Variation margins, fire sales, and information-constrained optimality*

Bruno Biais (HEC and TSE), Florian Heider (ECB), Marie Hoerova (ECB)

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Abstract

Protection buyers use derivatives to share risk with protection sellers, whose assets are only imperfectly pledgeable because of moral hazard. To mitigate moral hazard, privately optimal derivative contracts involve variation margins. When margins are called, protection sellers must liquidate some of their own assets. We analyse, in a general-equilibrium framework, whether this leads to inefficient fire sales. If investors buying in a fire sale interim can also trade ex ante with protection buyers, equilibrium is information-constrained efficient even though not all marginal rates of substitution are equalized. Otherwise, privately optimal margin calls are inefficiently high. To address this inefficiency, public policy should facilitate ex-ante contracting among all relevant counterparties.

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1 Introduction

Following the 2007-09 financial crisis, regulators and law-makers promote the use of margins or collateral in derivatives markets to avoid counterparty risk.[1] As McDonald and Paulson (2015, p.92) explain:

> "By construction, many derivatives contracts have zero market value at inception ... [But] as time passes and prices move ... [derivatives'] fair value [becomes] positive for one counterparty and negative ... for the other. In such cases it is common for the negative value party to make a compensating payment to the positive value counterparty. Such a payment is referred to as *[variation] margin* or *collateral*..."

At the same time, there is the concern that such variation margins will lead to a pecuniary externality in the form of fire sales. Variation margins may force counterparties to sell assets so that they can make the required compensating cash payment. A forced sale of assets, however, lowers their price in the market, which reduces the value of assets for everyone else who is selling. The presence of such a negative pecuniary externality raises the issue whether margin-setting, left to the discretion of market participants, leads to a socially inefficient outcome that requires regulation.[2]

Our goal is to shed light on the tradeoff between the benefits and costs of variation margins. We take a general equilibrium approach, with several markets and several types of agents, which enables us to analyse pecuniary externalities. To study the welfare consequences of margin-induced fire sales, we consider an environment in which all agents optimise, in particular when designing contracts and margins.

[1] For an account of the counterparty risk in derivatives created by the Lehman bankruptcy, see Fleming and Sarkar (2014). Post-crisis reform efforts feature moving certain derivative trades to market infrastructures where margin-setting can be administered (e.g., Title VII of the Dodd-Frank Act in the U.S., and the European Market Infrastructure Regulation (EMIR) in the EU).

[2] For regulators' concern about a fire-sale externality in margin-setting, see, for example, the Committee on the Global Financial System (2010), the Committee on Payment and Settlement Systems (2012), or Constâncio (2016).

We show that while variation margins create a fire-sale externality, this does not automatically lead to an inefficient outcome. As long as those who end up buying assets in a fire sale, and thus make a profit, can contract ex ante with those who are hurt by the possibility of a fire sale in the future, the market equilibrium is Pareto efficient. When such ex-ante contracting is not possible, the market equilibrium requires too much variation margining, which leads to too much asset sales and inefficiently low asset prices.

The analysis suggests that regulators can ensure that margin-setting among market participants is optimal by facilitating ex-ante contracting among all relevant counterparties, and by creating contracts contingent on the events triggering variation-margin calls, rather than by focusing on regulating margin-setting as such.

In our model, there are three types of agents. First, there is a mass-one continuum of risk-averse agents endowed with a risky asset. They seek to hedge the risk of their asset, and we refer to them as "protection buyers." For example, protection buyers can be commercial banks seeking to insure mortgage-related assets.

Second, there is a mass-one continuum of risk-neutral agents. As they are risk-neutral, they are natural providers of insurance for protection buyers, and we refer to them as "protection sellers." For example, protection sellers can be investment banks, broker-dealers, or specialised firms such as AIG.[3] Protection sellers have limited liability and hold productive assets, e.g., portfolios of loans or financial securities, whose output can be used to make insurance payments. Protection sellers must exert costly risk-management effort to limit the downward risk of their productive assets and avoid defaulting on their counterparties. Examples of costly risk-management effort are the conduct of due diligence to assess creditworthiness, the minimization of transaction costs, or the appropriate management of collateral and custody.

