Social Networks, Promotions, and the Glass-Ceiling Effect

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Abstract

Empirical studies show that women are under-represented in highly paid top management positions of firms (glass-ceiling effect) which could be a cause of the gender wage gap. In order to study women's career paths, we develop an equilibrium search and matching model where job ladders consist of three hierarchical levels and workers can progress in the career by means of internal promotions and job-to-job mobility. Both, formal applications and referral hiring via endogenous social networks, can be used for external moves between firms. We show that when female workers are the minority in the occupation and social link formation is gender-biased (homophily) there are too few female contacts in the social networks of their male colleagues. This disadvantage implies that female workers are referred less often for the jobs and thereby they become under-represented in top level management positions of firms. Our results suggest that endogenously forming homophilous social networks when female workers are the minority can explain a substantial part of the empirically observable total wage gap stemming from the glass-ceiling effect. In addition, we demonstrate that the effects of the mechanism are amplified by stronger clustering of social networks, and gender-biased promotion times. Furthermore, deeper hierarchical firm structures amplify the network-driven gender inequality but mitigate the inequality from direct discrimination in internal promotions.

Keywords: glass-ceiling effect, networks, discrimination, theory of the firm, promotions, search-and-matching labor market

JEL-classification: D21, D85, J31, J63, J71

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1 Introduction

Women are much less represented on the managerial level of firms than men. For the U.S., a report by McKinsey&Company (2017), based on data of 222 U.S. companies employing more than 12 million people, finds that women are significantly under-represented in the corporate pipeline, and that at every higher level the representation of women declines. By the time women reach the senior-vice president level, they only hold 21% of these positions. Similarly for Europe, only 15.8% of the decision making positions were held by women in the largest listed companies in 28 countries of the European Union in 2017. Moreover, women earn substantially less than men. The International Labor Office ILO (2016, p.82) reports an overall gender pay gap close to 20%, based on data of Eurostat's Structure of Earnings Survey. For Chief Executive Officers (CEOs) the gap is twice as large and it continues to wide to 50% among the top 1% earners.

Our paper addresses the gender wage gap arising from females not making it to the higher managerial positions in firms – known as the glass-ceiling effect. In the literature, there exist many explanations for the gender wage gap ranging from occupational segregation whereby women are sorted into low wage occupations (Erosa et al., 2017) to more frequent career interruptions by women (Winter-Ebmer and Zweimüller, 1997). Nevertheless, a recent overview study by Blau and Kahn (2017) shows that there remains a substantial unexplained gender wage gap even after most of the conventional explanations including human capital, occupation and industry dummies, and other variables are controlled for. Similarly, Albrecht et al. (2003) report a large unexplained gender wage gap associated with a glass-ceiling effect in their data. As an alternative explanation several studies mention the fact that women have a lack of "old-boys-club" connections as another factor for worse positioning of women in the job ladders and lower wages (Milgrom and Oster, 1987; Cassidy et al., 2016; Bertrand et al., 2018), but a proper analytical investigation of the underlying mechanism is still missing in the literature.[1] Our study attempts to fill this gap by developing a search and matching model with two gender groups, hierarchical firms giving rise to job ladders, and referral hiring via professional networks.

The primary goal of our theoretical approach is to analyze differences in the career progression of men and women and relate them to the overall gender wage gap. More specifically, we develop a model with job ladders consisting of three/four hierarchical levels. Male and female workers can progress in their careers by means of internal promotions and by moving to new employers. In modeling internal promotions, we follow the seminal approach by Gibbons and Waldman (1999) and its recent application in Dawid et al. (2019): workers need to accumulate a specific level of experience/human capital in order to be considered for a promotion, and there should be an open senior position in the firm to which a worker can be promoted. If there are no open senior positions in the firm experienced workers start searching on-the-job. This means that firms may fill a vacant position with an internal or an external candidate who accumulated sufficient experience. External candidates may be hired from the market which is subject to search and matching frictions, or recommended by senior managers who draw on their social network. Importantly, social networks form endogenously. Link formation is subject to gender homophily implying that similar workers are more likely to form social links (McPherson et al., 2001). Existing links are destroyed at an exogenous rate. To the best of our knowledge this is a first theoretical study analyzing gender differences in internal promotions and external mobility across firms in a unified framework with search frictions and endogenous network formation.

We use our model to shed light on gender inequality generated by differences in the endogenous formation of professional networks between men and women, and evaluate the magnitude of this effect. We ask the following questions: (a) What is the impact of referral hiring on the relative proportions of men and women along the job hierarchy of firms? (b) What is the effect of network characteristics such as gender homophily and clustering on females' employment shares in managerial positions? (c) What is the impact of the overall female participation in the profession/occupation, and is there a role to be played for the depth of firms' hierarchies with

[1] Some exceptions include Rubineau and Fernandez (2013, 2015) which we review later in the paper.

respect to gender representation? And finally (d) Is there an interaction between the network effects, hierarchical structures of firms and gender-based discrimination in promotions?

Several new results can be obtained from our framework. We find that a disproportionately low representation of women in senior positions, that is the glass-ceiling effect, can emerge without occupational segregation, direct discrimination in formal hiring, or unequal promotion chances. For this to occur in our set-up, women need to be a minority in the profession and network formation has to be homophilous. Note, that being a minority per se does not automatically lead to the glass-ceiling effect. For example, there is no glass-ceiling effect if the share of women reaching senior positions is the same as a share of men reaching this level even if women are a small group in the profession. So the glass-ceiling effect in our model is driven by the combination of low female participation and network homophily. Intuitively, network homophily implies that individuals from the same gender group are more likely to communicate with each other and establish a network link. Later, this network link may lead to a job referral which helps workers to progress in their careers by means of job-to-job transitions. The fact that referral hiring exhibits a gender bias is empirically documented by Fernandez and Sosa (2005). Again we show that gender-biased referring alone is not sufficient to generate gender inequality. For example, there is no glass-ceiling effect if the two gender groups are equally large. It is only if women are a minority in their profession, lower probability of creating social links with the majority group of men generates smaller professional networks of women, and leads to the disadvantage in the career progression as long as referral hiring is at place.

This result is inline with the idea by Marsden and Gorman (2001) who write that "women's networks are less likely to include contacts in possession of valuable job information ... [because] there are more other women (who, when employed, are less likely to hold ownership or authority positions) in women's networks then in men's" (p. 476). One way to check this hypothesis empirically is to compare the fractions of referral hires between men and women since the model predicts a lower fraction of referral hires among newly hired female workers compared to males. Corcoran et al. (1980) reports that in her sample men were more likely than women to first hear about a job through a contact, to have known others at the workplace prior to being hired, and to have received aid in getting the job from an influential person in the workplace. A more recent overview study by Topa (2011) comes to the conclusion that empirical "evidence suggests that women are less likely to use informal contacts than men (with regard to both information about vacancies and direct influence" (p. 1201) even though several studies find insignificant effects. Also Behtoui (2008) and Alaverdyan and Zaharieva (2019) support the view that women use informal methods of job-finding less often than men in Sweden and Germany.

Further, we find an unexpected result that the relationship between network homophily and gender inequality is not monotone. Rather the gender wage gap is maximized for an intermediate level of network homophily. In the extreme case, when social networks are fully segregated, meaning that employees always recommend friends of the same gender, referral hiring is simply reproducing the same shares of males and females on all hierarchical levels and the glass-ceiling effect disappears. One necessary condition for this effect is that (fully segregated) social networks of both gender groups should be sufficiently large. In this case fully balanced and fully segregated social networks do not give rise to gender inequality, whereas a moderate gender bias in network formation close to the empirical estimates reported in Fernandez and Sosa (2005) is associated with a maximal inequality.

We consider three extensions of our benchmark model. First, we want to know whether a more clustered network formation as compared to random networks has an effect on the gender wage gap. This extension is motivated by the fact that social networks are often characterized by triadic closures (Simmel, 1908) meaning that the probability that two individuals will form a social link is increasing if these two individuals have a common friend. Second, we introduce direct discrimination of women along the career path by reducing the experience requirement for men necessary to obtain a promotion. Even though empirical evidence on gender differences in promotion rates is generally mixed, there is a large group of studies reporting lower probabilities of internal promotions for women indicating some degree of discrimination (see, e.g., Winter-Ebmer and Zweimüller (1997), Ward (2001), Blau and DeVaro (2007), Johnston and Lee (2015), Kauhanen and Napari (2015) and Cassidy et al. (2016)). This extension allows us to evaluate the relative contribution of network hiring to the gender wage gap in the presence of another strong factor responsible for the gender wage differentials. Last, we analyze if the gender wage gap changes as firm hierarchies are becoming deeper. This extension captures an empirically well documented changing structure of corporate hierarchies (see, e.g., Bloom et al. (2010), Rajan and Wulf (2006), or Guadalupe and Wulf (2010)) and allows us to compare gender wage gaps across occupations with different hierarchical composition of firms. In this part of the paper, we rely on the simulation framework since none of the three extensions appears to be analytically tractable. In order to validate the simulation approach, we first replicate the benchmark analytical distributions and show that the simulation replicates the analytical results with a high degree of precision and reliability. In the next step, we perform the extensions.

In the first extension, we can see that triadic closures serve as an amplifying device for gender inequality. The reason is that with triadic closure there are less links of male workers with female workers, and more links of male workers with other male workers which makes job-to-job moves for females less likely. In the second extension, lower promotion chances of women amplify the gender wage gap, but there is an important difference. While referral hiring reduces the access of women to career ladders and acts at the bottom of the hierarchy, direct discrimination in the promotion times does not influence the access but the speed at which female workers climb the ladder and acts at the top level of the hierarchy. This difference explains the next result when we increase the number of hierarchical levels from three to four. The "bottom-level" inequality generated by referral hiring is amplified and there is a strong interaction effect between the number of hierarchical steps and network clustering. In contrast, the "top-level" inequality generated by direct discrimination in the promotion times is mitigated since differences in the speed of promotions are smoothed across a larger number of career steps. Overall, this shows that the impact of deeper firm hierarchies can amplify or mitigate the inequality depending on the exact reason giving rise to the unequal situation of men and women.

Numerically, we find that network homophily combined with a minority status of women (30% of the labor force in the occupation) may explain up to 42% of the wage gap attributed to the glass-ceiling effect. This means up to 2.7% out of 6.4% in a country like Germany. Next to proposing empirically based alternative channels explaining the gender wage gap, our analysis implies, as we believe, important policy implications. As homophilous networks are one driver behind the dis-proportionate gender distribution in managerial jobs, it occurs to be advisable to establish instruments that are conducive to gender-mixing of networks. Policies that encourage women's only networks at workplaces, as can be often observed nowadays, seem to be ineffective in reducing the gender inequality – at least if they result in some intermediate degree of homophily.

We proceed with a review of the related literature. In Section 3 we conduct an Oaxaca-Blinder decomposition of the gender wage differential for high skilled workers working in Germany in order to motivate empirically that an unequal representation of women in top management jobs explains a sizeable part of the overall gender wage gap. In Section 4, we introduce our analytical apparatus, and in Section 5, we present the results with respect to the network formation and the gender distribution along the hierarchical levels of the firm. In Section 6, we introduce extensions to our analytical framework and simulate their effects. The last section concludes.

2 Literature review

We are not aware of formal investigations of how networks, referrals, and the depth of firm hierarchies are associated with each other in search and matching labor markets. Each of these topics has received considerable attention on its own, however. Since M. Granovetter's assertion that "Careers are not made up of random jumps from one job to another, but rather that individuals rely on contacts acquired at various stages of their work-life, and before. (1995, p.85)" various empirical studies confirmed that a large fraction – sometimes close to and above 50% – of the employees found their jobs via personal contacts.[2]

From a theoretical perspective the seminal model on referral hiring was developed by Montgomery (1991) who formalized the idea of network homophily. In particular, he described homophily by ability, when high ability employees recommend high ability contacts from their network. This approach was incorporated into a search and matching model by Galenianos (2013), however, none of the two studies considers gender homophily and hierarchical firms. Other theoretical contributions on referral hiring in a search and matching framework include Calvó-Armengol and Zenou (2005), Ioannides and Soetevent (2006), Fontaine (2008), Galenianos (2014), and Stupnytska and Zaharieva (2017). Galenianos (2014) investigates cross-sectional frequency of referrals and shows that more intensive referral hiring is associated with more efficient matching in a given sector. Ioannides and Soetevent (2006) and Fontaine (2008) show that larger social networks improve job-finding chances of unemployed workers as well as their wage bargaining position in the negotiation with firms. This mechanism implies that heterogeneity in the composition of networks is translated into the equilibrium wage inequality. Hence, their focus is on network-driven differences in wages earned by workers performing identical jobs. In contrast to this approach, we assume that wages in the same jobs are identical for all workers and investigate the role of social networks for the distribution of workers across different hierarchical positions. The particular mechanism of referral hiring that we use is similar to Calvó-Armengol and Zenou (2005) and Stupnytska and Zaharieva (2017).

A considerable part of the literature on labor markets and social networks assumes that the use of contacts is exogenous to labor market conditions. Notable exceptions are Boorman (1975), Calvó-Armengol (2004), or Galeotti and Merlino (2014). Galeotti and Merlino (2014), for instance, explores the effect of labor market conditions on the use of social networks and their effectiveness on matching job seekers to vacancies. Our study adds to this strand of literature, notably in a context where the formation of homophilous networks is related to gender differences in labor market participation rates.

To the best of our knowledge there are only three studies that analyze the implications of social networks in the market with job-to-job mobility. These are Horvath (2014), Zaharieva (2015), and Arbex et al. (2018). The latter paper builds on the early work by Mortensen and Vishwanath (1994) and assumes that employees refer their friends to jobs with the same wage as their own. In this case, the distribution of network offers is superior to the standard wage offer distribution. A different approach is undertaken in Horvath (2014) and Zaharieva (2015). In these studies employees forward job offers that are (weakly) worse then their own, so network job offers are negatively selected. This selection effect is also present in our model since workers refer their network contacts to lower hierarchical positions than their own, but there is an additional competition effect: if one worker group moves faster in the job ladder it reduces the number of senior positions available to the other group, because the two groups are directly competing for a fixed number of jobs. This effect is absent in the previous work.

Next, our study is related to the literature analysing job search via social networks in a simulation framework. This group of studies includes Calvó-Armengol and Jackson (2004), Bramoullé and Saint-Paul (2010), or König et al. (2014), as well as work that has taken the agent-based simulation approach as, among others, Tassier and Menczer (2008), Gemkow and Neugart (2011), or Dawid and Gemkow (2013). None of these contributions, however, modeled the hiring and promotion decisions of firms with hierarchies in connection with endogenously evolving social networks.

Even though differences in professional networks between men and women are often mentioned as a factor for observed diverging labor market performances (Milgrom and Oster, 1987; Cassidy et al., 2016; Bertrand et al., 2018), so far there were hardly any attempts to investigate

[2]Pistaferri (1999) shows it for Italy, Addison and Portugal (2002) for Portugal, Bentolila et al. (2010) and Pellizzari (2010) for the United States and the European Union, and Rebien et al. (2017) for Germany.

the underlying mechanism with notable exceptions by Rubineau and Fernandez (2013, 2015). They analyze the interaction of the supply and demand side for hiring decisions of firms theoretically with Markov switching models. Contrary to their analyses, we place hierarchical firms in a labor market with search frictions. Empirically, the demand side perspective was investigated by Kmec (2005) who look into the organizational practices to locate and hire workers, or by Fernandez and Abraham (2010, 2011), Fernandez and Campero (2017) and Fernandez-Mateo and Fernandez (2016) who show that the gendered composition across levels of the organization can be traced back to the gendered nature of the candidate pools for jobs at the different levels. Given that referral hiring is an important feature of the labor market and social networking becomes more relevant with a rapid development of communication software (e.g. Facebook, LinkedIn) our study attempts to investigate the role of social and professional networks for gender inequality in the presence of hierarchical firms. Our contribution seeks to offer a theoretical model that clarifies the mechanisms that may underlie these empirical findings. In a setting with hierarchical firms, initial segregation of homophilous referral networks leads to a smaller fraction of women reaching senior positions. This implies that women have lower chances of recommending new applicants to mid-level positions contrary to Rubineau and Fernandez (2015). So demand-side effects are endogenous and amplify the segregating effect of homophilous network recruitment in our model.

Related to the hierarchical nature of the firms, one may also mention studies that document gender-based discrimination in the process of formal hiring, see Firth (1982) for the UK, Neumark et al. (1996) for the US, and Petit (2007) for France, or along the promotion path of firms. Blau and DeVaro (2007), Kauhanen and Napari (2015) and Cassidy et al. (2016) show that women have lower promotion probabilities within firms in the US and in Finnland. However, unequal promotion chances are not supported for countries like Germany and the United Kingdom, see Chadi and Goerke (2018) and Booth et al. (2018), respectively. We take up the issue of discrimination along the promotion path of firms in one of our simulation-based extensions.

Finally, the search and matching framework introduced by Diamond (1982), Mortensen (1982), and Pissarides (1985) within which we model firms' recruitment behavior and workers' network formation has become one of the workhorse models in labor economics.

3 Empirical motivation

In how far under-representation of women in decision making positions contributes to the gender wage gap empirically can be exemplified with data from the German Socio-Economic Panel (GSOEP). The GSOEP not only provides information on respondents' qualification and gross monthly wages but also on the type of job the person holds. For our analysis, we restrict the sample to full-time employed high skill men and women with more than 13 years of schooling. This sample includes 1446 observations for men and 957 observations for women. Even though women are not a minority in the German population and have the same average education level as men, the fact that many women work in permanent part-time jobs without career prospects makes career-oriented full-time employed women a minority comprising 39.8% of our sample. This fraction varies very strongly with the industry and occupation starting from 18-20% in the energy and construction sector, and reaching 45-48% in retail trade and non-financial services.

Table 1 shows the male and female employment shares and gross monthly wages by hierarchical level. Even though the data includes information about four hiererachical levels, we merged the upper two (middle and top management jobs) into one group for the purpose of our research. The reason is that there are three hierarchical levels in the benchmark model, and we use this data to provide a realistic numerical example illustrating our results. Table 1 shows that females are less likely to occupy the top and middle management positions with only 13.9% of women reaching the upper level and 64.3% remaining in non-management jobs. Moreover, positions in top and middle management pay on average 71% more than non-management positions.

