists\delta' \in Triggered$, such that $\delta' \succ \delta$ and $Conclusion(\delta') \vdash \sim Conclusion(\delta)\}$.
$Binding_{W,\mathcal{S}}$	*Triggered*, not *Defeated*.

Note: It is possible for non-inferential justification to be defeated as well. To handle this, we must record non-inferential justification as a kind of default on its own (e.g. $\{\top\} \hookrightarrow \phi$). $\top$ is always accepted, so this default, if active, is always triggered.

Acceptance

$$Accepted_{W,\mathcal{S}} \quad \{\phi \mid \exists\delta, \delta \in Binding, \phi = Conclusion(\delta)\}.$$

This set contains all accepted propositions, whether inferentially or not.

Aggregation Framework: usual definitions

An epistemic state $\langle W, S \rangle$ is *fully rational* iff $Accepted_{W,S}$ is consistent and complete relative to the agenda.

A profile $\vec{j}$ is a vector of fully rational epistemic states.

aggregation rule: $\mathcal{R}$ maps profiles onto judgment sets.

***proper* aggregation rule**: $\mathcal{R}$ maps profiles to epistemic states.

Strong Cohesiveness.

Definition

$F \subseteq \mathcal{G}$ is **strongly cohesive** w.r.t. ϕ (in $\vec{j}$) iff there is a default δ such that:

- Conclusion(δ)=ϕ and
- For every $i \in F$, δ is Binding (for i).

Definition

$F \subseteq \mathcal{G}$ is **perfectly cohesive** w.r.t. ϕ (in $\vec{j}$) iff there is a non-circular default proof $\langle \delta_1, ..., \delta_k \rangle$ of ϕ such that:

- For every $i \in F$ and $x \in \{1, ..., k\}$, δ_x is Binding (for i).

Cohesive Majority

Definition (Cohesive Majority)

$\phi \in \mathcal{CM}(\vec{j})$ iff there is a set of judges $S \subseteq \mathcal{G}$, such that S strongly cohesively supports ϕ and S is a majority of $\mathcal{G}$.

- CM is more demanding than Majority.
- CM is not independent.
- CM is not guaranteed to preserve consistency.
- CM is not guaranteed to preserve deductive closure.

PBP And All That

Is there a clear relation between *PBP* and *CM*?

Yes, in a sense *PBP* is a special case of *CM*.

Definition (Premise-centered)

(i) j_m is *premise-centered* iff j_m's judgment on the "conclusion" is grounded on the individual judgment on the "premises", and no other propositions are inferentially grounded.

(ii) $\vec{j}$ is *premise-centered* iff every epistemic state in it is.

Theorem

On conjunctive/disjunctive agendas, if $\vec{j}$ is premise-centered, $\phi \in Th(CM(\vec{j}))$ iff $\phi \in PBP(\vec{j})$.

Vindicates motivating points:

> *(i) a Premise-Based approach captures a special kind of reason-sensitivity.*
> *(ii) puts resolution of 'instability' in the hands of the judges themselves.*

CM and Consistency.

In the standard framework, there is a supermajoritarian fix to the consistency problem [Pettit (2006), List (2007)].

[List (2007): $t_{\mathcal{A}} = (x-1)/x$, where x is the size of the largest minimally inconsistent subset of the agenda]

The same threshold also works in this context.

Definition (Cohesive Supermajority)

$\phi \in CSM(\vec{j})$ iff there is a set of judges $S \subseteq \mathcal{G}$, such that S strongly cohesively supports ϕ and $|S|/|\mathcal{G}| > t_{\mathcal{A}}$

CSM is incomplete, but so was *CM*.

The need for Proper Rules

There is a gap in the account I have been sketching.

Although the rules *input* a pair of judgments and reasons, the output is just a set of judgments.

Luckily, we can construct a proper rule that matches *CM*.

Cohesive Majority: Default version

$\mathcal{S}_{\vec{j}} = \{\delta \mid \delta$ is Binding for every member of a majority M_δ of judges in $\vec{j}\}$

$W_{\vec{j}} = \{\phi \mid \{\top\} \hookrightarrow \phi \in \mathcal{S}_{\vec{j}}\}$.

$CM_{proper}(\vec{j}) = \langle W_{\vec{j}}, \mathcal{S}_{\vec{j}} \rangle$.

Theorem

$\phi \in CM(\vec{j})$ *iff* $\phi \in Accepted_{W_{\vec{j}}, \mathcal{S}_{\vec{j}}}$

Advantage: $CM_{proper}(\vec{j})$ is richer than $CM(\vec{j})$. Contains both triggered and untriggered defaults.

Dynamics: a preliminary story.

We are interested in the idea that cohesiveness makes the group judgment *more* stable.

$$R[\text{update}(\vec{j})] = \text{update}[R(\vec{j})]$$

If a group (under an aggregation rule) satisfies the schema (w.r.t a kind of update), it means that the group can respond systematically to the particular update without need for re-aggregation.

A kind of update

Update rule: $+\delta$.

> *When $\langle W, \mathcal{S}\rangle$ is an epistemic state, $\langle W, \mathcal{S}\rangle_\delta$ consists in:*
>
> - *updating to $\langle W, \mathcal{S} \cup \{\delta\}\rangle$*
> - *modifying $>$ so that δ has the highest priority of the defaults in $\mathcal{S}$.*

Interpretationally, this consists in the idea that δ is announced as an acceptable pattern, not defeated by any current information.

Commuting with $+\delta$.

- On the negative side:
  CM_{proper} does not commute with $+\delta$.
- On the positive side:
  remember our notion of perfect cohesiveness. That notion can be used to define a more stringent rule we might call Perfect Cohesive Majority (PCM).
  The proper version of PCM commutes with $+\delta$

Conclusions

There are some expressive advantages to the 'input rich' framework we are developing.

We can define a (broadly) majoritarian rule that:

- incorporates an entrenched model of reasons-dependence.
- has premise-based majority as a natural special case.
- showcases a new kind of responsiveness.
- can be made consistent by the usual quota-raising maneuvers.

Additionally: the rule can easily be given a 'proper' version:

- gets us collective reasons, alongside collective judgments.
- allows analysis of stability properties.