[3] In the run up to the 2007-09 crisis, Lehman Brothers, AIG and other financial institutions became major sellers of CDS and more complex derivatives linked to mortgages. At the same time, commercial banks were buying these derivative to insure against low values of their mortgage-related assets (see Harrington, 2009). Duffie et al. (2015) analyse empirically the risk exposure of broker-dealers and their customers in the CDS market. These broker-dealers (which are often subsidiaries of large investment banks) and their customers are another example of the protection sellers and protection buyers in our model.

Third, there is a mass-one continuum of risk-averse agents, each endowed with one unit of a safe asset. We refer to them as "investors." Investors can also manage the assets held by protection sellers, but they are less efficient at that task than protection sellers. Investors can be thought of as hedge funds, investment funds, or sovereign wealth funds. It is natural to assume that they are less efficient at monitoring loans or managing trading strategies than the originators of those loans and strategies.

The key friction in our model is that protection sellers' risk-management effort is unobservable. Without this friction, protection sellers fully insure protection buyers, and investors do not participate in the optimal provision of insurance. With this friction, there is a moral hazard problem and full insurance is no longer feasible.[4]

To model a change in the value of derivative contracts over time, we assume there is a publicly observable signal on the future value of protection buyers' risky assets. The signal occurs after initial contracting, but before protection sellers decide on risk-management effort. When the signal reveals bad news about the future value of protection buyers' assets, this renders the derivative position of protection sellers, who sold insurance to protection buyers, an expected liability. The corresponding debt overhang reduces protection sellers' incentives to exert costly risk-management effort.[5]

The first contribution of this paper is to characterise the incentive-constrained Pareto set, i.e., the second best. It is the set of consumptions and asset allocations, contingent on all publicly observable information, that maximise a weighted average of the three types of agents' expected utilities, subject to incentive, participation and resource constraints.

The constrained-efficient outcome has two key characteristics. First, the marginal rates of substitution (MRS) between consumption after a good signal and consumption after a bad signal are not equalised across all agents. They are equalised between protection buyers

[4]Biais, Heider and Hoerova (2017) offer a partial equilibrium analysis of risk-sharing under moral hazard, with one protection buyer and one protection seller, and with an exogenous liquidation value for the protection seller's asset. Bolton and Oehmke (2015) use a similar moral-hazard framework to analyse whether derivative contracts should be priviledged in bankruptcy.

[5]The seminal contribution on debt overhang is Myers (1977).

and investors who share risk optimally. But the MRS is different for protection sellers. Their moral hazard problem limits their ability to share risk with others and hence, prevents the equalization of their MRS with that of protection buyers and investors.

The second characteristic of the second best concerns the allocation of assets. The larger is the amount of assets managed by protection sellers, the larger is their cost of effort and the more severe is the moral-hazard problem. After a bad signal, it can therefore be optimal to transfer assets from protection sellers to investors. The asset transfer improves risk-sharing by relaxing protection sellers' incentive constraint. But it also generates a productive inefficiency because investors are less good at managing production sellers' assets. In the constrained-efficient outcome, the optimal asset transfer equalizes the marginal benefit of better risk-sharing and the marginal cost of inefficient asset transfers.

Our second contribution is to analyse the market equilibrium in this environment. We assume market participants can write and trade contracts contingent on all observable variables, but are subject to incentive and participation constraints.

Privately optimal contracts between protection buyers and protection sellers involve derivatives and variation margins. After a bad signal, the value of protection sellers' derivative position turns into an expected liability. They must sell a fraction of their productive assets in the market, and use the cash proceeds as collateral in case they default, e.g., by depositing the cash on a margin account. Using cash as collateral against possible default on a derivative position after bad news constitutes a variation margin as McDonald and Paulson (2015) explain:

> "Payments due to market value changes are variation margins. ... Collateral is held by one party against the *prospect* of a loss at the future date when the contract matures. ... If the contract ultimately does not generate the loss implied by the market value change, the collateral is returned."