The average gender wage gap in the group of full-time high skilled men and women in

Table 1: Employment shares and wages

	Fractions in %			Predicted wages, all	
Hierarchical level	Females	Males	All	in €	Normalized
Middle/top management jobs	13.94	25.78	21.09	5579.0	1.713
Lower management jobs	21.76	27.32	25.12	4295.0	1.319
Non-management jobs	64.30	46.90	53.79	3255.4	1.000

Data source: SOEP (2013), wave 2013.

Germany is equal to 31%. We decompose this difference in wages between men and women by using a triple Blinder-Oaxaca decomposition (Oaxaca, 1973). Accordingly, the wage gap can be decomposed into endowment, coefficient, and interaction effects:

$$\begin{aligned}\text{Wage gap} &= E[Y^M] - E[Y^F] = E[X^M]'\beta^M - E[X^F]'\beta^F \\ &= \underbrace{(E[X^M]' - E[X^F]')\beta^F}_{\text{Endowment effect}} + \underbrace{E[X^F]'(\beta^M - \beta^F)}_{\text{Coefficient effect}} + \underbrace{(E[X^M]' - E[X^F]')(\beta^M - \beta^F)}_{\text{Interaction effect}}\end{aligned}$$

with Y_M and Y_F being male and female wages, respectively. X is a vector of explanatory variables that includes information on education, age, tenure, migration background, industry, size of the firm, location with respect to East or West Germany, and hierarchical position in the firm. β^F and β^M are the vectors of estimated coefficients. The first term in the decomposition is the "Endowment effect". It shows which part of the wage gap can be explained by between-group differences in the observable characteristics. The second term is the "Coefficient effect". It shows which part of the wage gap can be explained by different pricing of female and male characteristics in the market. The last effect shows the interaction between the endowment and the coefficient effects. Our decomposition results are summarized in Table 9 in the Appendix. In the second column of that table the contribution of every explanatory variable k to the total endowment effect $(E[X_k^M] - E[X_k^F])\beta_k^F$ is shown.

We find that 16.4% of the total wage gap equal to 31% is explained by differences in the observable characteristics of men and women. More specifically, 10% of the endowment effect are driven by the fact that females are younger, less experienced, over-represented in service occupations, and employed in smaller firms. In addition, the female participation rate is higher in Eastern Germany and the average salary level is lower in this region which contributes significantly to the gender wage gap. We do not find significant differences in formal qualification and education, indicating that a conventional human capital explanation for the gender wage gap vanished in Germany as it did in other developed economies (see Blau and Kahn, 2017, for the US). Importantly, and in relation to our analysis, differences in the hierarchical levels between the two gender groups explain 6.4% out of the endowment effect (16.4%). We interpret this difference of 6.4% as the glass-ceiling effect, i.e. differences in the gender wage gap driven by women being under-represented in the better paying top jobs in firms.

4 The Model

The primary goal of our theoretical approach is to analyze possible differences in the career progression of men and women. Note, that career growth can be achieved by internal promotions within firms and by external job-to-job transitions to higher hierarchical positions. Since internal and external mobility is likely to be influenced by different factors we claim that it is important to distinguish between the two in a theoretical framework. For example, the speed of internal promotions within firms is likely to depend on firm-specific experience and motivation of the employee, whereas the speed of external mobility across firms is likely to depend on formal qualification, search frictions and social networks. In addition, both types of career progression depend on the hierarchical composition of firms and can be associated with direct discrimination of one worker group. Since empirical research in the recent years shows insignificant differences in the formal qualification between men and women, our study is focused on the remaining factors such as experience accumulation, search frictions and social networks. In the extension of the model we also analyze the role of firm hierarchies and direct discrimination as well as possible interaction effects between several of these factors. To the best of our knowledge this is a first theoretical study analyzing gender differences in internal promotions and external mobility across firms in a unified framework with search frictions.

In a nutshell our model has the following characteristics: time is continuous and workers enter and exit the market at an exogenous rate ρ. The total population size is normalized to 1 and there is no population growth. There are two types of agents: female workers (F) and male workers (M). Fraction $h \leq 0.5$ of workers are of type F and fraction $1 - h \geq 0.5$ are of type M. This parameter allows us to capture the lower participation rate of women in a number of occupations, especially in professional full-time jobs. All workers are identical with respect to their education and productivity.

Job ladders consist of three hierarchical levels: low-level jobs, middle management and senior management. Positions in the first hierarchical level are freely available to all workers without search frictions; these workers form a pool of applicants for middle management jobs. Since there are infinitely many firms offering low-level jobs we do not model these firms explicitly. All career firms consist of one middle management job and one senior management job. This is the simplest firm structure allowing to study internal promotions from mid-level to senior positions. Another advantage of having two-position firms is that senior managers may recommend their social contacts for mid-level jobs in the same firm which opens room for modelling referral hiring. In addition, this firm structure gives rise to two separate submarkets with search frictions: in the first submarket young workers employed in low-level jobs apply for positions in middle management. In these positions, workers accumulate experience x and as they have reached a sufficient level ($\overline{x}$) they qualify for a senior managerial job. They are promoted by their current employer if there is a vacancy on the senior level in their firm, otherwise they start applying for senior positions in the second submarket. Having a sufficient level of experience $\overline{x}$ is a necessary requirement for an application to the senior position, which leads to the complete segregation of the two submarkets. Hiring to middle-level positions is based on the pool of formal applicants and candidates recommended by the senior manager of this firm. Referral hiring is based on social networks which form endogenously and can be homophilous.

In Section 4.1 we describe how social networks are formed and continue with the referral process via networks in Section 4.2. Further, in Section 4.3 we describe the structure of firms and workers' mobility patterns within and between firms.

4.1 Social networks

At rate ϕ every worker can be randomly matched with another worker. Formation of social links is subject to (gender) homophily, that is workers are more likely to create social links with others of the same type (see McPherson et al., 2001). Let τ_0 denote the probability of creating a social link with a worker of a different type and $\tau \geq \tau_0$ be the probability of creating a link with another worker of the same type (conditional on matching). Note, that the special case when $\tau = \tau_0$ corresponds to the situation without homophily. We consider directed links. This means that, if two workers A and B are randomly matched, it is possible that B becomes a social contact of A but not vice versa. The reason behind this assumption is that we only keep track of professional contacts rather than friendship ties, and assume that the person is ready to give a job referral/recommendation to each of his/her contacts at any point in time. Thus, our setting captures situations where person A is ready to refer person B for the job but not necessarily the other way round. Intuitively, this is similar to the directed links in citation networks. Every social link can be destroyed at rate δ.

Let ξ_k^{ij} denote a fraction of type i workers with exactly k social contacts of type j, $i, j \in \{M, F\}$. This is a fraction out of all type i workers. Consider some type M worker without contacts of his type. With our notation this worker belongs to the group ξ_0^{MM}. At rate ϕ this worker is matched with some other worker. With probability $1 - h$ this worker is of the same type M, and the social link is created with probability τ. Next, consider a worker of type M with only one contact of his type belonging to the group ξ_1^{MM}. This person may lose his contact at rate δ. In the steady state (when variables ξ_k^{MM} are constant), the propensity for the worker to make transition between the two states $k-1$ and k will be equalized, this means:

$$\begin{aligned}
\xi_0^{MM}\phi(1-h)\tau &= \delta\xi_1^{MM} \qquad \Rightarrow \quad \xi_1^{MM} = \frac{\xi_0^{MM}\phi(1-h)\tau}{\delta} \\
\xi_1^{MM}\phi(1-h)\tau &= 2\delta\xi_2^{MM} \qquad \Rightarrow \quad \xi_2^{MM} = \xi_0^{MM}\Big(\frac{\phi(1-h)\tau}{\delta}\Big)^2\frac{1}{2} \\
\xi_{k-1}^{MM}\phi(1-h)\tau &= k\delta\xi_k^{MM} \qquad \Rightarrow \quad \xi_k^{MM} = \xi_0^{MM}\Big(\frac{\phi(1-h)\tau}{\delta}\Big)^k\frac{1}{k!}
\end{aligned}$$

Let $\psi_{MM} \equiv \phi(1-h)\tau/\delta$. Since all fractions ξ_k^{MM} should add up to 1 for $k = 0...\infty$ we find that $\xi_0^{MM} = e^{-\psi_{MM}}$, which allows us to characterize the full distribution of male contacts ξ_k^{MM} depending on the network formation parameters ϕ, τ, δ and the fraction of male workers in the occupation $1-h$. Alternatively, the type M worker can be matched with another worker of type F, which happens at rate ϕh. So the social link is formed at rate τ_0. In this case let $\psi_{MF} \equiv \phi h\tau_0/\delta$. Repeating the same steps as before we find that the fraction of type M workers with 0 type F contacts in their professional network is given by $\xi_0^{MF} = e^{-\psi_{MF}}$. This allows us to formulate the first result in the following way:

Result 1: *The number of type M contacts in the network of a male worker has a Poisson distribution with parameter ψ_{MM}, whereas the number of type F contacts of a male worker has a Poisson distribution with parameter ψ_{MF}. So ψ_{MM} and ψ_{MF} are the average numbers of type M and type F contacts in the professional network of a male worker:*

$$\xi_k^{MM} = e^{-\psi_{MM}}\Big(\psi_{MM}\Big)^k\frac{1}{k!} \qquad \text{and} \qquad \xi_k^{MF} = e^{-\psi_{MF}}\Big(\psi_{MF}\Big)^k\frac{1}{k!}$$

Further, let n_M denote the average network size for type M workers including both types of professional contacts and γ_M be the fraction of type M contacts in the network, so we get:

$$n_M = \psi_{MM} + \psi_{MF} = \frac{\phi}{\delta}[(1-h)\tau + h\tau_0] \qquad \gamma_M = \frac{(1-h)\tau}{(1-h)\tau + h\tau_0}$$

This equation shows the following. If $\tau_0 < \tau$, then a larger fraction of female workers in the occupation h is reducing the average size of social networks for men since $\partial n_M/\partial h = \phi(\tau_0 - \tau)/\delta < 0$. The reason is that cross-gender links are less likely to be formed with gender homophily.

Next, we repeat the same approach for female workers and denote $\psi_{FF} \equiv \phi h\tau/\delta$ and $\psi_{FM} \equiv \phi(1-h)\tau_0/\delta$. So ψ_{FF} and ψ_{FM} are the average numbers of type F and type M contacts in the professional network of a female worker. Hence the total number of professional contacts in the network of a female worker can be found as:

$$n_F = \psi_{FF} + \psi_{FM} = \frac{\phi}{\delta}[h\tau + (1-h)\tau_0] \qquad \gamma_F = \frac{h\tau}{h\tau + (1-h)\tau_0}$$

where n_F is the total network size of female workers and γ_F is a fraction of type F contacts in their network. One can see that $\partial n_F/\partial h = \phi(\tau - \tau_0)/\delta > 0$ if $\tau_0 < \tau$, so the average size of female social networks is increasing with a larger fraction of females h in the occupation. Moreover, the case of full homophily ($\tau_0 = 0$) leads to the complete segregation of social networks between the two genders, that is $\gamma_M = \gamma_F = 1$. In the opposite case without homophily ($\tau_0 = \tau$), the fraction of contacts of the same type is equal to the fraction of this type in the total population, that is $\gamma_M = 1-h$ and $\gamma_F = h$. Comparing the average sizes of social networks for male and female workers one can show the following:

$$n_M - n_F = \frac{\phi}{\delta}[(1-h)\tau + h\tau_0 - h\tau - (1-h)\tau_0] = \frac{\phi}{\delta}(1-2h)(\tau - \tau_0)$$

Thus male workers have larger networks on average ($n_M > n_F$) for $h < 0.5$ and $\tau_0 < \tau$. This allows us to formulate the second result:

Result 2: *Professional networks of women are smaller compared to men if women are the minority in the occupation ($h < 0.5$) and social connections exhibit some degree of homophily ($\tau_0 < \tau$). Under these conditions female contacts are underrepresented in the networks of male workers ($1-\gamma_M < h$), whereas they are overrepresented in the networks of female workers ($\gamma_F > h$).*

The second part of this result follows directly from the following observation:

$$\begin{aligned}
1-\gamma_M &= \frac{h\tau_0}{(1-h)\tau + h\tau_0} = h\frac{1}{(1-h)\frac{\tau}{\tau_0} + h} < h \quad \text{for} \quad \tau_0 < \tau \\
\gamma_F &= \frac{h\tau}{h\tau + (1-h)\tau_0} = h\frac{1}{(1-h)\frac{\tau_0}{\tau} + h} > h \quad \text{for} \quad \tau_0 < \tau
\end{aligned}$$

Recall that h is the fraction of women participating in the occupation. This finding forms the ground for gender-biased referrals in our model which is presented in the following section.

4.2 Labor market

There are three types of jobs in the market: low-level, mid-level and senior-level jobs. For simplicity we assume that there are no frictions in the market for low-level jobs and there are infinitely many of these jobs available to both worker groups. For the purpose of tractability, we assume that there is no unemployment in the model, however, low-level jobs are intuitively similar to the state of unemployment, hence all workers in low-level jobs are searching and applying for better positions[3]. Let e_0^i denote the measure/number of type i workers employed in low-level jobs, $i = M, F$. Workers do not gain any professional experience by performing low-level jobs.

Mid-level and senior jobs are provided by firms operating in a frictional market. The total number of these firms is fixed and denoted by d. All these firms are identical and every firm is a dyad consisting of two positions: one middle level position (supervisor) and one senior level position (manager). Thus job ladders consist of three hierarchical levels in total. Here we build on the model by Dawid et al. (2019). In Section 6 we extend our benchmark model to more than three levels and analyze the implications of deeper firm hierarchies. At rate ρ every worker may exit the market for exogenous reasons and is substituted with a new agent of the same gender who enters the pool of young inexperienced workers $e_0 = e_0^M + e_0^F$. So the total population is constant over time. From the perspective of firms, ρ is the job destruction shock. Let e_1^i denote the number of type i workers employed in mid-level jobs and e_2^i – the number of type i workers employed in senior jobs, $i = M, F$, so:

$$e_0^F + e_1^F + e_2^F = h \qquad \text{and} \qquad e_0^M + e_1^M + e_2^M = 1-h$$

Once accepted in the mid-level position, workers start accumulating professional experience $x \geq 0$ with $\dot{x} = 1$. All workers in mid-level jobs have to accumulate an exogenously given experience level $\bar{x}$ to become eligible for senior positions. Here, we follow the human capital approach to promotions developed by Gibbons and Waldman (1999). Experience accumulation

[3]Our assumption is partially justified by the empirical evidence showing that the risk of unemployment is very low for high skill workers in professional occupations

is costly for workers so it stops at $\bar{x}$ since there are no incentives for workers to accumulate more human capital than required by firms. This is also a proxy for the decreasing returns to learning-by-doing. If the senior position is open, firms commit to promote their employees with experience $\bar{x}$ to senior positions. If there is no worker eligible for promotion the firm is posting an open high-level vacancy on the external market. If there is no open senior position in the firm, the worker with experience $\bar{x}$ starts applying to senior positions in other firms. This is the process of on-the-job search. Experience x is observable and can be transferred to other firms if the worker changes the job voluntarily. Workers with experience $x < \bar{x}$ are not eligible for senior positions in any company.

Note, that the experience requirement $\bar{x}$ and the probability of internal promotion are identical for male and female workers. We make this assumption as the evidence on promotion chances for men and women within firms is inconclusive. This assumption also allows us to focus on the role of social networks and investigate this channel in the isolation from other factors generating the glass-ceiling effect. We extend our benchmark framework and consider unequal promotion chances (direct discrimination) and their interaction with the network channel in Section 6.

There are two separate matching markets in our model, one where firms post mid-level positions and anticipate inexperienced workers with $x = 0$ and another one where firms post senior positions and anticipate workers with experience $x = \bar{x}$. Variable d_{00} denotes the stock of empty firms in the market, whereas d_{01} is the stock of firms with a senior manager but no supervisor. Since all these firms have an open mid-level position the total stock of open mid-level positions available for matching is equal to $d_{00} + d_{01}$. These positions are randomly matched with $z_1 e_0$ searching workers employed in low-level jobs, where z_1 denotes exogenous search effort of workers applying to mid-level jobs. More precisely, z_1 is the fraction of searching workers who prepare and send an application at every point in time. To determine the number of matches in the submarket for mid-level positions we use an urn-ball matching mechanism (see, e.g., Petrongolo and Pissarides, 2001), which yields the formal matching rate q_1 for firms:

$$q_1 = 1 - \left(1 - \frac{1}{d_{00} + d_{01}}\right)^{z_1 e_0} \tag{1}$$

The reason why we use an urn-ball matching mechanism rather than a more traditional Cobb-Douglas matching function, is that the urn-ball matching can be directly implemented in the simulation whereas the Cobb-Douglas approach is a "black box" from the perspective of practial implementation. In Section 6, we perform several extensions of our model by using a simulation framework and the urn-ball matching mechanism leads to the close replication of the analytical model in the simulation. So we can be sure that the results in Section 6 are fully driven by the respective extension rather than discrepancies in the approximation of the matching technology. None of our qualitative results is driven by this assumption.

Next, consider a candidate who was matched and chosen for the mid-level position, with probability $\alpha_1 = e_0^M / e_0$ the chosen candidate is of type M and with a counterprobability $(1 - \alpha_1) = e_0^F / e_0$ the person is of type F. Note, that:

$$\alpha_1 = \frac{(1-h)\mu_M}{(1-h)\mu_M + h\mu_F}$$

where $\mu^i = e_0^i/(1-h)$ is the equilibrium fraction of type i workers employed in low-level jobs, $i = M, F$. This equation shows the following. If the distribution of workers across the hierarchical levels is identical for male and female workers, then $\mu_M = \mu_F$ and $\alpha_1 = 1 - h$. So the probability that the hired job candidate is of type M is equal to the population average $1 - h$. However, if female workers are overrepresented at the bottom ($\mu_F > \mu_M$) a randomly matched job candidate is more likely to be a female and $\alpha_1 < 1 - h$.

In addition to the formal application process some mid-level positions can be filled by referrals. Consider firms of type d_{01} consisting of d_{0F} and d_{0M}, depending on the type of the senior manager. With probability s in both types of firms the senior manager is asked to recommend a contact for the open mid-level position. On average, type M managers have $n_M \gamma_M$ type M contacts. So with probability $(1-\mu^M)^{n_M \gamma_M}$ the senior manager does not know any type M candidate for the mid-level position. In addition, type M managers have $n_M(1-\gamma_M)$ type F social contacts, but with probability $(1-\mu^F)^{n_M(1-\gamma_M)}$ all of them are already employed in professional jobs. With this information, we obtain the following probability that there is at least one social contact recommended by the male senior manager:

$$\tilde{q}_1^M = s\Big(1 - (1-\mu_M)^{n_M \gamma_M}(1-\mu_F)^{n_M(1-\gamma_M)}\Big)$$

If the manager has several social contacts employed in low-level jobs, the manager randomly chooses one of them *independent of the gender* and refers this contact for the open position in his firm. The referred candidate is of type M with probability $\tilde{\alpha}_1^M$ and of type F with probability $1-\alpha_1^M$, where $\tilde{\alpha}_1^M$ depends on the composition of the network:

$$\tilde{\alpha}_1^M = \frac{\gamma_M \mu_M}{\gamma_M \mu_M + (1-\gamma_M)\mu_F} = \frac{(1-h)\mu_M}{(1-h)\mu_M + h\mu_F \cdot \frac{(1-\gamma_M)(1-h)}{\gamma_M h}}$$

This equation shows that $\tilde{\alpha}_1^M > \alpha_1$ in homophilous networks because $\frac{(1-\gamma_M)(1-h)}{\gamma_M h} < 1$ for $\tau_0 < \tau$. Intutively, this means that a candidate referred by the male manager is more likely to be a male worker compared to the formal channel even if the manager does not have any taste for discrimination and randomizes between all of his social contacts interested in the mid-level job. Following the same logic we define $\tilde{q}_1^F$ – the probability that there is at least one social contact recommended by the female senior manager:

$$\tilde{q}_1^F = s\Big(1 - (1-\mu_M)^{n_F(1-\gamma_F)}(1-\mu_F)^{n_F \gamma_F}\Big)$$

Further, $\tilde{\alpha}_1^F$ – probability for a type F manager of recommending a type M candidate from the network, so that:

$$\tilde{\alpha}_1^F = \frac{(1-\gamma_F)\mu_M}{(1-\gamma_F)\mu_M + \gamma_F \mu_F} = \frac{(1-h)\mu_M}{(1-h)\mu_M + h\mu_F \cdot \frac{\gamma_F(1-h)}{(1-\gamma_F)h}}$$

This shows that a candidate recommended by the female manager is less likely to be a male worker compared to the formal hiring channel, that is $\tilde{\alpha}_1^F < \alpha_1$ for $\tau_0 < \tau$. Gender homophily in the process of network formation leads to gender-biased referrals even if workers are fair and neglect gender when recommending their social contacts for jobs. We summarize these results in the following way:

Result 3 *For $\tau_0 < \tau$ a job candidate referred by the male senior manager to the mid-level job is more likely to be a male worker compared to the formal channel ($\tilde{\alpha}_1^M > \alpha_1$) even if the manager does not have any taste for discrimination and randomizes between all of his social contacts interested in the mid-level job. A candidate recommended by the female manager is less likely to be a male worker compared to the formal hiring channel ($\tilde{\alpha}_1^F < \alpha_1$).*

Next, we can see that firms with an open mid-level position and a type M senior manager will fill their position with a type M candidate at rate $q_1\alpha_1$ via the formal application process and via the network at rate $\tilde{q}_1^M \tilde{\alpha}_1^M$. We do not assume that the recommended candidate is preferred to the external candidates. Rather all applicants for a given mid-level position are pooled together and a random draw is made. So the recommended applicant has the same chances as external applicants given that all of them have the same qualification. Assuming preference for the recommended candidate would amplify the network effect in the quantitative estimation. Let the total job-filling rate with a type M candidate be denoted by $\bar{q}_1^{MM}$. In addition, the open position can be filled with a type F candidate at the total rate $\bar{q}_1^{FM}$:

$$\bar{q}_1^{MM} = q_1\alpha_1 + \tilde{q}_1^{M}\tilde{\alpha}_1^{M} \quad \text{and} \quad \bar{q}_1^{FM} = q_1(1-\alpha_1) + \tilde{q}_1^{M}(1-\tilde{\alpha}_1^{M})$$

Finally, consider firms with an open mid-level position and a type F senior manager. These firms will fill their position with a type M candidate at rate $q_1\alpha_1$ via the formal application process and via the network at rate $\tilde{q}_1^{F}\tilde{\alpha}_1^{F}$. Let the total job-filling rate with a type M candidate be denoted by $\bar{q}_1^{MF}$. In addition, the open position can be filled with a type F candidate at the total rate $\bar{q}_1^{FF}$:

$$\bar{q}_1^{MF} = q_1\alpha_1 + \tilde{q}_1^{F}\tilde{\alpha}_1^{F} \quad \text{and} \quad \bar{q}_1^{FF} = q_1(1-\alpha_1) + \tilde{q}_1^{F}(1-\tilde{\alpha}_1^{F})$$

Notice the following, when referral hiring is not used, that is $s = 0$, the rate at which a male candidate is hired is equal to $q_1\alpha_1$, and the rate at which a female candidate is hired is equal to $q_1(1-\alpha_1)$. Both are independent of the gender of the senior manager.

Further, let $d_{10} = d_{F0} + d_{M0}$ denote firms with a middle-level worker but no senior manager. This means that the total number of open managerial positions is given by $d_{00} + d_{10}$. Finally, let $d_{11}^N = d_{MF}^N + d_{MM}^N + d_{FF}^N + d_{FM}^N$ denote the stock of full firms with both employees, where the worker in the mid-level position is not yet eligible for promotion ($x < \bar{x}$). In a similar way, $d_{11}^S = d_{MF}^S + d_{MM}^S + d_{FF}^S + d_{FM}^S$ – is the stock of full firms, where the mid-level worker is already eligible for senior positions ($x = \bar{x}$) and searching on-the-job. This means that the stock of applicants in the managerial market is given by $z_2 d_{11}^S$, where z_2 is the exogenous search intensity of experienced workers. So the urn-ball matching rate in the managerial market q_2 is given by:

$$q_2 = 1 - \Big(1 - \frac{1}{d_{00} + d_{10}}\Big)^{z_2 d_{11}^S}$$

With probability α_2 the firm will be matched with a type M experienced mid-level worker and hire him for the manager position and with a counter-probability $1-\alpha_2$ the firm will be matched with a type F experienced mid-level worker and hire her as a manager:

$$\alpha_2 = \frac{d_{MF}^S + d_{MM}^S}{d_{11}^S}$$

So the total job-filling rate with a type M candidate is given by $q_2\alpha_2$ and $q_2(1-\alpha_2)$ with a type F candidate. Note, that we assume that workers do not recommend their contacts for senior positions, so there are no referrals on this level. Also in our setting it is not rational for mid-level workers to refer their contacts for senior positions, since they are hoping to be promoted themselves in the future. Moreover, there is no favoritism and gender-based discrimination in the process of formal hiring since at this stage we seek to identify a separate effect of homophilous networks on labor market outcomes in isolation from other factors. In the later extensions we consider a combination of different factors including discrimination and their interaction effects.

4.3 Firm Dynamics

In this section, we analyze the transformation of firms as workers enter and exit jobs as well as the steady-state distributions of workers and firms. Consider changes in the stock of empty firms d_{00}. Since every empty firm posts both the mid-level and the senior position in the respective submarkets it exits the state d_{00} whenever it finds the first employee. So the outflow of firms from d_{00} takes place at rate $q_1 + q_2$. The inflow into this state consists of all firms with only one employee experiencing the job destruction/exit shock ρ. These are the firms d_{F0} d_{M0}, d_{0F} and d_{0M}. We restrict our analysis to the steady states and consider a stationary distribution of workers and firms across states. This means that $\dot{d}_{00} = 0$ in the steady state:

$$0 = \dot{d}_{00} = \rho(d_{F0} + d_{M0} + d_{0F} + d_{0M}) - (q_1 + q_2)d_{00} \tag{2}$$

Further, consider changes in the stocks of firms $d_{F0}(x)$, $d^N_{FM}(x)$ and $d^N_{FF}(x)$. Note that workers with experience $0 \leq x \leq \bar{x}$ are not yet searching on-the-job since their experience is not sufficient for managerial positions and there are no gains from changing to another mid-level job. This means that the inflow of firms into state $d_{F0}(x)$ is equal to $\rho(d^N_{FM}(x) + d^N_{FF}(x))$. These are the firms where the manager exits at rate ρ and they are left with only one mid-level worker of type F. If the manager exits firms post the open position in the second submarket for experienced workers and find a manager at rate q_2. This means that the outflow of workers from the state $d_{F0}(x)$ is equal to $(q_2 + \rho)d_{F0}(x)$ where the term $\rho d_{F0}(x)$ corresponds to the job destruction shock ρ of the mid-level position. So we get the following differential equation[4]:

$$\frac{\partial d_{F0}(x)}{\partial x} = \rho(d^N_{FM}(x) + d^N_{FF}(x)) - (\rho + q_2)d_{F0}(x)$$

Next, we take into account changes in the stock of firms $d^N_{FM}(x)$ and $d^N_{FF}(x)$. Each of these firms has exactly two filled positions, so the job destruction shock arrives at the increased rate 2ρ. The inflow of firms into category $d^N_{FF}(x)$ is equal to $q_2(1 - \alpha_2)d_{F0}(x)$. These are the firms $d_{F0}(x)$ filling their senior position with a type F candidate. In a similar way, the inflow of firms into category $d^N_{FM}(x)$ is equal to $q_2\alpha_2 d_{F0}(x)$. These are the firms $d_{F0}(x)$ filling their senior position with a type M candidate. So we get the following two differential equations:

$$\begin{aligned} \partial d^N_{FF}(x)/\partial x &= q_2(1 - \alpha_2)d_{F0}(x) - 2\rho d^N_{FF}(x) \\ \partial d^N_{FM}(x)/\partial x &= q_2\alpha_2 d_{F0}(x) - 2\rho d^N_{FM}(x) \end{aligned}$$

The coefficient matrix of the three first order linear differential equations for $\{d_{F0}(x), d^N_{FF}(x), d^N_{FM}(x)\}$ has three eigenvalues equal to: -2ρ, $-\rho$ and $-(2\rho + q_2)$. So the general solution is given by:

$$\begin{aligned} d_{F0}(x) &= k^F_2 \frac{\rho^2}{q_2} e^{-\rho x} - k^F_3 q_2 e^{-(2\rho+q_2)x} \\ d^N_{FF}(x) &= k^F_1 e^{-2\rho x} + k^F_2 \rho(1 - \alpha_2)e^{-\rho x} + k^F_3 q_2(1 - \alpha_2)e^{-(2\rho+q_2)x} \\ d^N_{FM}(x) &= -k^F_1 e^{-2\rho x} + k^F_2 \rho\alpha_2 e^{-\rho x} + k^F_3 q_2\alpha_2 e^{-(2\rho+q_2)x} \end{aligned}$$

In order to find the constant terms k^F_1, k^F_2 and k^F_3, we use the following initial conditions: $q_1(1-\alpha_1)d_{00} = d_{F0}(0)$, $\bar{q}^{FF}_1 d_{0F} = d^N_{FF}(0)$ and $\bar{q}^{FM}_1 d_{0M} = d^N_{FM}(0)$. The first condition implies that the stock $d_{F0}(0)$ consists of firms d_{00} finding a mid-level worker of type F, that is $q_1(1-\alpha_1)d_{00}$. The second condition implies that the stock of firms $d^N_{FF}(0)$ consists of firms d_{0F} who find a mid-level worker of type F at rate $\bar{q}^{FF}_1$. The third condition implies that the stock of firms $d^N_{FM}(0)$ consists of firms d_{0M} who find a mid-level worker of type F at rate $\bar{q}^{FM}_1$. Exact expressions for k^F_1, k^F_2 and k^F_3 are provided in the Appendix.

Note, that in all three states $d_{F0}(x)$, $d^N_{FF}(x)$, $d^N_{FM}(x)$ female workers employed in mid-level positions remain inactive and accumulate experience till it reaches the minimum level $\bar{x}$ necessary for the senior position. If the senior position is free, the mid-level worker is immediately promoted, so the stock of firms $d_{F0}(\bar{x})$ is one of the entries into the stock d_{0F}. However, if the senior position is not vacant, then mid-level workers start searching and applying for senior

[4]In general the stock variable $d_{F0}(x,t)$ may depend on time t, so the total derivative is given by:

$$\frac{\partial d_{F0}(x,t)}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial d_{F0}(x,t)}{\partial t} = \rho(d^N_{FM}(x) + d^N_{FF}(x)) - (\rho + q_2)d_{F0}(x)$$

Since the distribution of firms $d_{F0}(x,t)$ is stationary in the steady state we set the time derivative $\dot{d}_{F0} = \frac{\partial d_{F0}(x,t)}{\partial t}$ equal to zero. Moreover, experience x is accumulating one to one with the time because $\dot{x} = \partial x/\partial t = 1$).

positions in other firms. This means that stocks of firms $d^N_{FM}(\bar{x})$ and $d^N_{FF}(\bar{x})$ are the entries into d^S_{FM} and d^S_{FF} respectively. This meachanism allows us to obtain the total stocks of firms d_{F0}, d^N_{FF} and d^N_{FM} be integrating from $x = 0$ till $x = \bar{x}$. This yields the following:

$$\begin{aligned} d_{F0} &= \frac{k^F_2\rho}{q_2}(1-e^{-\rho\bar{x}}) - \frac{k^F_3 q_2}{2\rho+q_2}(1-e^{-(2\rho+q_2)\bar{x}}) && (3) \\ d^N_{FF} &= \frac{k^F_1}{2\rho}(1-e^{-2\rho\bar{x}}) + k^F_2(1-\alpha_2)(1-e^{-\rho\bar{x}}) + \frac{k^F_3 q_2(1-\alpha_2)}{2\rho+q_2}(1-e^{-(2\rho+q_2)\bar{x}}) && (4) \\ d^N_{FM} &= -\frac{k^F_1}{2\rho}(1-e^{-2\rho\bar{x}}) + k^F_2\alpha_2(1-e^{-\rho\bar{x}}) + \frac{k^F_3 q_2\alpha_2}{2\rho+q_2}(1-e^{-(2\rho+q_2)\bar{x}}) && (5) \end{aligned}$$

Next, we repeat our analysis with the stocks of firms $d_{M0}(x)$, $d^N_{MM}(x)$ and $d^N_{MF}(x)$, where there is a male worker employed in the mid-level position. This yields the following system of differential equations:

$$\begin{aligned} \partial d_{M0}(x)/\partial x &= \rho(d^N_{MF}(x) + d^N_{MM}(x)) - (\rho+q_2)d_{M0}(x) \\ \partial d^N_{MM}(x)/\partial x &= q_2\alpha_2 d_{M0}(x) - 2\rho d^N_{MM}(x) \\ \partial d^N_{MF}(x)/\partial x &= q_2(1-\alpha_2)d_{M0}(x) - 2\rho d^N_{MF}(x) \end{aligned}$$

Firms of the type $d_{M0}(x)$ are searching for a senior manager and find one at rate q_2. With probability α_2 the chosen candidate is a male worker, so the firm makes transition into the state $d^N_{MM}(x)$. Here, the mid-level employee is also a male worker with experience $x < \bar{x}$. With the counter-probability $1-\alpha_2$ the new senior manager is a female worker, so the firm makes a transition into the state $d^N_{MF}(x)$. The three eigenvalues of this system of differential equations are again -2ρ, $-\rho$ and $-(2\rho+q_2)$, so the general solution is:

$$\begin{aligned} d_{M0}(x) &= k^M_2\frac{\rho^2}{q_2}e^{-\rho x} - k^M_3 q_2 e^{-(2\rho+q_2)x} \\ d^N_{MM}(x) &= k^M_1 e^{-2\rho x} + k^M_2\rho\alpha_2 e^{-\rho x} + k^M_3 q_2\alpha_2 e^{-(2\rho+q_2)x} \\ d^N_{MF}(x) &= -k^M_1 e^{-2\rho x} + k^M_2\rho(1-\alpha_2)e^{-\rho x} + k^M_3 q_2(1-\alpha_2)e^{-(2\rho+q_2)x} \end{aligned}$$

In order to find the constant terms k^M_1, k^M_2 and k^M_3 we use the following initial conditions: $q_1\alpha_1 d_{00} = d_{M0}(0)$, $\bar{q}^{MM}_1 d_{0M} = d^N_{MM}(0)$ and $\bar{q}^{MF}_1 d_{0F} = d^N_{MF}(0)$. The first condition implies that the stock $d_{F0}(0)$ consists of firms d_{00} finding a mid-level worker of type M, that is $q_1\alpha_1 d_{00}$. The second condition implies that the stock of firms $d^N_{MM}(0)$ consists of firms d_{0M} who find a mid-level worker of type M at rate $\bar{q}^{MM}_1$. The third condition implies that the stock of firms $d^N_{MF}(0)$ consists of firms d_{0F} who find a mid-level worker of type M at rate $\bar{q}^{MF}_1$. Exact expressions for k^M_1, k^M_2 and k^M_3 are again provided in the Appendix. Finally, integrating variables $d_{M0}(x)$, $d^N_{MM}(x)$ and $d^N_{MF}(x)$ from $x = 0$ till $x = \bar{x}$ we get the following:

$$\begin{aligned} d_{M0} &= \frac{k^M_2\rho}{q_2}(1-e^{-\rho\bar{x}}) - \frac{k^M_3 q_2}{2\rho+q_2}(1-e^{-(2\rho+q_2)\bar{x}}) && (6) \\ d^N_{MM} &= \frac{k^M_1}{2\rho}(1-e^{-2\rho\bar{x}}) + k^M_2\alpha_2(1-e^{-\rho\bar{x}}) + \frac{k^M_3 q_2\alpha_2}{2\rho+q_2}(1-e^{-(2\rho+q_2)\bar{x}}) && (7) \\ d^N_{MF} &= -\frac{k^M_1}{2\rho}(1-e^{-2\rho\bar{x}}) + k^M_2(1-\alpha_2)(1-e^{-\rho\bar{x}}) + \frac{k^M_3 q_2(1-\alpha_2)}{2\rho+q_2}(1-e^{-(2\rho+q_2)\bar{x}}) && (8) \end{aligned}$$

To close the model, consider the stocks of firms d^S_{FF}, d^S_{FM}, d^S_{MM} and d^S_{MF}. In all these firms the mid-level worker has experience more than $\bar{x}$ and is already searching for a senior position. We already know that $d^N_{FM}(\bar{x})$ and $d^N_{FF}(\bar{x})$ are the only entries into d^S_{FM} and d^S_{FF} respectively. In a similar way, variables $d^N_{MF}(\bar{x})$ and $d^N_{MM}(\bar{x})$ are the only entries into d^S_{MF} and d^S_{MM}. There are three possible events that can alter the state of these firms. Either one of the two employees is dismissed from the job at rate ρ, or the mid-level worker finds another employment as a senior manager and quits the firm at rate λ_2. Thus we get:

$$\dot{d}^S_{FF} = d^N_{FF}(\bar{x}) - (2\rho + \lambda_2)d^S_{FF} \qquad \dot{d}^S_{MM} = d^N_{MM}(\bar{x}) - (2\rho + \lambda_2)d^S_{MM} \tag{9}$$

$$\dot{d}^S_{FM} = d^N_{FM}(\bar{x}) - (2\rho + \lambda_2)d^S_{FM} \qquad \dot{d}^S_{MF} = d^N_{MF}(\bar{x}) - (2\rho + \lambda_2)d^S_{MF} \tag{10}$$

Finally, consider the stock of firms d_{0F}. We already know that $d_{F0}(\bar{x})$ is one of the entries into d_{0F}, because mid-level workers are promoted to senior positions upon reaching experience $\bar{x}$. Also the firms d^S_{FM} and d^S_{FF} promote their female mid-level employees to senior positions in the event when the senior manager is dimissed, which happens at rate ρ. So the inflow of firms into d_{0F}, which is *due to immediate or delayed promotions* is given by $d_{F0}(\bar{x}) + \rho(d^S_{FM} + d^S_{FF})$.