When asset sales occur, protection sellers' productive assets are purchased by investors. Since investors are less efficient than protection sellers at managing these assets, variation margins trigger price drops, which can be interpreted as fire sales.[6] By triggering price drops, variation margins generate pecuniary externalities. When one protection seller liquidates some of his assets to respond to the variation margin, he contributes to depressing the price, which generates a negative externality on the other protection sellers, also selling at that price.[7]

The third contribution of our paper is to analyse whether margin-induced pecuniary externalities lead to inefficiencies. We show that the market equilibrium is constrained efficient. In particular, the privately optimal variation margins requested by protection buyers in the market equilibrium implement second-best asset transfers. This is striking, given the fire-sale externality.

The intuition for constrained efficiency despite pecuniary externalities is the following. The fire sales triggered by variation-margin calls generate profit opportunities for investors, who can buy assets at a low price. Thus, while a negative signal constitutes a negative shock for protection buyers, it is a positive shock for investors. Since protection buyers and investors have different exposures to this shock, they benefit from mutually insuring against it. They fully exploit this risk-sharing opportunity until their marginal rates of substitution are equalised, just as in the second best. Still, and also as in the second best, the marginal rates of substitution of investors/protection buyers differs from that of protection sellers because moral hazard generates endogenous market incompleteness.

The above discussion underscores that investors play two important roles in our model. At time 1, they buy in the fire sale triggered by negative news, and at time 0, they provide insurance against this negative news. It is possible, however, that they do not do so in practice. Consider a situation, in which investors are pension funds or sovereign funds, and

[6]As discussed in Shleifer and Vishny (1992, 2011), price drops occur because assets are bought by non-specialists. Bian et al. (2017) document margin-induced fire sales triggering price drops in equity markets. Ellul et al (2011) find that fire sales of downgraded corporate bonds by insurance companies trigger price declines. Merrill et al. (2014) document fire sales of residential mortgage-backed securities (RMBS).

[7]The empirical evidence in Chernenko and Sundaram (2017) suggest that there is indeed an externality. Mutual funds belonging to the same fund family try to mitigate the fire-sale externality by holding back on asset sales.

protection sellers are investment banks. In normal circumstances, these funds typically do not invest in the specialised assets initially held by investment banks. But, in extraordinary circumstances such as in a fire sale, the funds will buy these assets, because they are offered at a discount. Given that these circumstances at t=1 are extraordinary, the funds may, however, not anticipate at time 0 to factor them into their investment strategies, and hence, would not provide insurance at time 0. We show that in such an incomplete-market setting the equilibrium is inefficient. Because they cannot obtain insurance from investors, protection buyers request large insurance from protection sellers. This in turn implies excessively large variation margins.[8]

Our theoretical analysis helps to interpret the policy implications of empirical studies on fire sales. For example, Meier and Servaes (2015) find that firms buying distressed assets in fire sales earn excess returns, which is consistent with our theoretical model. To assess the welfare implications of fire sales, they argue, one should compare the profits of asset buyers to the losses of asset sellers. According to our theoretical analysis, comparing the profits of asset buyers to the losses of asset sellers is not enough. To conduct a welfare analysis of fire sales, one must consider both their ex-post and ex-ante implications. From a welfare point of view, for ex-post profits of buyers to offset the losses of sellers, it is necessary that investors anticipate fire sales and sell insurance against them.