However, also empty firms d_{00} are searching for senior managers and find one at rate q_2. With probability α_2 the new manager is a male worker, so the firm d_{00} becomes d_{0M}, but with probability $1 - \alpha_2$ the new manager is a female worker, so the firm enters the state d_{0F}. Hence the entry of firms into state d_{0F}, which is *due to outside hiring*, is equal to $q_2(1 - \alpha_2)d_{00}$.

In addition, we know that any of the firms d^N_{FF}, d^N_{MF}, d^S_{FF} and d^S_{MF} may lose their mid-level employees at rate ρ *due to the exogenous exit* and therefore enter the state d_{0F} as the only remaining worker in these firms is a senior female manager. So the next entry is $\rho(d^N_{FF} + d^N_{MF} + d^S_{FF} + d^S_{MF})$. Moreover, it can also happen that mid-level employees in firms d^S_{FF} and d^S_{MF} separate from their employers *due to quitting* and taking employment in other firms, which happens at rate λ_2. This yields the last entry into the state d_{0F}, namely, $\lambda_2(d^S_{FF} + d^S_{MF})$. Summarizing, we find that the entry of firms into the state d_{0F} is given by $d_{F0}(\bar{x}) + \rho(d^S_{FM} + d^S_{FF}) + q_2(1 - \alpha_2)d_{00} + \rho(d^N_{FF} + d^N_{MF} + d^S_{FF} + d^S_{MF}) + \lambda_2(d^S_{FF} + d^S_{MF})$.

Next, we investigate the exits of firms from the state d_{0F}. On the one hand, senior managers may exit the market at rate ρ rendering the firm empty (d_{00}). On the other hand, firms may fill their open mid-level position with a female worker, which happens at rate $\bar{q}^{FF}_1$, or with a male worker, which happens at rate $\bar{q}^{MF}_1$. Note, that these rates include the possibility of formal and referral hiring to mid-level positions. So the exit of firms from the state d_{0F} is given by: $(\rho + \bar{q}^{FF}_1 + \bar{q}^{MF}_1)d_{0F}$. This yields the following differential equations for d_{0F} and d_{0M}:

$$\begin{aligned}
\dot{d}_{0F} &= \underbrace{d_{F0}(\bar{x}) + \rho(d^S_{FM} + d^S_{FF})}_{\text{promotions of mid-level workers}} + \underbrace{q_2(1 - \alpha_2)d_{00}}_{\text{outside hiring into senior position}} \\
&+ \underbrace{\rho(d^N_{FF} + d^N_{MF} + d^S_{FF} + d^S_{MF})}_{\text{exit of mid-level workers}} + \underbrace{\lambda_2(d^S_{FF} + d^S_{MF})}_{\text{mid-level workers quitting}} \qquad (11) \\
&- \underbrace{\rho d_{0F}}_{\text{exit of senior managers}} - \underbrace{(\bar{q}^{FF}_1 + \bar{q}^{MF}_1)d_{0F}}_{\text{formal and referral hiring of mid-level workers}} \\
\dot{d}_{0M} &= d_{M0}(\bar{x}) + \rho(d^S_{FM} + d^S_{MM}) + q_2\alpha_2 d_{00} + \rho(d^N_{MM} + d^N_{FM} + d^S_{MM} + d^S_{FM}) \qquad (12) \\
&+ \lambda_2(d^S_{MM} + d^S_{FM}) - \rho d_{0M} - (\bar{q}^{MM}_1 + \bar{q}^{FM}_1)d_{0M}
\end{aligned}$$

We restrict our attention to steady state equilibria, so we set $\dot{d}_{00} = \dot{d}^S_{MF} = \dot{d}^S_{FM} = \dot{d}^S_{MM} = \dot{d}^S_{FF} = \dot{d}_{0F} = \dot{d}_{0M} = 0$. Given that this system of equations is over-identified, we substitute one of the equations by fixing the total number of firms d. This is an exogenous parameter, which yields:

$$d_{00} + d_{F0} + d_{M0} + d_{0F} + d_{0M} + d^N_{MM} + d^N_{MF} + d^N_{FM} + d^N_{FF} + d^S_{MM} + d^S_{MF} + d^S_{FM} + d^S_{FF} = d \tag{13}$$

Solving equations (3)-(13) in the steady state, we find the equilibrium distribution of firms $\{d_{00}, d_{F0}, d_{M0}, d_{0F}, d_{0M}, d^N_{FM}, d^N_{FF}, d^N_{MF}, d^N_{MM}, d^S_{FM}, d^S_{FF}, d^S_{MF}, d^S_{MM}\}$. Based on the above distribution of firms, we can immediately calculate the distribution of male and female workers in different job levels. Recall that the absolute numbers of workers in different job levels are denoted by e_0^j, e_1^j and e_2^j, $j = M, F$, so we get:

$$\begin{aligned} e_1^F &= d_{FF}^N + d_{F0} + d_{FM}^N + d_{FM}^S + d_{FF}^S & e_1^M &= d_{MM}^N + d_{M0} + d_{MF}^N + d_{MF}^S + d_{MM}^S \quad (14) \\ e_2^F &= d_{FF}^N + d_{0F} + d_{MF}^N + d_{MF}^S + d_{FF}^S & e_2^M &= d_{MM}^N + d_{0M} + d_{FM}^N + d_{FM}^S + d_{MM}^S \quad (15) \end{aligned}$$

Finally, let variables p_0^j, p_1^j and p_2^j , $j = M, F$ denote the distribution of workers across different hierarchical levels, that is $p_2^M = e_2^M/(1-h)$, $p_1^M = e_1^M/(1-h)$, $p_0^M = 1 - p_1^M - p_2^M$. And same for female workers: $p_2^F = e_2^F/h$, $p_1^F = e_1^F/h$, $p_0^F = 1 - p_1^F - p_2^F$. So p_2^F here is the fraction of female workers in senior managerial positions, whereas p_2^M is the corresponding fraction for male workers. Given that our framework leads to a general equilibrium model based on the large set of steady-state equations described above, we characterize our next results with a help of a numerical example.

5 Numerical results

5.1 Benchmark setting

This section is dedicated to the numerical analysis of the model. We start by analyzing the benchmark model – Model I – with equal shares of male and female workers ($h = 0.5$) and without referal hiring ($s = 0$). In this model the vector of exogenous parameters includes $\{\rho, h, d, \bar{x}, z_1, z_2\}$. Note, that this is a model without social networks and without worker heterogeneity, so it is closely related to the study by Dawid et al. (2019). Following this study we set $\rho = 0.015$ and $z_1 = 0.014$. For this parameter setting, Dawid et al. (2019) find that the optimal promotion time $\bar{x}$ is in the range [40..60] depending on the productivity of workers. Thus, we set $\bar{x} = 50$, but we also perform comparative statics with other values of $\bar{x} \in [40..60]$. Given that one period of time is often interpreted as one quarter in search theory, $\bar{x} = 50$ intuitively means that firms require at least $50/4 = 12.5$ years of professional experience for promoting mid-level workers to senior positions.

For the remaining three parameters we use data from the German Socio-Economic Panel. Table 1 shows that the average fraction of workers in low levels jobs is equal to $p_0 = 0.54$, whereas the fraction of workers performing senior management jobs is $p_2 = 0.21$. In order to target this cross-sectional distribution of workers across hierarchical levels we choose $d = 0.3$ and $z_2 = 0.2$. This means that there are 300 two-level firms in the labor market with 1000 workers. Potentially this generates $300 \cdot 2 = 600$ management jobs. In the model without search frictions and without experience requirement $\bar{x}$ this would imply that 60% of workers perform management jobs. However, in a model with search frictions and experience requirement not all of these positions are regularly filled and the average number of employees in mid-level and senior management jobs is only $(p_2 + p_1) \cdot 1000 = 460$. Intuitively, this implies that 140 out of 600 management jobs remain vacant on average due to the experience requirement in senior jobs and search frictions. Finally, from the German labor market data, SOEP (2013), we know that the fraction of high skill women performing full-time jobs varies between 18-20% in the energy and construction sector, and 45-48% in retail trade and non-financial services. In the middle range there is manufacturing, financial services and transportation with 26-33% of women. Following this empirical evidence, we also consider more realistic female participation rates in the range $h \in [0.3..0.5]$ in later model specifications. Table 2 provides an overview of our parameter choices.

Our benchmark model – Model I – is summarized in columns (1)-(2) of Table 3. This is the model without referrals ($s = 0$) and with equal participation of males and females ($h = 0.5$). We can also see that $q_1 = 0.1374$ is larger than $q_2 = 0.0181$. This means that in equilibrium it is much easier for firms to hire mid-level managers than senior managers. This is also reflected in the fact that there are more firms without a senior manager $d_{10} = 0.0806$ compared to the number of firms without a mid-level worker $d_{01} = 0.0385$. The numbers of firms are reported per

Table 2: Parameter values.

Parameter	Value	Definition and Explanation
h	[0.3..0.5]	Fraction of female workers. Benchmark $h = 0.5$. GSOEP data, 2013
ρ	0.015	Worker exit rate. Following Dawid et al. (2019)
$\bar{x}$	[40..60]	Experience level necessary for promotion.
		Benchmark $\bar{x} = 50$. Following Dawid et al. (2019)
z_1	0.014	Search intensity of workers in low level jobs.
		Following Dawid et al. (2019)
z_2	0.2	Search intensity of workers in mid-level jobs. Targeting the
		fraction of workers in senior jobs $p_2 = 0.21$, GSOEP data 2013
d	0.3	Total number of firms per worker. Targeting the
		fraction of workers in low level jobs $p_0 = 0.54$, GSOEP data 2013

worker. So $d_{10} = 0.0806$ implies approximately 81 firms in a labor market with 1000 workers. We can also see that $\lambda_2 = 0.1993$ indicating intensive job-to-job transitions of workers.

In the next step, we implement a more realistic participation rate of females and consider the case $h = 0.3$. This means that 70% of workers in the market are males M. This is Model II and it is summarized in columns (3)-(4) of Table 3. Recall that α_1 is a probability that the applicant to the mid-level position is of type M. Given that M workers are the majority in Model II it is intuitive that $\alpha_1 = 0.7$. This merely reflects the fact that there are 70% type M workers in the economy. There are also more firms with male managers: $d_{M0} = 0.0564 > d_{F0} = 0.0242$ and $d_{0M} = 0.0270 > d_{0F} = 0.0115$. However, this does not affect the chances of male and female workers in terms of upward mobility. So the distributions p_2, p_1 and p_0 are identical between male and female workers indicating equal career opportunities despite female workers being the minority. We formulate this result in the following way:

Result 4: *In the absence of referral hiring ($s = 0$) lower participation of women in the occupation ($h < 0.5$) reduces the absolute number of women in all jobs in comparison to men, $e_0^F < e_0^M$, $e_1^F < e_1^M$ and $e_2^F < e_2^M$, but does not change the relative distribution of women across different hierarchical levels. There are equal opportunities of career progression for both gender groups: $p_0^F = p_0^M$, $p_1^F = p_1^M$ and $p_2^F = p_2^M$.*

5.2 Network composition

In this section, we analyze the structure of social networks in our model. Social networks are driven by the following vector of parameters $\{\phi, \delta, \tau, \tau_0\}$. Recall that the average number of directed links of a male worker to other males is given by $\psi_{MM} = \frac{\phi}{\delta}\tau(1-h)$, whereas the average number of links to female workers is given by $\psi_{MF} = \frac{\phi}{\delta}\tau_0 h$. So the total number of contacts in the network of a male worker is given by $n_M = \psi_{MM} + \psi_{MF} = \frac{\psi}{\delta}[\tau(1-h) + \tau_0 h]$.

Here, again, we start by considering the situation with equal participation of males and females $h = 1 - h = 0.5$ and no gender homophily, that is $\tau = \tau_0$. So the total average number of contacts for male and female workers is given by $n_M = n_F = \frac{\psi}{\delta}\tau$. Cingano and Rosolia (2012) report that the number of social connections between individuals in Italy is about 32. Glitz (2017) reports a similar number for Germany with approximately 43 social contacts. Thus, we set parameters $\psi = 0.8$, $\delta = 0.01$ and $\tau = 0.5$ in order to obtain the average number of social links in the benchmark model equal to $0.8 \cdot 0.5/0.01 = 40$. This means that a given person meets another one with probability $\phi = 0.8$ per unit time and includes this new person into the network with probability 0.5. On average every person creates 0.4 social links per unit time. Every link is destroyed with probability $\delta = 0.01$. With an average number of social links equal to 40, this means that individuals lose $40 \cdot 0.01 = 0.4$ links per unit time. Thus, the social network is balanced in the equilibrium.

Table 3: Equilibrium values of endogenous variables. Parameters: $\rho = 0.015$, $\bar{x} = 50$, $z_1 = 0.0135$, $z_2 = 0.2$, $d = 0.3$

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		Model I		Model II		Model III		Model IV	
		$h = 0.5,\ s = 0$		$h = 0.3,\ s = 0$		$h = 0.3,\ s = 0.4$		$h = 0.3,\ s = 0.4$	
						$\tau = \tau_0 = 0.5$		$\tau = 0.5,\ \tau_0 = 0.25$	
M	F	M	F	M	F	M	F	M	F
Fractions of males and females among new hires									
α_1	$1-\alpha_1$	0.5000	0.5000	0.7000	0.3000	0.7000	0.3000	0.6915	0.3085
α_2	$1-\alpha_2$	0.5000	0.5000	0.7000	0.3000	0.7000	0.3000	0.7078	0.2922
$\tilde{\alpha}_1^M$	$1-\tilde{\alpha}_1^M$					0.7000	0.3000	0.8175	0.1825
$\tilde{\alpha}_1^F$	$1-\tilde{\alpha}_1^F$					0.7000	0.3000	0.5284	0.4716
Job-finding and job-filling rates									
λ_1		0.0127		0.0127		0.0102		0.0102	
λ_2		0.1993		0.1993		0.1978		0.1978	
q_1		0.1374		0.1374		0.5044		0.5044	
q_2		0.0181		0.0181		0.0378		0.0378	
$\bar{q}_1^{MM}$	$\bar{q}_1^{FM}$					0.6331	0.2713	0.6758	0.2286
$\bar{q}_1^{MF}$	$\bar{q}_1^{FF}$					0.6331	0.2713	0.5601	0.3442
Numbers of firms with vacancies									
d_{00}		**0.0115**		**0.0115**		**0.0019**		**0.0019**	
d_{0M}	d_{0F}	0.0193	0.0193	0.0270	0.0115	0.0054	0.0023	0.0054	0.0023
d_{01}		**0.0385**		**0.0385**		**0.0077**		**0.0077**	
d_{M0}	d_{F0}	0.0403	0.0403	0.0564	0.0242	0.0425	0.0182	0.0428	0.0180
d_{10}		**0.0806**		**0.0806**		**0.0607**		**0.0608**	
Numbers of complete firms									
d_{MM}^N	d_{FM}^N	0.0403	0.0403	0.0790	0.0338	0.1067	0.0457	0.1140	0.0401
d_{MF}^N	d_{FF}^N	0.0403	0.0403	0.0338	0.0145	0.0457	0.0196	0.0403	0.0233
d_{11}^N		**0.1612**		**0.1612**		**0.2177**		**0.2177**	
d_{MM}^S	d_{FM}^S	0.0021	0.0021	0.0041	0.0018	0.0058	0.0025	0.0062	0.0023
d_{MF}^S	d_{FF}^S	0.0021	0.0021	0.0018	0.0008	0.0025	0.0011	0.0023	0.0012
d_{11}^S		**0.0084**		**0.0084**		**0.0119**		**0.0119**	
Distribution of workers across job levels									
p_2^M	p_2^F	0.2080	0.2080	0.2080	0.2080	0.2374	0.2374	0.2400	0.2313
p_1^M	p_1^F	0.2500	0.2500	0.2500	0.2500	0.2904	0.2904	0.2936	0.2830
p_0^M	p_0^F	0.5420	0.5420	0.5420	0.5420	0.4722	0.4722	0.4665	0.4857

With equal proportions of male and female workers $h = 0.5$ and no gender homophily ($\tau = \tau_0$) we get that half of the links are with workers of the same (opposite) gender. This is illustrated in panel (A) of Table 4. Next, we reduce participation of female workers, so that $h = 0.3$. This case is illustrated in panel (B). Since there is still no homophily in this setting, lower participation of female workers leads to a lower fraction of female workers in the networks of males and other females. For example, we can see that both genders now have only a fraction $1 - \gamma_M = \gamma_F = 12/40$ of female workers in their networks, which is exactly 30%. Figure 1 (left) shows the corresponding Poisson distributions of social networks, where the left density illustrates links of both genders to female workers and the right density illustrates links of both genders to male workers. These distributions only reflect the fact that female workers are the minority in panel (B).

Further we return to the setting with equal participation $h = 0.5$ and analyze the implications of gender homophily, illustrated in panel (C). Here, we set $\tau_0 = 0.25 < \tau = 0.5$. We justify our

Table 4: Network composition for different parameter values

	M	F	Total
M	20	20	40
F	20	20	40

(A): $h = 0.5$, $\tau = \tau_0 = 0.5$

	M	F	Total
M	20	10	30
F	10	20	30

(C): $h = 0.5$, $\tau = 0.5$, $\tau_0 = 0.25$

	M	F	Total
M	28	12	40
F	28	12	40

(B): $h = 0.3$, $\tau = \tau_0 = 0.5$

	M	F	Total
M	28	6	34
F	14	12	26

(D): $h = 0.3$, $\tau = 0.5$, $\tau_0 = 0.25$

choice of the homophily parameter τ_0 in the next section when we combine networks and referral hiring. Panel (C) reveals that gender homophily is erasing half of the cross-gender links and their average number falls from 20 down to 10. This also reduces the average total number of links in the social network down to 30. The corresponding Poisson densities are illustrated in Figure 1 (right) where the dashed curve corresponds to cross-gender connections. We can see that only 10/30, that is about 33% of links in the networks of male workers are with female workers, even though both gender groups have the same size in (C). The situation is symmetric and there are only 33% of links to male workers in the networks of females.

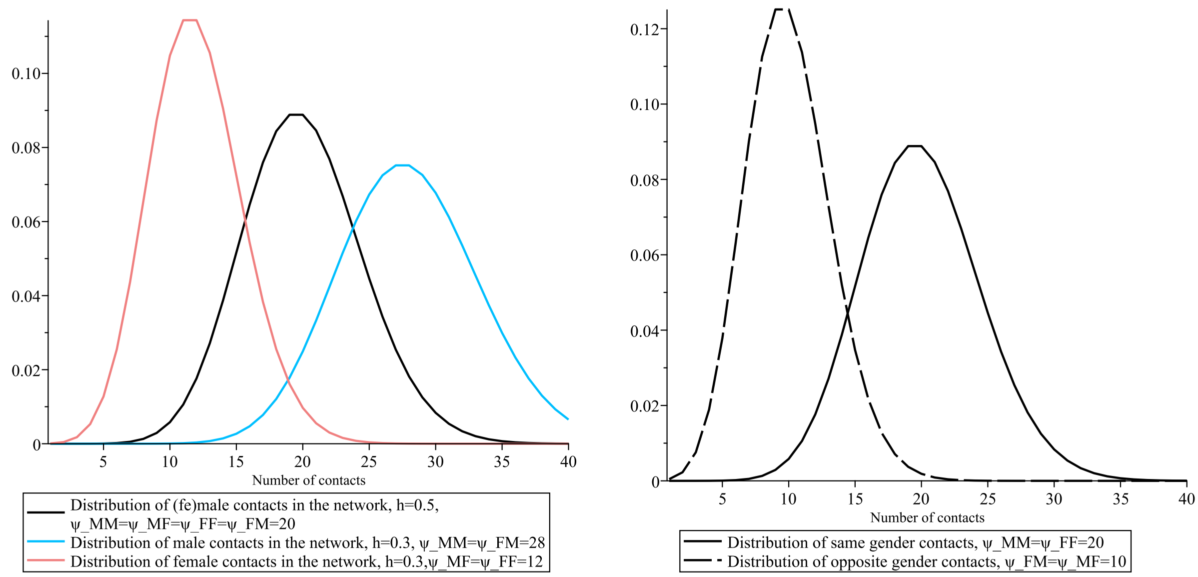


Figure 1: Left: Effect of the participation rate h. Right: Effect of the gender homophily τ_0

Finally, we combine the two effects and consider the situation with $h = 0.3$ (lower participation of females) and $\tau_0 = 0.25$ (gender homophily). This case is contained in part (D) of Table 4. We can see that lower participation of female workers leads to $\psi_{MM} = 28$ and $\psi_{FF} = 12$ as in case (B). At the same time gender homophily is erasing half of the potential cross-gender contacts, so that $\psi_{MF} = 6$ and $\psi_{FM} = 14$ as in case (C). One direct consequence of the combined effect is that female workers end up with smaller social networks ($n_F = 26 < n_M = 34$). Moreover, the fraction of female contacts in the networks of males is only 6/34, that is 17.6%, which is well below the population average of 30%. The reason is two-fold, on-the-one hand, female workers are the minority and, on-the-other hand, it is more difficult to create connections with the opposite gender. At the same time the fraction of male contacts in the networks of females is 14/26, that is 53.8%, which is well below the population average of males equal to 70%. The corresponding four densities are illustrated in Figure 2. Due to the inherent property of the

Poisson distribution, a larger mean is associated with a larger variance. The figure reveals that the standard deviation of the male distribution of male contacts $\sqrt{\psi_{MM}} = \sqrt{28} \approx 5.3$ is much larger than the standard deviation in the distribution of female contacts $\sqrt{\psi_{MF}} = \sqrt{6} = 2.4$. So the former distribution is more dispersed.

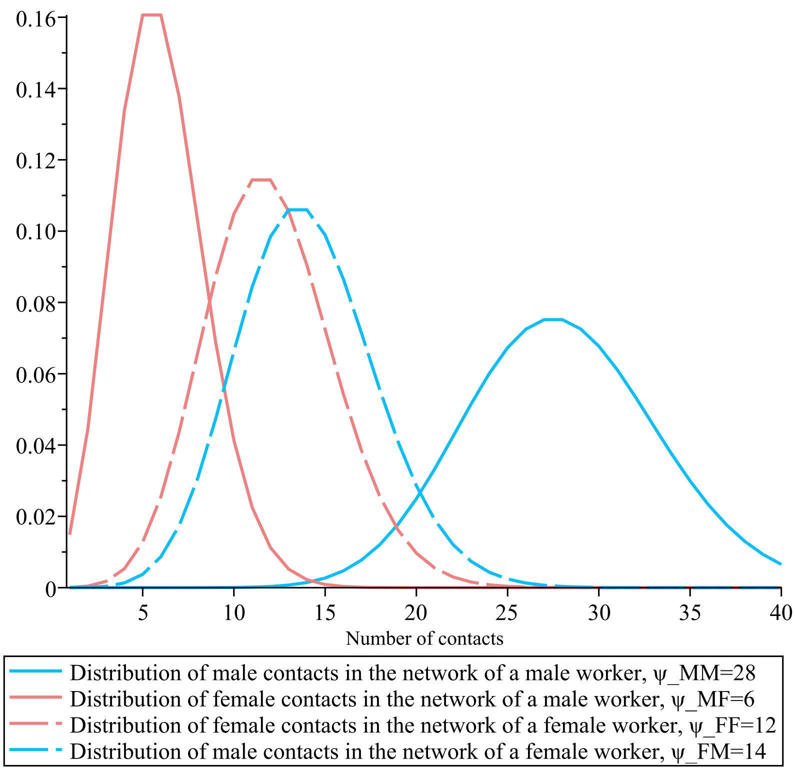


Figure 2: Network densities with gender homophily and lower participation of female workers

5.3 Referral hiring

In this section, we incorporate social networks into the model and analyse the implications of referrals. First, we set $s = 0.4$, which is the probability that the senior manager is asked to recommend a contact for the open mid-level position. This parameter is driving the frequency of referral hiring. Here, we follow the empirical literature (Pistaferri, 1999; Addison and Portugal, 2002; Bentolila et al., 2010; Pellizzari, 2010), which reports that 30%-50% of employees are hired via referrals. For Germany Rebien et al. (2017) show that the average fraction of referral hires in the years 2012-2015 is equal to 37%.

Model III in Table 3 (columns (5)-(6)) shows the equilibrium with referral hiring, lower participation of female workers ($h = 0.3$) but no gender homophily ($\tau = \tau_0 = 0.5$). This network type corresponds to case (B) in the previous section. Intensive referral hiring has a strong impact on the labor market. We can see that the total job-filling rate is relatively high: $\bar{q}_1^{MM} + \bar{q}_1^{FM} = 0.9044$, indicating that firms with open mid-level positions enjoy 90.44% probability of filling their position per unit time. This number does not depend on the gender of the senior manager since $\bar{q}_1^{MF} + \bar{q}_1^{FF} = 0.9044$ and it is much higher than the job-filling rate in the model without referrals (Model II) where the job-filling rate of mid-level positions was only 0.1374. Note, that the total job-filling rate $\bar{q}_1^{MM} + \bar{q}_1^{FM}$ consists of the formal hiring rate $q_1 = 0.5044$ and the referral hiring rate $\tilde{q}_1^M = 0.4$, so the average fraction of employees hired by recommendation can be evaluated at $0.4/0.9044 \approx 0.44$, that is 44%. This value is inline with the empirical evidence presented in the previous paragraph.

Model III reveals that the general equilibrium effect is amplifying the consequences of referral hiring. Since many firms fill their positions via referrals there are less open vacancies for mid-level jobs $d_{00} + d_{01}$. This reduces the competition of firms in the formal hiring process and leads to the higher job-filling rate $q_1 = 0.5044 > 0.1374$ (see Equation (1)). So referral hiring reduces the number of firms with empty positions $d_{00} + d_{01} + d_{10}$ which was 0.1306 in Model II and raises the number of firms with full employment $d_{11}^N + d_{11}^S$ which was 0.1696. The numbers in Model III are now 0.0703 and 0.2297. Intuitively, this means that in equilibrium there are 60 firms more with full employment (per 1000 workers) due to referral hiring.

Network hiring also has strong implications for workers as shown in the last rows of Table 3. Workers move much faster from level 0 to level 1, so the average fraction of workers in the lowest level is reduced to 47.22%, while the fraction of workers in middle management is increased to 29.04%. Even though firms still require $\bar{x}$ periods of experience for senior management jobs, more workers start accumulating experience on average and so more workers reach the highest positions. Finally, Model III reveals that neither differences in the participation rates nor referral hiring give rise to position differences between male and female workers. Even though female workers are the minority and both genders are involved into referral hiring, upward mobility patterns and opportunities are still the same for both groups of workers. We formulate these results in the following way:

Result 5: *Let $\tau = \tau_0$, then referral hiring increases the absolute numbers of male and female workers in senior positions and reduces the absolute numbers of male and female workers in low-level positions, but it preserves equal opportunities of career progression for both gender groups: $p_0^F = p_0^M$, $p_1^F = p_1^M$ and $p_2^F = p_2^M$, irrespective of the female participation rate h.*

Further, we investigate the consequences of network hiring with homophilous social networks in Model IV (columns (7)-(8) of Table 3). Conditional on the probability of referral hiring, male senior managers are more likely to recommend male applicants which happens with probability $\tilde{\alpha}_1^M = 0.8175$. This probability is substantially higher than the population average of male workers 0.7. In a similar way, female senior managers tend to over-recommend female job candidates with the corresponding probability $1 - \tilde{\alpha}_1^F = 0.4716$, which is well above the population average of female workers equal to 0.3. These numbers are consistent the empirical findings of Fernandez and Sosa (2005). They find that both genders tend to over-recommend their own types by about 10% compared to the fraction of their gender among external applicants. This justifies our choice of the homophily parameter $\tau_0 = 0.25$. Hence, at this point, we have a complete set of network parameters which can be summarized in the following way – see Table 5.

Table 5: Network parameters.

Parameter	Value	Definition and Explanation
$\psi\tau/\delta$	40	Average network size. Consistent with the evidence in Cingano and Rosolia (2012), Glitz (2017) and Fontaine (2008)
s	0.4	Probability of recommendation. Consistent with a share of referral hires equal to 44%, Bentolila et al. (2010), Pellizzari (2010)
τ_0/τ	0.5	Gender homophily ratio. Consistent with the evidence in Fernandez and Sosa (2005) that both genders over-recommend their own types by about 10% relatively to the pool of formal applicants

Next, we describe the main result of our study which is illustrated by Model IV. If the senior manager in the firm is a male worker, then the rate at which the open mid-level job is filled with a male applicant is increasing to $\bar{q}_1^{MM} = 0.6758$ compared to Model III, and the rate at which this position is filled with a female worker is decreasing to $\bar{q}_1^{FM} = 0.2286$. In contrast, if the senior manager is a female worker, then the rate at which the job is filled with a male candidate is decreasing to $\bar{q}_1^{MF} = 0.5601$, while the rate at which this position is filled with another female candidate is increasing to $\bar{q}_1^{FF} = 0.3442$. One important point to be emphasized is that there are relatively many firms with a male senior manager $d_{0M} = 0.0054$ and relatively few firms with a female senior manager $d_{0F} = 0.0023$. This is due to the fact that female workers are the minority. Thus, lower participation of female workers, combined with intensive referral hiring and network homophily benefits and improves career opportunities of the majority male group, while worsening the chances of the minority female group. This leads to the more beneficial distribution of male workers with 46.65% of male workers remaining in low level jobs on average and 24% becoming senior managers compared to 48.57% of female workers in the bottom and 23.13% in senior management. Therefore, we draw the following conclusion:

Result 6: *A combination of three factors – lower participation of females ($h < 0.5$), referral hiring ($s > 0$) and network homophily ($\tau_0 < \tau$) violate the equal career opportunities for men and women leading to the glass-ceiling effect: $p_2^F < p_2^M$ and $p_0^F > p_0^M$. The glass-ceiling effect can arise even in the absence of direct discrimination.*

5.4 Comparative statics

In this subsection, we perform comparative statics analyses and investigate which factors mitigate or amplify the glass-ceiling effect generated in Model IV. First, we vary the female participation rate h in the range [0.3..0.5] and calculate the fractions of male and female workers in lowest and highest positions. The left panel of Figure 3 shows fractions p_0^F and p_0^M. We can see that these fractions are the same and equal to 0.4722 as a long as both groups are equally large ($h = 0.5$). The difference appears as soon as h falls below 0.5 and the gap is increasing with lower values of h, which means that more and more female workers remain in low level jobs compared to male workers. For $h = 0.3$ we reach the values of Model IV with $p_0^F = 0.4857$ and $p_0^M = 0.4665$. The right panel of Figure 3 shows fractions of male and female worker in senior positions, that is p_2^F and p_2^M. Again there are no differences between the two gender groups as long as $h = 0.5$. The gap is generated as soon as $h < 0.5$ and it is increasing with lower h reaching the levels $p_2^F = 0.2313$ and $p_2^M = 0.2400$ for $h = 0.3$. These numbers correspond to Model IV in Table 3. Overall, we conclude that lower participation of female workers generates a stronger disadvantage in terms of professional networks for female workers and amplifies the glass-ceiling effect.

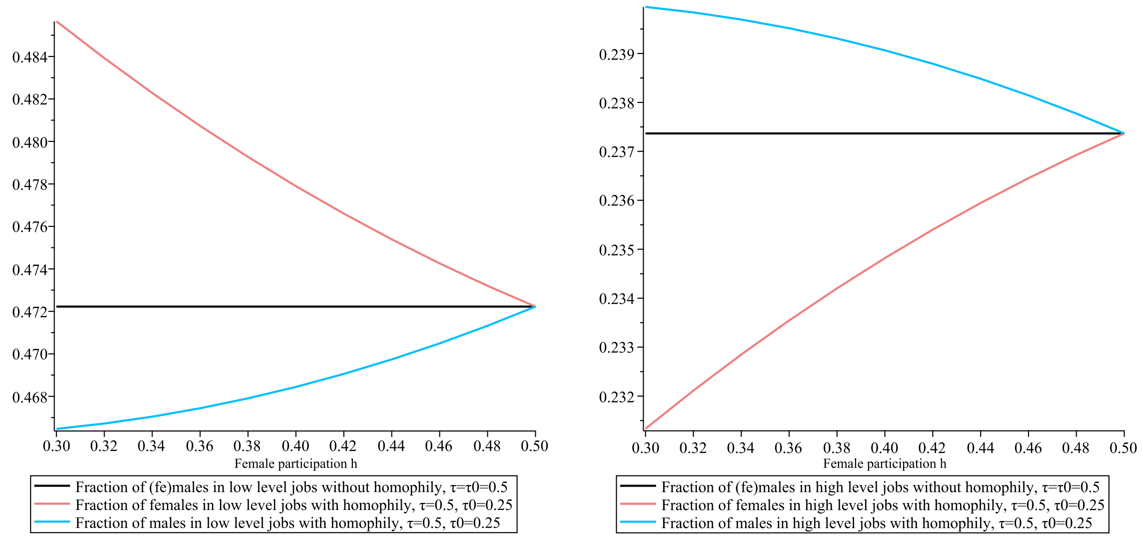


Figure 3: Comparative statics with respect to female participation fraction h. Left: fraction of employees on level 0 within their gender group, p_0^F and p_0^M. Right: fraction of employees on level 2 within their gender group, p_2^F and p_2^M.