Stiglitz (1982), Greenwald and Stiglitz (1986), and Geanakoplos and Polemarchakis (1986) analyse how market incompleteness leads to inefficient information-constrained equilibria, while Prescott and Townsend (1984) analyse economies, in which the equilibrium is constrained efficient even though there is asymmetric information or moral hazard.[9] Our paper contributes to this debate by analysing a setting, where in spite of the ability to trade complete contracts, contingent on all observable variables, the market is endogenously in-

[8]Hence, when the market is incomplete, a desire to insure against bad outcomes via derivatives and variation margins can lead to inefficient asset-price drops should the situation worsen (i.e., after bad news that make bad outcomes more likely). In that sense, agents' individual "flight-to-safety" via insurance can be destabilizing. Caballero and Krishnamurthy (2008) provide an account of inefficient flight-to-safety based on Knightian uncertainty and liquidity hoarding.

[9]See also Kilenthong and Townsend (2014) and Kocherlakota (1998).

complete. The endogenous incompleteness leads to discrepancies between marginal rates of substitutions and yet, does not preclude the equilibrium from being constrained efficient.

Recent analyses of equilibrium inefficiency under financial constraints and fire sales consider risk-sharing (Gromb and Vayanos, 2002), lending (Lorenzoni, 2008, Kuong, 2016), or both (Davila and Korinek, 2017).[10]

Gromb and Vayanos (2002) analyse how financially constrained arbitrageurs supply insurance to hedgers. When arbitrageurs suffer losses, their leverage constraints tighten, and they have to liquidate their trading positions.[11] Because hedgers cannot directly trade with one another, markets are incomplete. Lorenzoni (2008) considers entrepreneurs borrowing to fund investment projects. Because entrepreneurs are financially constrained, they must sell assets after negative shocks. Moreover, markets are incomplete so that entrepreneurs cannot insure against these shocks. In both papers, the combination of financial constraints and market incompleteness generates pecuniary externalities that render the equilibrium constrained inefficient. In Gromb and Vayanos (2002) arbitrageurs' positions are excessively large, while in Lorenzoni (2008) entrepreneurs' investment is excessively large.

Davila and Korinek (2017) offer a general framework to analyze these issues. They distinguish two types of pecuniary externalities: distributional externalities that arise from incomplete insurance markets, and collateral externalities that arise from price-dependent financial constraints. In their analysis, when agents can trade securities contingent on all observable states of the world, distributional externalities vanish and marginal rates of substitution are equalised across agents. In our model in contrast, even when agents can trade contracts contingent on all observable states of the world, moral hazard limits risk-sharing

[10]Examples of fire-sale externalities in other contexts include Caballero and Krishnamurthy (2003), who show how financial constraints and depressed asset prices lead to firms' excessive borrowing in foreign currency; Stein (2012), who considers banks' excessive creation of safe short-term debt, the need for asset sales to honor this debt in bad states of the world, and the role of monetary policy to restore efficiency; and He and Kondor (2016), who show how financial constraints and a two-sided pecuniary externality lead to inefficient investment waves.

[11]Brunnermeier and Pedersen (2009) examine a similar feedback between arbitrageurs' financial constraints and asset prices in a static multi-asset model, in which the financial constraint itself depends on asset prices. However, they do not examine welfare.

and prevents an equalisation of marginal rates of substitution.

Kuong (2016) shows that self-fulfilling, constrained-inefficient equilibria can occur due to a feedback between risk-taking incentives of borrowers and fire sales of collateral. In his analysis, lenders who worry about borrowers' risk-taking request collateral which, if liquidated, leads to fire sales. When creditors anticipate fire sales and depressed collateral valuations, they request more collateral to prevent borrowers' risk-taking. If, however, there is not enough collateral, borrowers engage in more risk-taking and are more likely to default. In aggregate, both more collateral and more defaults of borrowers lead to more collateral being liquidated in the market, confirming the anticipation of a fire sale.

Gromb and Vayanos (2002), Lorenzoni (2008), Kuong (2016), and Davila and Korinek (2017) all consider initial collateral (or initial margins), which occurs at the inception of a loan. Initial collateral limits the extent to which agents can lever up their investment in risky assets. We in contrast consider variation margins, which occur during the life of a derivative contract (there is no investment). Variation margins react to the implicit leverage of agents whose derivative position has become loss-making. Correspondingly, our productive inefficiency concerns time-1 asset allocations instead of time-0 allocations as in these other papers.