Next we perform comparative statics analysis with respect to parameter s, which is the probability that the senior manager is asked to recommend one of his/her contacts for the open mid-level position. Indirectly, this parameter is driving the frequency of referral hires. Figure 4 shows fractions p_0^F and p_0^M and p_2^F and p_2^M for s in the interval [0..0.4]. On the left panel, we can see that there is no difference between the two gender groups for $s = 0$. This is the case without referrel hiring, where $p_0^F = p_0^M = 0.5422$. The difference appears as soon as $s > 0$ and the gap is increasing for larger values of s indicating more intensive referrals. When $s = 0.4$ we obtain the values of Model IV with $p_0^F = 0.4857$ and $p_0^M = 0.4665$. The right panel shows fraction p_2^F and p_2^M. Here we can see that fewer female workers reach senior positions with more intensive referral hiring.

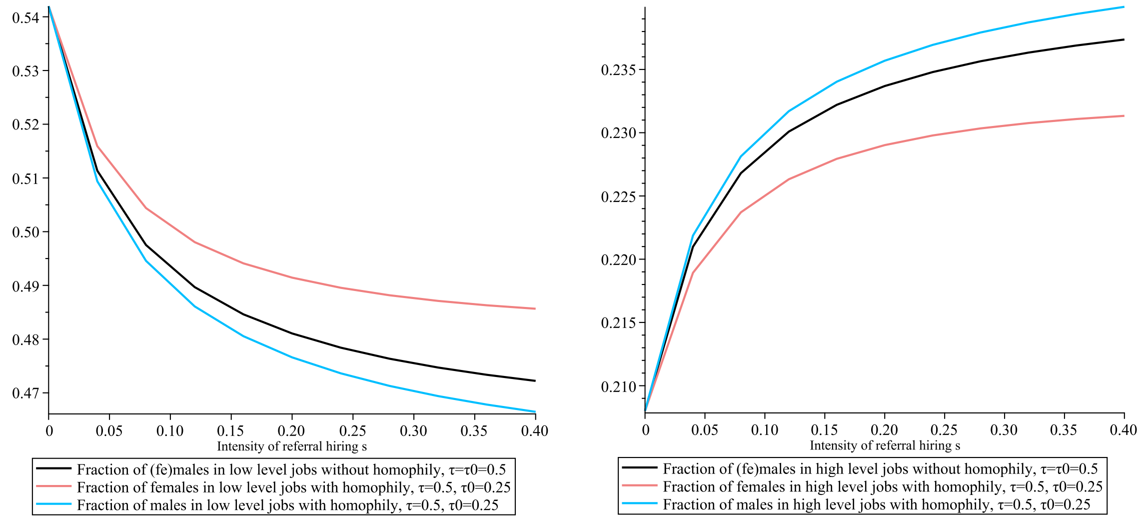


Figure 4: Comparative statics with respect to the referral intensity s. Left: fraction of employees on level 0 within their gender group, p_0^F and p_0^M. Right: fraction of employees on level 2 within their gender group, p_2^F and p_2^M.

Another key parameter in our model is τ_0. This parameter is equal to 0.25 in the benchmark specification and captures the degree of network homophily. In the next step, we perform comparative statics analysis with respect to τ_0 by varying it in the range [0..0.5]. The first extreme case $\tau_0 = 0$ corresponds to fully segregated social networks, where workers of different genders never form social links. In contrast, $\tau_0 = \tau = 0.5$ corresponds to the case of balanced networks where every worker has a fraction h of female contacts and a fraction $1 - h$ of male contacts. Our results are illustrated in Figure 5. The solid curves correspond to the benchmark case $s = 0.4$, while the dashed curves are presented for less intensive referral hiring $s = 0.2$. We can see that the impact of network homophily on gender inequality in not monotone and has an "eye"-shape. In particular, the most unequal distribution of males and females is observed for $\tau_0 = 0.13$. Even stronger network segregation corresponding to $\tau_0 < 0.13$ does not increase, but reduces the inequality!

To understand this result, consider a firm in which the senior manager is a male with probability $1-h$ and a female with probability h. So the gender distribution in this firm corresponds to population averages on all hierarchical levels. Suppose the senior manager is asked to recommend a friend for the job. If social networks are sufficiently large, which is the case for our parameter values, it is almost always the case that the senior manager has at least one friend interested in the job. Then the recommended applicant is a male worker with probability $\tilde{\alpha}_1^M(1 - h) + \tilde{\alpha}_1^F h$ and a female worker with the opposite probability. Note that $\tilde{\alpha}_1^M = 1$, while $\tilde{\alpha}_1^F = 0$ in the case of full segregation. This means that male senior managers always recommend only male applicants, whereas female senior managers never recommend male applicants. In Table 6 we compare the cases with segregated networks and with balanced networks. This table shows that in both cases the probability of a male applicant being recommended is equal to $1 - h$ and the probability of a female applicant being recommended is equal to h. In both cases, referral hiring reproduces the initial gender distribution in the occupation $[1 - h, h]$ and does not change the

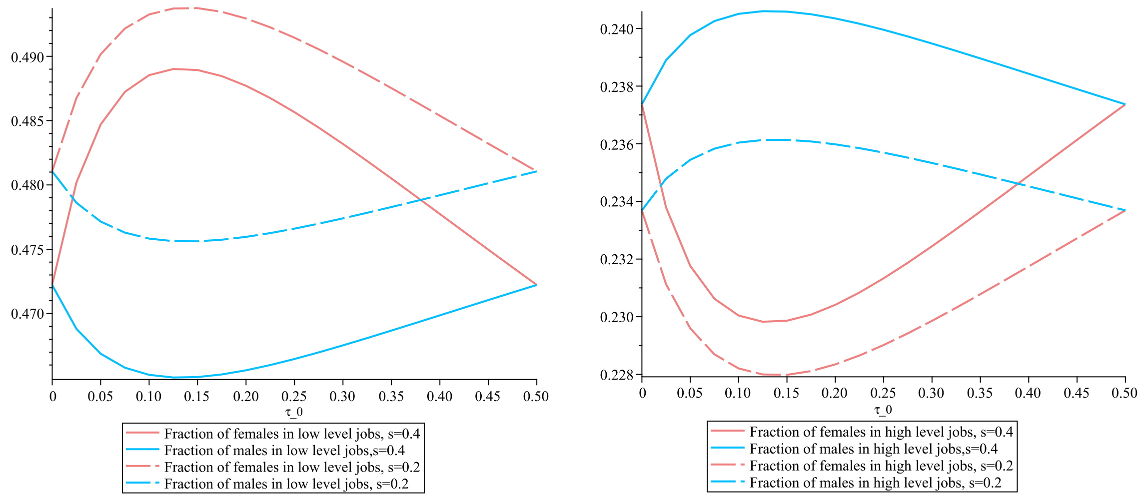


Figure 5: Comparative statics with respect to the network homophily τ_0. Left: fraction of employees on level 0 within their gender group, p_0^F and p_0^M. Right: fraction of employees on level 2 within their gender group, p_2^F and p_2^M.

gender composition of the firm. This explains the observation that gender inequality tends to disappear in the extreme cases of fully segregated and balanced networks. So it is maximized for intermediate values of the homophily parameter τ_0.

Table 6: Comparison between segregated and balanced networks

	Segregated networks $\tau_0 = 0$ $\tilde{\alpha}_1^M = 1, \quad \tilde{\alpha}_1^F = 0$	Balanced networks $\tau_0 = \tau$ $\tilde{\alpha}_1^M = 1 - h, \quad \tilde{\alpha}_1^F = 1 - h$
Probability of a male applicant being referred	$\tilde{\alpha}_1^M(1-h) + \tilde{\alpha}_1^F h$ $= 1 - h$	$\tilde{\alpha}_1^M(1-h) + \tilde{\alpha}_1^F h$ $= (1-h)^2 + (1-h)h = 1 - h$
Probability of a female applicant being referred	$1 - (\tilde{\alpha}_1^M(1-h) + \tilde{\alpha}_1^F h)$ $= h$	$1 - (\tilde{\alpha}_1^M(1-h) + \tilde{\alpha}_1^F h)$ $h(1-h) + h^2 = h$

In addition, Figure 5 shows that both curves p_0^F and p_0^M shift upwards for lower values of s. This means that workers stay longer in low level jobs with less intensive referral hiring. Moreover, we can see that the "eye" is closing for lower s and it closes completely for $s = 0$. This is the case when both curves coincide, so that $p_0^F = p_0^M = 0.542$ for $s = 0$ (Model II). We summarize these results in the following way:

Result 7: *In the model with referral hiring $s > 0$ and low participation of females in the occupation $h < 0.5$ there is a non-monotone relationship between the network homophily parameter τ_0 and gender inequality such that the inequality is maximized for some intermediate level of τ_0. Full network segregation by gender does not lead to the maximal inequality.*

Our previous analysis shows that lower participation of female workers and more intensive referral hiring contribute to larger differences in the distributions of male and female workers. The question remains, how much of the empirically observed wage gap due to the glass-ceiling effect in Germany (6.4%) can be explained by Model IV? To answer this question, we take normalized wages from Table 1 and assign these wages to the three hierarchical levels in our model, so that $w_0 = 1$, $w_1 = 1.319$ and $w_2 = 1.713$. With these values we can calculate the average wages of male and female workers in our model:

$$
\begin{aligned}
w^F &= p_0^F w_0 + p_1^F w_1 + p_2^F w_2 = 0.4857 \cdot 1 + 0.2830 \cdot 1.319 + 0.2313 \cdot 1.713 = 1.254 \\
w^M &= p_0^M w_0 + p_1^M w_1 + p_2^M w_2 = 0.4665 \cdot 1 + 0.2936 \cdot 1.319 + 0.2400 \cdot 1.713 = 1.264
\end{aligned}
$$

The wage gap generated by Model IV is equal to $1.264/1.254 - 1 = 0.008$, that is 0.8%.

Another measure of inequality which is often used in gender studies is a Duncan segregation index (Duncan and Duncan, 1955). Typically, this index compares the distribution of male and female employees in different occupations. In our model all workers are employed in the same occupation, so in the following we will use this index to compare the distributions of male and female workers across different hierarchical levels. This yields:

$$\begin{aligned} \text{Duncan Index} &= 0.5\sum_{i=0}^{2}|p_i^M - p_i^F| = 0.5(|0.4665 - 0.4857| + |0.2936 - 0.2830| \\ &+ |0.2400 - 0.2313|) = 0.5(0.0192 + 0.0106 + 0.0087) = 0.0193 \end{aligned}$$

The Duncan index generated by Model IV is equal to 1.93%. This is the percentage of men (women) who would have to change jobs in order to eliminate segregation. The index also shows that the gender inequality generated by our model is a "bottom-level" inequality which is propagating to the top because $0.0192 > 0.0106 > 0.0087$. The difference in the proportions of male and female workers is strong on level 0 and decays as we move to level 2. The reason is that referral hiring in our model operates between levels 0 and 1 since senior workers recommend their social contacts for lower hierarchical positions. So referral hiring reduces the access of female workers to professional career jobs in our setting.

Both inequality measures – the gender wage gap and the Duncan segregation index – can be used to measure the degree of inequality between the two groups. There is, however, also an important difference between the two. Whereas the gender wage gap combines information on the distributions p_i^F and p_i^M and wages w_i, the Duncan index is independent of wages w_i. For example, if wages on all levels were the same but the distributions of male and female workers were different, this type of inequality would be revealed by the Duncan index even if the gender wage gap is equal to zero in this setting.

In the next section, we perform several extensions of our model capturing some prevasive features of the labor market and compare the implications of these extensions for gender inequality by using both inequality measures.

6 Extensions

6.1 Deeper firm hierarchies, triadic closure, and unequal promotion times

In the preceding part of the paper, we could show that the glass ceiling effect may arise even in the absence of discriminatory behavior if women are under-represented in the occupation and network formation is homophilous. The goal so far was to develop a minimally sufficient model with referral hiring, endogenous professional networks, and hierarchical firms in order to isolate and illustrate the new mechanism giving rise to unequal career advancement of men and women. Nevertheless, the model presented in the previous sections is very stylized and builds on a number of simplifying assumptions. In order to obtain a better assessment of the gender wage gap, also in quantitative terms, generated by referral hiring and homophilous networks we extend our model in three dimensions: (a) we consider network formation following the idea of triadic closures, (b) we introduce gender specific promotion times, and (c) we add one more hierarchical level to the structure of firms. All these three extensions follow up on our general approach to study gender inequality in a framework that combines a labor market characterized by search and matching with a more elaborate representation of firms' organizational structures and hiring practices.

Triadic closures: In our analytical framework we started off by considering a random network of workers. This is in line with many studies that assume topologies to be either completely regular or completely random. It has been claimed, however, that many networks actually lie in between the two extremes (see, e.g., Watts and Strogatz, 1998). Evidence for clustered networks of various degrees spans from co-authorship networks of scientists (Opsahl and Panzarasa, 2009;

Barabâsi et al., 2002), to corporate elite networks in the US during the 1980s and 1990s (Davis et al., 2003), and Internet dating communities (Holme et al., 2004), to only name a few. In order to deviate from a random network analysis towards the effects of a more structured network on gender segregation, we employ the idea of triadic closures in networks in the spirit of sociologist Georg Simmel (Simmel, 1908). Here, it is claimed that if individuals A and B know each other, and at the same time B and C know each other, then it is very likely that A and C get to know each other. Thus, the likelihood that A and C are connected is higher than the probability that a tie between these two is established randomly. Then, the primary question in this extension is whether network clustering will amplify or mitigate the effects of referrals on the gender wage gap. In particular, we will extend the model in order to take into account triadic closure by introducing a probability with which two workers meet randomly, and a counterprobability with which a particular worker meets someone from the list of contacts of one of his or her friends.

Gender specific promotion times: Even though empirical evidence on gender differences in promotion rates is generally mixed, there is a large group of studies reporting lower probabilities of internal promotions for women indicating some degree of discrimination (see, e.g., Winter-Ebmer and Zweimüller (1997), Ward (2001), Blau and DeVaro (2007), Johnston and Lee (2015), Kauhanen and Napari (2015) and Cassidy et al. (2016)). Following this evidence, we extend our model by incorporating gender-specific promotion times and compare the estimated wage gaps. This extension allows us to evaluate the relative contribution of network hiring to the gender wage gap in the presence of another strong factor responsible for the gender wage differentials.

The study by Winter-Ebmer and Zweimüller (1997) is using Austrian microcensus data from 1983. The probability of internal promotion in their sample is 44.1% for men and 25.3% for women indicating that women are confronted with a 43% lower probability of promotions. Moreover, only a small part of this gap is explained by endowment differentials and career discontinuities of women due to childbearing. Johnston and Lee (2015) estimate gender-specific promotion rates based on an Australian longitudinal survey for the years 2001-2008. They focus on university-educated men and women. The probability of internal promotion for men is 23.9% in their sample, while it is only 16.5% for women. This finding implies that women are facing about 31% lower chances of internal promotions compared to men. The gap in promotion rates is reduced but remains significant once differences in observable characteristics of workers are controlled for. Finally, Cassidy et al. (2016) use Finnish employment data for the period 2002-2012, and find that the probability of internal promotion is 25% lower for women compared to men. The advantage of their approach, compared to previous estimates, is that differences in performance and productivities are controlled for. Following these studies, we set a minimum human capital level necessary for promotion $\bar{x}$ for men that is 25% lower than that for women. As the average duration until promotion is the inverse of the probability of promotion, we set $\bar{x}$ for men to 18.75 and leave it for women equal to 25 periods. Thus, our numerical exercise that follows is not specific to any particular country but rather reflects a general situation observed in a number of developed economies reviewed above.

Firm hierarchies: This extension is, first, motivated by our empirical data (GSOEP) which actually includes four levels of job hierarchies. For the purpose of the previous analysis, we merged the middle and top management into one category of a senior management level (see Table 1). In what follows, we perform a more detailed quantitative evaluation by separating the two levels, in line with the empirical data which leads to firms with three management positions and four hierarchical levels in total in the extended model. Second, this extension also reflects the empirical evidence from other sources. For example, it was found for France that a vast majority of French firms have a hierarchical structure with up to four distinct layers (Caliendo et al. (2015)). Moreover, deeper firm hierarchies also capture an empirically well documented changing structure of corporate hierarchies (see, e.g., Bloom et al. (2010), Rajan and Wulf (2006), or Guadalupe and Wulf (2010)) that may have an effect on gender segregation along firm hierarchies. Thus, the primary question in this extension is whether there is an interaction effect between the impact of referral hiring and deeper hierarchical structures of firms.

More specifically, we proceed as follows. Top level managers may refer their contacts for mid-level positions and for senior management positions. Accordingly, we split the overall experience requirement $\bar{x}$ of 50 time periods into two parts, so that firms require 25 periods of experience for workers to be promoted from mid-level positions to senior management and another 25 periods to become a top manager. As a result, there are three separate markets in this extended setting. Moreover, management jobs are kept fixed when moving from three to four hierarchical levels for comparability. While in the benchmark setting there were 300 firms each with two management jobs, in the present extension we will consider 200 firms with three jobs in each firm. So the total number of management jobs remains 600. Recall that the average wage in the senior level was $w_2 = 1.71$ with a wage of non-management workers w_0 normalized to 1. In the evaluation of the extended model, i.e. to which extent it can explain the gender wage differentials, we set $w_2 = 1.68$ and $w_3 = 1.87$ in line with the GSOEP data.

As these extensions occur to be intractable in an analytical framework we program a simulation model. Recurring to simulation models in order to analyze labor market behavior in more encompassing settings was already proposed by Freeman (1998) some time ago. In particular, simulation models suit well for the formalization of set-ups which are characterized by heterogenous interacting agents. One of the earliest attempts to build such labor market models can be found in Bergmann (1990). Others followed with applications that included network structures, see, e.g., by Tassier and Menczer (2008), Stovel and Fountain (2009), Gemkow and Neugart (2011), or Dawid and Gemkow (2013). These and other contributions are surveyed in Neugart and Richiardi (2018). In order to lend credibility to our simulation model we show in Section 6.3 that the analytical results can be replicated by our simulation before we present the results on the three extensions.

6.2 Simulation code

Algorithm (1) outlines the pseudocode of the simulation model.[5] In each of the 1000 iterations the following steps are taken. First, network links are established. For the baseline simulation model that replicates the analytical results a worker meets another worker with probability $\phi = 0.8$. If they are of the same gender a network link is established with probability $\tau = 0.5$, and if they are of opposite gender a link is established with $\tau_0 = 0.25$. In the extension incorporating triadic closures, there is a probability with which two workers meet randomly, and a counter-probability with which a particular worker meets someone from the list of contacts of one of his or her own contacts. We set these probabilities to 0.5. After networks are formed, firms promote workers who reached the human capital level $\bar{x}$ for a vacancy (if there is one) on the next hierarchical level. The human capital level $\bar{x}$ is set to 50 for the model with three hierarchical levels, and to 25 for the simulation model with four hierarchical levels. These human capital levels are set equal for men and women except for the extension within which we analyze discrimination along promotional paths. Here, $\bar{x}$ becomes 18.75 and 37.5 for men in the three and four hierarchical levels models, respectively. Once the internal labor markets are closed, workers apply with probability $z_1 = 0.0135$ for vacancies that do not require a certain human capital endowment, and with probability $z_2 = 0.2$ for vacancies for which they meet the human capital requirement, i.e. the senior and the top-level management positions. When hiring workers firms may consider workers with a referral. Referrals can be given by supervisors on all higher hierarchical levels. With probability $s = 0.4$ supervisors on the senior and top-level management positions are invited to refer someone who is a member of their network. Next, workers' human capital is updated, and by the end of the iteration, jobs are destroyed at rate $\rho = 0.015$ and network links are dissolved at rate $\delta = 0.01$. At iteration 1000 model outcomes are saved, and what we call one run is completed. In total, we simulate 50 runs which gives us a distribution of 50 observations for every outcome variable that we may analyze.

[5]The simulation code is written in Repast – a Java based platform.

Algorithm 1 Pseudocode of simulation model

```
for number of runs < 51 do
    for number of iterations < 1001 do
        New network links are established at rate φ
        Firms promote workers above x̄
        Workers apply for vacancies with probability z_1 (z_2)
        Firms hire (in)experienced worker on the external market
        Update of workers' human capital x
        Jobs are destroyed at rate ρ
        Network links are dissolved at rate δ
    Recording of model outcomes
```

6.3 Results

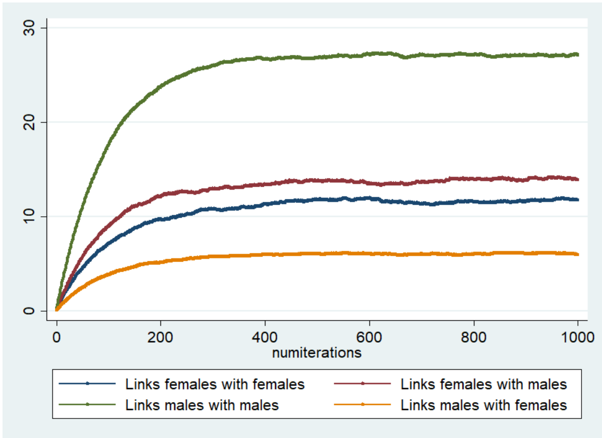
First, we show that our simulation provides a close replication of the analytical model. For this comparison we focus on specification IV (as shown in Table 3). Figure 6 illustrates the dynamics of one of the 50 runs showing the time series for selected variables over the 1000 iterations. At the beginning of the run, all positions in the firms are vacant (panel a) and no network links exist (panel b). Eventually, firms hire workers and most of the firms have both of the positions (middle and high) filled after a while. Moreover, as time evolves network links are established. Panels (c) and (d) show the shares of male and female workers on the low and high positions, respectively. Again, there is convergence over time. Making use of the 50 replications we calculate the average values of some of the key variables in the simulation model and compare them to the outcomes of the analytical Model IV, see last columns of Table 3. The means of the outcome variables of the simulation model with respect to the number of firms having two, one, or no vacancy, and the shares of workers on the different jobs by gender are close to the theoretical model. In fact, they are not different from a statistical point of view. Moreover, the numbers for the average network links in the simulation model quickly converge to the – by the analytical model – predicted numbers of links that male workers should have with male workers (28) or with female workers (6). The same is true for the network links of female workers.

Overall, the simulation model produces results that match those of the analytical model. This makes us confident that the results of any of the three extensions to which we turn now are not due to the transition from an analytical to a simulation framework.

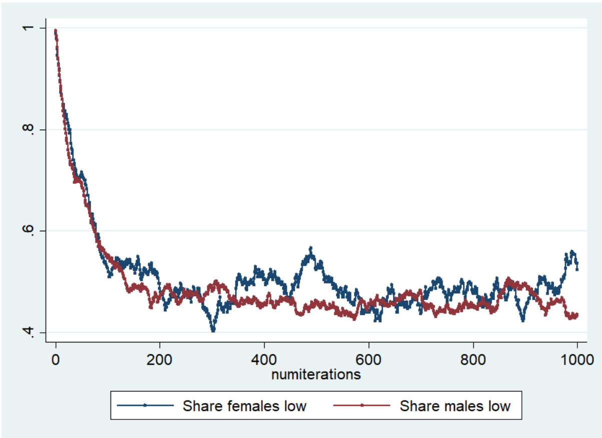
In Tables 7 and 8, we summarize the simulation outcomes. Table 7 shows the employment patterns for the extensions of triadic closure (EI), gender specific promotion times (EII), and the interaction of the two (EIII) for a simulation model with three hierarchies. For ease of comparison, we include the gender-specific employment patterns without any of the extension (E0). Thus, simulation E0 is a replication of the analytical Model IV. Note, that the gender inequality in columns E0 and EI is network-driven while it is combined with direct discrimination in internal promotions in specifications EII and EIII. First, we can see that triadic closures serve as an amplifying device for gender inequality, the gender wage gap is almost doubled and reaches 1.55%. Also the Duncan index is showing higher inequality in model EI (3.88%). Next, we consider the impact of direct discrimination. As expected model EII shows more female workers in low level jobs compared to male workers, moreover, male workers are overrepresented in high level jobs. The Duncan index increases to 7.18% for the inclusion of gender-specific promotion times, and to 7.37% if both extensions apply.

Note that gender-specific promotion times generate a different type of inequality compared to social networks. It is a "top-level" inequality because $|0.410 - 0.444| < |0.279 - 0.317| < |0.311 - 0.240|$. The difference in the proportions of male and female workers is low on level 0 but gets stronger as we move to level 2. The reason is that internal promotions operate inside companies between levels 1 and 2, whereas referral hiring operates on the external market between levels 0 and 1 and affects the access to career jobs rather than internal advancement. In

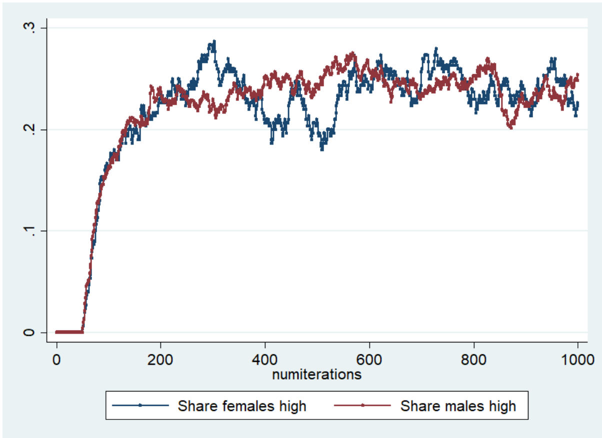
(a)



(b)



(c)



(d)

Figure 6: Time series

Notes: Time series on (a) number of firms by positions filled, (b) average number of network links by gender and direction, (c) share of employment on low level positions by gender, and (d) share of employment on high level positions by gender. Single run, 1000 iterations, Model IV.

Table 7: Employment shares by gender and wage gap with three hierarchical levels

	E0		EI		EII		EIII	
	M	F	M	F	M	F	M	F
low	0.462	0.485	0.455	0.494	0.410	0.444	0.400	0.462
	(0.002)	(0.004)	(0.003)	(0.005)	(0.002)	(0.004)	(0.002)	(0.004)
middle	0.294	0.287	0.298	0.277	0.279	0.317	0.287	0.298
	(0.001)	(0.003)	(0.001)	(0.003)	(0.002)	(0.004)	(0.001)	(0.003)
high	0.243	0.229	0.247	0.230	0.311	0.240	0.313	0.240
	(0.002)	(0.004)	(0.002)	(0.003)	(0.002)	(0.003)	(0.002)	(0.003)
Duncan index	0.0220		0.0388		0.0718		0.0737	
Wage gap in %	0.8		1.55		3.09		3.86	

Notes: The table shows the mean employment shares and standard errors over 50 runs by gender and hierarchical level for the replication of the analytical model (E0), a network formation with triadic closures (EI), gender specific promotion times (EII) and a combination of both (EIII).

addition, we can analyze if there is an interaction effect between the two extensions – clustered networks and unequal promotion-times. The separate contributions of the two extensions are given by $1.55 - 0.8 = 0.75\%$ and $3.09 - 0.8 = 2.29\%$ respectively. So the predicted wage gap generated by the combination of the two effects should be $0.8 + 0.75 + 2.29 = 3.85\%$. Table 7 shows that the actual wage gap in model EIII is 3.89%, this is about the same magnitude as the sum of the individual effects indicating that there are rather no interaction effects between the two extensions.

Table 8 shows the effects of our final extension, namely deeper firm hierarchies. We observe more gender segregation (3.15%) for the model with random networks and four hierarchical levels (H0) compared to a model with random networks and three hierarchical levels (E0). As before, triadic closure increases employment segregation compared to random networks also for four hierarchical levels (HI). The gender wage gap in model HI is equal to 2.7%. This means that in a developed country like Germany our model captures about 42% ($= 100 \cdot 2.7/6.4$) of the gender wage differential that arises empirically due to a relatively larger representation of women on lower managerial levels. This gender wage gap is fully attributed to referral hiring, deep firm hierarchies and homophilous social networks with triadic closures but it is not driven by explicit discrimination.

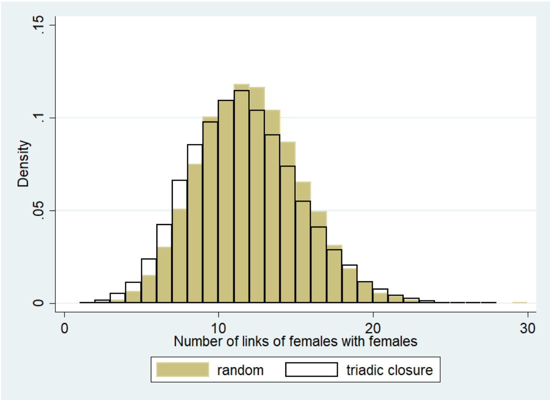
Table 8: Employment shares by gender and wage gap with four hierarchical levels

	H0		HI		HII		HIII	
	M	F	M	F	M	F	M	F
low	0.432	0.464	0.426	0.480	0.406	0.441	0.395	0.463
	(0.002)	(0.004)	(0.002)	(0.004)	(0.002)	(0.004)	(0.002)	(0.003)
middle	0.196	0.182	0.197	0.181	0.184	0.203	0.187	0.196
	(0.001)	(0.002)	(0.001)	(0.003)	(0.001)	(0.003)	(0.001)	(0.003)
high	0.169	0.163	0.172	0.153	0.195	0.190	0.204	0.176
	(0.002)	(0.003)	(0.002)	(0.003)	(0.001)	(0.002)	(0.001)	(0.003)
top	0.203	0.192	0.206	0.186	0.214	0.166	0.215	0.165
	(0.001)	(0.003)	(0.001)	(0.003)	(0.001)	(0.003)	(0.001)	(0.003)
Duncan index	0.0315		0.0545		0.0540		0.0775	
Wage gap in %	1.3		2.7		2.95		4.5	

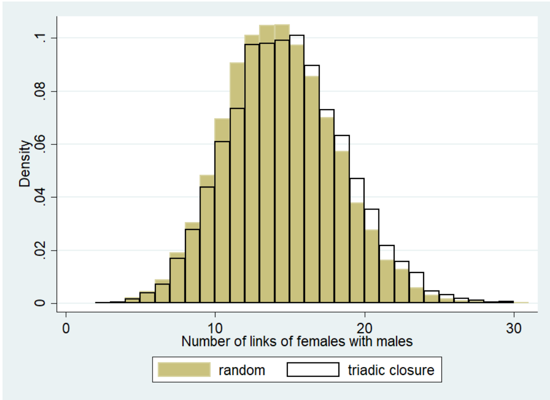
Notes: The table shows the mean employment shares and standard errors over 50 runs by gender and hierarchical level for the extension to four hierarchical levels (H0), a network formation with triadic closures (HI), gender specific promotion times (HII) and a combination of both (HIII).

In the next step, we analyze if there is an interaction effect between deeper firm hierarchies and network clustering. The separate contributions of each extension to the gender wage gap can be calculated as: $1.3 - 0.8 = 0.5\%$ and $1.55 - 0.8 = 0.75\%$. So the predicted wage gap generated by the combination of the two effects should be $0.8 + 0.75 + 0.5 = 2.05\%$. The actual wage gap in model HI is equal to 2.7%. Hence, the sum of the individual effects is substantially lower than the actual wage gap indicating a strong interaction effect between firm hierarchies and triadic closures equal to 0.65%. Recall that network driven inequality is generated at the bottom. So we conclude that this "bottom-level" inequality is amplified by deeper firm hierarchies. Intuitively, this means that the disadvantage of female workers in the access to career jobs is more severe if job ladders have more hierarchical levels.

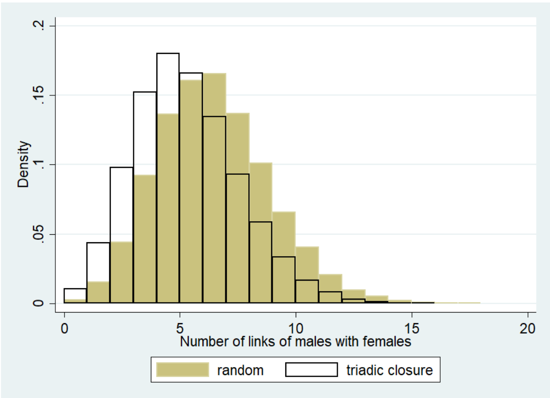
The reason for this interaction effect can be better understood with a help of Figure 7. This figure shows the distribution of network links comparing the simulation models with random selection and triadic closure (for four hierarchical levels). In panel (a) the links of female workers with female workers is shown, and in panel (b) the links of female workers with male workers. Analogously, panels (c) and (d) give insight into the distribution of links for male workers. The distributions for the male workers shift such that female workers are disadvantaged when it comes to recommendations. With triadic closure there are less links of male workers with female workers and more links of male workers with male workers which makes job-to-job moves for female workers less likely. Thus, male workers are more likely than female workers to show up in the top jobs. This network effect is amplified by deeper firm hierarchies as moving to higher level jobs via the external labor market is hampered twice by network driven recommendations that work against under-represented female workers.



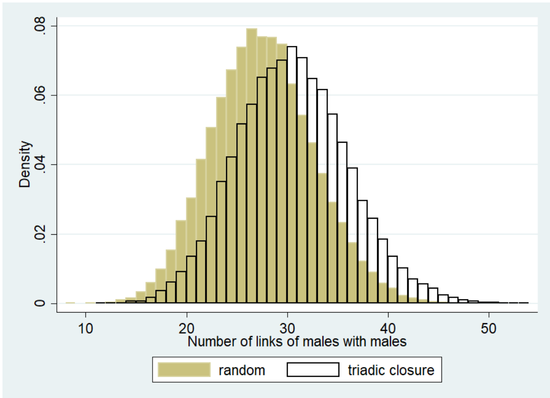
(a) Mean = 11.6 ; Variance = 11.0



(b) Mean = 13.9 ; Variance = 13.7



(c) Mean = 5.9 ; Variance = 5.9



(d) Mean = 27.1 ; Variance = 25.8

Figure 7: Distribution of network links

Notes: Histograms show directed links per worker by gender for models H0 (4 hierarchial levels with random network - grey bars) and H1 (4 hierarchical levels with triadic closure - black lines). Distributions are based on workers' network links at the 1000th iteration for 50 runs. Means and variances are stated for the distributions of model H0. We calculated the global clustering coefficients which is defined as the number of closed triplets over the number of all triplets (both open and closed). The global clustering coefficient increases from 0.032 to 0.041 when going from model H0 to model H1.

Finally, we consider models HII and HIII. Interestingly, as more hierarchies are added to the model with gender-specific promotion times (EII compared to HII) the effect on gender-segregation is mitigated. The Duncan index drops from 7.18% to 5.40%. Also the gender wage gap is reduced from 3.09% to 2.95%. The observation that gender segregation decreases as more hierarchies are added to a model with gender-specific promotion times is due to a reduction in the "top-level" inequality. The reason for this unexpected effect can be better understood with the help of Figure 8. In panel (a), we consider the model without network homophily, that is $\tau_0 = 0.5$, so there are no asymmetric network effects on panel (a). Next, we plot the Duncan indices over various levels of gender specific promotion times. The combination $[25, 50]$

corresponds to the situation without discrimination. Here, both gender groups are promoted after 50 time periods in the model with three hierarchies, and after 25 periods in the model with four hierarchies. The Duncan index for this point is very close to zero indicating equal chances for both gender groups. Moving further to the left of the graph, we introduce discrimination in the promotion rates, for example, the point $[18.75, 37.5]$ means that male workers are promoted already after 37.5 time periods in the model with three hierarchies, and after 18.75 time periods in the model with four hierarchies. At the same time, promotion requirements for women remain unchanged, so the degree of discrimination is increasing as we move from the right of the x-axis to the left. We can see that the model with four hierarchies reveals lower inequality as measured by the Duncan index compared to the model with three hierarchies. This supports our result that deeper firm hierarchies mitigate the "top-level" inequality generated by direct discrimination in internal promotions.

With panel (b) we illustrate the same effect in a model with network homophily $\tau_0 = 0.25$. In the absence of discrimination (point [25, 50]), the Duncan indices coincide with our calculations for models E0 and H0. This means that deeper firm hierarchies amplify the inequality. As we move to the left of this figure the discrimination is increasing and the "bottom-level" inequality generated by homophilous networks, and referral hiring is gradually substituted by the "top-level" inequality generated by the unequal promotion times. For the extreme point [18.75, 37.5], we replicate the Duncan indices from Models EII and HII. At this point, deeper firm hierarchies mitigate the inequality. This explains the twist in the Duncan index in the middle of Figure 8 (b). We summarize these results in the following way:

Result 8 *More clustered networks lead to higher inequality. Deeper firm hierarchies amplify the "bottom-level" gender inequality generated by referral hiring and homophilous social networks. On the contrary, deeper firm hierarchies mitigate the "top-level" gender inequality generated by unequal experience requirements in internal promotions. The effect is ambiguous when both sources of inequality are combined.*

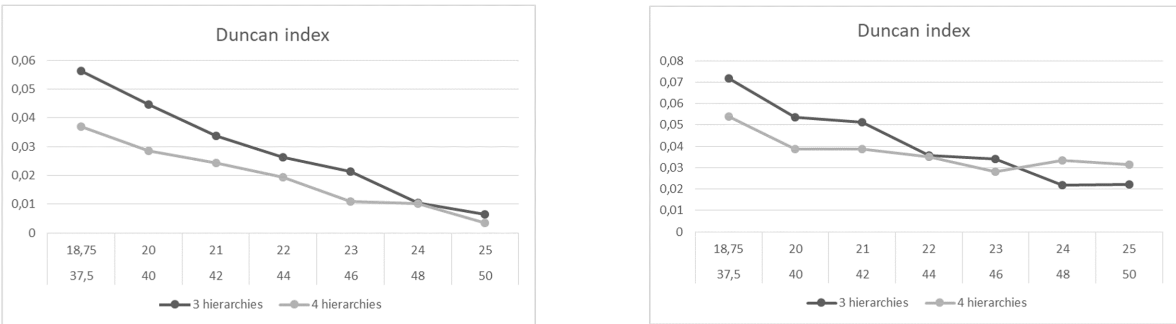


Figure 8: Duncan index comparing three and four hierarchical models for different gender-specific promotion times (left): model without homophily $\tau_0 = 0.5$ (right): model with homophily $\tau_0 = 0.25$

7 Conclusions

In this paper, we were interested in the extent to which the gender wage gap can be explained by high-skilled female workers not making it to the higher managerial positions in firms. We addressed the question by, first, setting up and analyzing an analytical model that, later on, was accompanied by a simulation to take into account further explanatory factors. Our model extends the existing literature on search and matching models by considering endogenous network formation and referral hiring, internal promotions, and job-to-job mobility of male and female workers in hierarchical firms. Our model is producing a number of new and interesting results. We show that a disproportionately low representation of women in managerial positions of firms may emerge without discrimination. Two requirements have to be fulfilled for this to occur: women need to be under-represented in the labor market and network formation has to be homophilous. If these two requirements are fulfilled, the disproportional representation of women on top managerial positions becomes, moreover, stronger as networks are more clustered (e.g. by triadic closures), and discrimination takes place for promotions within firms. In addition, we find that deeper firm hierarchies amplify gender inequality generated by referral hiring but mitigate the inequality from direct discrimination.

Thus, part of the gender pay gap is reflected in women being under-represented in better paying managerial jobs, and there are reasons why women do not make it to the top positions which are not necessarily related to discrimination or occupational segregation. In fact, it may suffice that the participation of women is lower compared to men – which is the case in many occupations (ILO, 2017)– and that workers are more inclined to form professional network links with others of the same gender. Whether both requirements fulfilled could already be the result of discrimination needs discussion. If, for example, lower female participation in the occupation is the result of young women not investing into human capital because they are systematically kept out of higher education in general or certain fields, as it is often observed in engineering studies or the natural sciences, then discrimination would already have taken place before our analysis starts. Similarly, one could rightly argue that the formation of networks along the gender-dimension is already the result of discrimination. Men might want to form a network link with another men rather than a women because they want to keep women out of their relevant labor market. We would speak of discrimination here in the sense of Borjas (2013, p.367) who defines it as something that "... occurs when participants in the marketplace take into account such factors as race and sex when making economic exchanges." Such a reasoning, however, would disregard all other factors why men like to hook up more with men, and women more with women which are orthogonal to getting a referral sometimes in the future.

In empirical terms our analysis suggests that as we observe flatter firm hierarchies in the future (Rajan and Wulf, 2006; Guadalupe and Wulf, 2010) also the gender wage differential generated by the gender-bias in referral hiring should decline but not necessarily the gender wage gap attributed to discrimination in internal promotions. Moreover, our analysis bears some interesting policy implications. As homophilous networks are one driver behind the disproportionate gender distribution in managerial jobs, it occurs to be advisable to establish instruments that are conducive to gender-mixing of networks. Policies that encourage women's only networks at workplaces, as can be often observed nowadays, seem to be the wrong way to go – at least if they result in some intermediate level of homophily. Our analysis also suggests that policies which successfully raise female participation to the levels of male workers will also erase wage differentials even if network formation remains homophilous.

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Appendix

Analytical derivations:

We consider the system of differential equations for female workers $\dot{d}_{F0}$, $\dot{d}^N_{FF}$ and $\dot{d}^N_{FM}$, first. The coefficient matrix and the characteristic equation for r are given by:

$$\begin{pmatrix} -(\rho+q_2) & \rho & \rho \\ q_2(1-\alpha_2) & -2\rho & 0 \\ q_2\alpha_2 & 0 & -2\rho \end{pmatrix}$$

$$(-\rho - q_2 - r)(-2\rho - r)(-2\rho - r) - \rho q_2(1-\alpha_2)(-2\rho - r) - \rho q_2\alpha_2(-2\rho - r) = 0$$

The first eigenvalue is given by $r_1 = -2\rho$. The remaining quadratic term is:

$$r^2 + r(q_2 + 3\rho) + 2\rho^2 + 2\rho q_2 - \rho q_2 = 0$$

The discriminant of this quadratic equation is $(q_2+\rho)^2$, so the second and the third eigenvalues are given by $r_2 = -\rho$, $r_3 = -(q_2 + 2\rho)$. The corresponding three eigenvectors are given by:

$$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \qquad \begin{pmatrix} \frac{\rho^2}{q_2} \\ \rho(1-\alpha_2) \\ \rho\alpha_2 \end{pmatrix} \qquad \begin{pmatrix} -q_2 \\ q_2(1-\alpha_2) \\ q_2\alpha_2 \end{pmatrix}$$

The general solution is given by:

$$\begin{aligned} d_{F0}(x) &= k_2^F \frac{\rho^2}{q_2} e^{-\rho x} - k_3^F q_2 e^{-(2\rho+q_2)x} \\ d^N_{FF}(x) &= k_1^F e^{-2\rho x} + k_2^F \rho(1-\alpha_2) e^{-\rho x} + k_3^F q_2(1-\alpha_2) e^{-(2\rho+q_2)x} \\ d^N_{FM}(x) &= -k_1^F e^{-2\rho x} + k_2^F \rho\alpha_2 e^{-\rho x} + k_3^F q_2\alpha_2 e^{-(2\rho+q_2)x} \end{aligned}$$

The three constant terms k_1^F, k_2^F and k_3^F can be found from the following initial conditions: $q_1(1-\alpha_1)d_{00} = d_{F0}(0)$, $\bar{q}_1^{FF} d_{0F} = d^N_{FF}(0)$ and $\bar{q}_1^{FM} d_{0M} = d^N_{FM}(0)$:

$$\begin{aligned} d_{F0}(0) &= k_2^F \frac{\rho^2}{q_2} - k_3^F q_2 = q_1(1-\alpha_1)d_{00} \\ d^N_{FF}(0) &= k_1^F + k_2^F \rho(1-\alpha_2) + k_3^F q_2(1-\alpha_2) = \bar{q}_1^{FF} d_{0F} \\ d^N_{FM}(0) &= -k_1^F + k_2^F \rho\alpha_2 + k_3^F q_2\alpha_2 = \bar{q}_1^{FM} d_{0M} \end{aligned}$$

Adding the latter two equations we can express $k_3^F q_2 = \bar{q}_1^{FF} d_{0F} + \bar{q}_1^{FM} d_{0M} - k_2^F \rho$. Then inserting it into the first equation we get:

$$\begin{aligned}
k_2^F &= \frac{q_2}{\rho(\rho+q_2)}\Big[q_1(1-\alpha_1)d_{00} + \bar{q}_1^{FF} d_{0F} + \bar{q}_1^{FM} d_{0M}\Big] \\
k_3^F &= \frac{\rho}{q_2(\rho+q_2)}\Big[\bar{q}_1^{FF} d_{0F} + \bar{q}_1^{FM} d_{0M}\Big] - \frac{q_1(1-\alpha_1)}{\rho+q_2} d_{00} \\
k_1^F &= \alpha_2 \bar{q}_1^{FF} d_{0F} - (1-\alpha_2)\bar{q}_1^{FM} d_{0M}
\end{aligned}$$

Integrating $d_{F0}(x)$ over x in the interval $[0..\bar{x}]$ we get the total stock of firms d_{F0}:

$$d_{F0} = \int_0^{\bar{x}} \Big[k_2^F \frac{\rho^2}{q_2} e^{-\rho x} - k_3^F q_2 e^{-(2\rho+q_2)x}\Big] dx = \frac{k_2^F \rho}{q_2}(1-e^{-\rho\bar{x}}) - \frac{k_3^F q_2}{2\rho+q_2}(1-e^{-(2\rho+q_2)\bar{x}})$$

Integrating $d_{FF}^N(x)$ over x in the interval $[0..\bar{x}]$ we get the total stock of firms d_{FF}^N:

$$\begin{aligned}
d_{FF}^N &= \int_0^{\bar{x}} \Big[k_1^F e^{-2\rho x} + k_2^F \rho(1-\alpha_2)e^{-\rho x} + k_3^F q_2(1-\alpha_2)e^{-(2\rho+q_2)x}\Big] dx \\
&= \frac{k_1^F}{2\rho}(1-e^{-2\rho\bar{x}}) + k_2^F(1-\alpha_2)(1-e^{-\rho\bar{x}}) + \frac{k_3^F q_2(1-\alpha_2)}{2\rho+q_2}(1-e^{-(2\rho+q_2)\bar{x}})
\end{aligned}$$

Integrating $d_{FM}^N(x)$ over x in the interval $[0..\bar{x}]$ we get the total stock of firms d_{FM}^N:

$$\begin{aligned}
d_{FM}^N &= \int_0^{\bar{x}} \Big[-k_1^F e^{-2\rho x} + k_2^F \rho\alpha_2 e^{-\rho x} + k_3^F q_2\alpha_2 e^{-(2\rho+q_2)x}\Big] dx \\
&= -\frac{k_1^F}{2\rho}(1-e^{-2\rho\bar{x}}) + k_2^F \alpha_2(1-e^{-\rho\bar{x}}) + \frac{k_3^F q_2\alpha_2}{2\rho+q_2}(1-e^{-(2\rho+q_2)\bar{x}})
\end{aligned}$$

Next we consider the system of differential equations for male workers $\dot{d}_{M0}$, $\dot{d}_{MM}^N$ and $\dot{d}_{MF}^N$. The coefficient matrix is given by:

$$\begin{pmatrix} -(\rho+q_2) & \rho & \rho \\ q_2\alpha_2 & -2\rho & 0 \\ q_2(1-\alpha_2) & 0 & -2\rho \end{pmatrix}$$

The eigenvalues are the same, but the eigenvectors are slightly different and given by:

$$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \qquad \begin{pmatrix} \frac{\rho^2}{q_2} \\ \rho\alpha_2 \\ \rho(1-\alpha_2) \end{pmatrix} \qquad \begin{pmatrix} -q_2 \\ q_2\alpha_2 \\ q_2(1-\alpha_2) \end{pmatrix}$$

So the general solution becomes:

$$\begin{aligned}
d_{M0}(x) &= k_2^M \frac{\rho^2}{q_2} e^{-\rho x} - k_3^M q_2 e^{-(2\rho+q_2)x} \\
d_{MM}^N(x) &= k_1^M e^{-2\rho x} + k_2^M \rho\alpha_2 e^{-\rho x} + k_3^M q_2\alpha_2 e^{-(2\rho+q_2)x} \\
d_{MF}^N(x) &= -k_1^M e^{-2\rho x} + k_2^M \rho(1-\alpha_2)e^{-\rho x} + k_3^M q_2(1-\alpha_2)e^{-(2\rho+q_2)x}
\end{aligned}$$

The initial conditions are: $q_1\alpha_1 d_{00} = d_{M0}(0)$, $\bar{q}_1^{MM} d_{0M} = d_{MM}^N(0)$ and $\bar{q}_1^{MF} d_{0F} = d_{MF}^N(0)$. So we find the three constant terms k_1^M, k_2^M and k_3^M from the following system of equations:

$$\begin{aligned}
d_{M0}(0) &= k_2^M \frac{\rho^2}{q_2} - k_3^M q_2 = q_1 \alpha_1 d_{00} \\
d_{MM}^N(0) &= k_1^M + k_2^M \rho \alpha_2 + k_3^M q_2 \alpha_2 = \bar{q}_1^{MM} d_{0M} \\
d_{MF}^N(0) &= -k_1^M + k_2^M \rho (1-\alpha_2) + k_3^M q_2 (1-\alpha_2) = \bar{q}_1^{MF} d_{0F}
\end{aligned}$$

Adding the latter two equations we can express $k_3^M q_2 = \bar{q}_1^{MM} d_{0M} + \bar{q}_1^{MF} d_{0F} - k_2^M \rho$. Then inserting it into the first equation we get:

$$\begin{aligned}
k_2^M &= \frac{q_2}{\rho(\rho+q_2)} \left[q_1 \alpha_1 d_{00} + \bar{q}_1^{MM} d_{0M} + \bar{q}_1^{MF} d_{0F} \right] \\
k_3^M &= \frac{\rho}{q_2(\rho+q_2)} \left[\bar{q}_1^{MM} d_{0M} + \bar{q}_1^{MF} d_{0F} \right] - \frac{q_1 \alpha_1}{\rho + q_2} d_{00} \\
k_1^M &= (1-\alpha_2) \bar{q}_1^{MM} d_{0M} - \alpha_2 \bar{q}_1^{MF} d_{0F}
\end{aligned}$$

Table 9: Regression results of Blinder-Oaxaca decomposition

N=2403	(1) LNincome	(2) Endowments	(3) Coefficients	(4) Interaction
$Male$	0.154***			
	(8.76)			
$Level$	0.196***	-0.0639***	-0.0700	0.0117
	(18.17)	(-8.07)	(-1.68)	(1.65)
$Education$	0.0746***	-0.00711	0.104	-0.000646
	(16.61)	(-1.24)	(0.71)	(-0.61)
Age	0.0634***	-0.187***	-1.058	0.0581
	(10.90)	(-4.69)	(-1.84)	(1.74)
Age^2	-0.000605***	0.149***	0.440	-0.0420
	(-9.31)	(4.38)	(1.39)	(-1.34)
$Tenure$	0.00518***	-0.00374	0.0596*	-0.00522
	(4.84)	(-1.76)	(2.05)	(-1.62)
$Native$	0.0813	0.000278	0.183	0.00180
	(1.58)	(0.45)	(1.69)	(1.11)
$Agriculture$	–	0.00224	-0.000297	0.000175
		(1.77)	(-0.11)	(0.11)
$Energy$	0.307**	-0.0000435	0.00269	-0.00150
	(2.90)	(-0.06)	(0.96)	(-0.89)
$Mining$	0.831***	-0.000620	-0.000190	0.0000945
	(3.85)	(-0.63)	(-0.22)	(0.21)
$Manufacturing$	0.250**	0.000715	0.00219	-0.000845
	(3.13)	(0.30)	(0.18)	(-0.18)
$Construction$	0.325***	-0.00605	-0.00459	0.00299
	(4.02)	(-1.64)	(-0.39)	(0.39)
$Trade$	-0.0573	-0.00308	-0.00189	-0.000327
	(-0.69)	(-0.98)	(-0.37)	(-0.35)
$Transportation$	0.192*	0.000593	-0.00104	0.000229
	(2.20)	(0.83)	(-0.25)	(0.24)
$Banking$	0.300***	-0.000737	0.00141	-0.000474
	(3.56)	(-0.66)	(0.23)	(-0.23)
$Services$	0.179*	-0.0150*	0.00189	0.000738
	(2.32)	(-2.25)	(0.06)	(0.06)
$Firmsize \leq 5$	–	-0.000602	0.00144	0.0000767
		(-0.34)	(0.36)	(0.25)
$5 < Firmsize \leq 10$	0.167***	0.000670	-0.00551	-0.00128
	(3.63)	(0.90)	(-1.66)	(-1.00)
$10 < Firmsize \leq 20$	0.160***	-0.000604	0.00128	0.000958
	(3.53)	(-0.39)	(0.45)	(0.44)
$20 < Firmsize \leq 100$	0.136***	-0.000705	-0.000136	-0.0000197
	(3.65)	(-0.91)	(-0.02)	(-0.02)
$100 < Firmsize \leq 200$	0.192***	0.0000718	0.00330	0.000182
	(4.58)	(0.30)	(0.73)	(0.35)
$200 < Firmsize \leq 2000$	0.191***	-0.000289	0.00466	-0.000269
	(5.29)	(-0.59)	(0.54)	(-0.45)
$2000 < Firmsize$	0.280***	-0.00813**	0.000196	-0.0000384
	(7.88)	(-2.99)	(0.02)	(-0.02)
$West$	0.231***	-0.0200***	-0.0652*	0.00616
	(11.73)	(-3.82)	(-2.06)	(1.84)
$Constant$	4.362***			
	(26.41)			
Adj. R^2	0.4662			

Data source: SOEP (2013). Sample is restricted to high-skill full-time employees. The dependent variable is the log of the gross monthly wages. t statistics in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.