Acharya and Viswanathan (2011) also focus on lending, which differs from our focus on risk-sharing. They consider asset sales instead of initial collateral, and this creates a similar incentive constraint to ours. A major difference is that we conduct a normative analysis, characterise the second best, and compare it to the market equilibrium.

In section 2, we present the model an discuss its mapping to real markets and institutions. In Section 3, we analyse the first best. In Section 4, we analyse the second best. In Section 5, we analyse the market equilibrium. In Section 6, we discuss empirical and policy implications.

2 Economic environment

2.1 Model

We consider an economy with three dates: time 0, time 1 and time 2; three types of agents: protection buyers, protection sellers and investors; one consumption good, consumed at time 2; and three types of assets generating output in terms of the consumption good at time 2.

Agents and endowments: There is a unit-mass continuum of protection buyers, each with utility u, increasing and concave, and endowed at time 0 with one unit of a risky asset, paying $\tilde{\theta}$ units of consumption good at time 2. There is also a unit-mass continuum of investors each with utility v, also increasing and concave, and endowed at time 0 with one unit of a safe asset, paying 1 unit of consumption good at time 2. Finally, there is a unit-mass continuum of risk-neutral protection sellers, each endowed with one unit of a productive asset, paying $\tilde{R}$ units of consumption good at time 2.

Assets payoffs: The exogenous realisation of the protection buyers' asset at time 2, $\tilde{\theta}$, can take on two values: $\bar{\theta}$ with probability π, or $\underline{\theta}$ with probability $1 - \pi$.

Each unit of the protection sellers' asset yields R at time 2 for sure if protection sellers exert risk-management effort, at cost ψ per unit, at time 1. When consuming c_S units of the consumption good and exerting effort over y units of the asset, a protection seller obtains utility $c_S - y\psi$. If a protection seller does not exert risk-management effort, his asset's payoff is R with probability μ and 0 with probability $1 - \mu$. We assume $R - \psi > \mu R$, so that protection seller's effort is efficient.

In most of our analysis, we assume that the effort a protection seller exerts is not observable by outside parties. Coupled with limited liability, unobservable effort generates a moral hazard problem for protection sellers. We follow Holmström and Tirole (1997) and define pledgeable return, i.e., the part of the physical return that can be promised to outsiders without jeopardising incentives, as

$$\mathcal{P} \equiv R - \frac{\psi}{1-\mu}. \tag{1}$$

Because effort is efficient, $\mathcal{P} > 0$.

Signals: While output and consumption occur at time 2, at time 1 an advanced signal $\tilde{s}$ about $\tilde{\theta}$ is publicly observed, before effort is exerted. When the final realisation of $\tilde{\theta}$ is $\bar{\theta}$, the signal is $\bar{s}$ with probability λ and $\underline{s}$ with probability $1-\lambda$. When the final realisation of $\tilde{\theta}$ is $\underline{\theta}$, the signal is $\bar{s}$ with probability $1-\lambda$ and $\underline{s}$ with probability λ. We assume $\lambda > \frac{1}{2}$ so that the signal is informative.

Asset transfers: Effort takes place at time 1, after the signal is publicly observed. Before effort is exerted (but after observing the public signal), a fraction α of the productive asset can be transferred from protection sellers to investors. This is costly, however, because investors are less efficient than protection sellers at managing assets. Whatever α, investors' per-unit cost of managing the asset is larger than that of protection sellers: $\psi_I(\alpha) > \psi, \forall\alpha$. When consuming c_I units of the consumption good and exerting effort over α units of asset, an investor obtains utility $v(c_I - \alpha\psi_I(\alpha))$. We assume investors' per-unit cost of handling the asset is non-decreasing, $\psi_I' \geq 0$, and convex, $\psi_I'' \geq 0$. Thus, investors' marginal cost, $\psi_I(\alpha)+\alpha\psi_I'$, is increasing. Yet, we assume it is efficient that investors exert effort even when holding all of the asset: $R - \psi_I(1) \geq \mu R$. We also maintain the following assumption:

$$\psi_I(1) + \psi_I'(1) > \frac{\psi}{1-\mu} > \psi_I(0). \tag{2}$$

As will be seen below, the right inequality in (2) will allow for asset transfers, by making those transfers not too inefficient when α is close to 0. At the same time, the left inequality in (2) will preclude the full transfer of assets ($\alpha = 1$) because this would be too inefficient.[12]

Risk-sharing and moral hazard: Risk-averse protection buyers seek insurance against the risk $\tilde{\theta}$ they hold. When seeking insurance, they can turn to protection sellers or to investors, facing the following trade-off. On the one hand, protection sellers are efficient providers of insurance, as they are risk-neutral, but they have a moral-hazard problem. If they do not exert effort, their asset's payoff can be zero and they cannot make insurance payments to protection buyers. On the other hand, investors are less efficient at managing the productive asset, and also at providing insurance since they are risk-averse. Risk aversion, however, suppresses the moral hazard problem when $v(0)$ is sufficiently low, which we hereafter assume. Under that assumption, threatening risk-averse investors to give them 0 consumption when the asset yields 0 is enough to induce effort (making the zero return on investors' assets an out-of-equilibrium event).[13] Thus, while we need to impose incentive-compatibility constraints for protection sellers, we do not need to do so for investors. Given this trade-off, we study the optimal risk-sharing arrangement between protection buyers, protection sellers and investors.

Sequence of events: Summarising, the sequence of events is as follows:

- At time 0, agents receive their endowments.
- At time 1, first the signal s is observed, then a fraction $\alpha(s)$ of the productive asset can be transferred from protection sellers to investors, and then holders of the productive asset decide whether to exert effort or not.
- At time 2, the output of the assets held by protection buyers, investors and protection sellers is realised and publicly observed, and consumption takes place.

[12] In general, assets could also be transferred to protection buyers. For simplicity we assume this is not possible as protection buyers do not have the technology to manage those assets.

[13] This is the case, for example, if $v(c) = \ln(c)$, since $\ln(c) \to -\infty$, when $c \to 0$.

For given effort decisions, $\tilde{\theta}$ and $\tilde{R}$ are independent. So there is no exogenous correlation between the valuations of the two assets. In spite of this simplifying assumption, we show below that moral hazard creates endogenous positive correlation. Exogenous positive correlation would only reinforce this effect.

2.2 Mapping the model to real markets and institutions

Protection buyers can be, for example, commercial banks, seeking protection against reductions in values of securities they hold. For instance, prior to the 2007-09 crisis, banks frequently bought protection against credit-related losses on corporate loans and mortgages. By buying protection, they were able to reduce or even eliminate regulatory capital requirements for holding the underlying securities under the first Basel Agreement. Indeed, out of $533 billion (net notional amount) of credit default swaps that AIG had outstanding at year-end 2007, 71 percent were categorized as such "Regulatory Capital" contracts (see Harrington, 2009).

Protection sellers can be investment banks or specialised firms, who must exert due dilligence effort to reduce downside risk on the assets they hold. The sellers' assets can be a loan portfolio, in which case the due dilligence effort reducing downside risk can be interpreted as the screening and monitoring of loans. Lack of screening and monitoring leads to a higher risk of losses. For example, the report of the Financial Crisis Inquiry Commission (2011) states that "investors relied blindly on credit rating agencies as their arbiters of risk instead of doing their own due dilligence" and "... Merrill Lynch's top management realized that the company held $55 billion in "super-senior" and supposedly "super-safe" mortgage-related securities that resulted in billions of dollars in losses".

Alternatively, the sellers' assets can be financial securities, and downside risk reduction effort concerns the management of these securities, in terms of collateral, liquidity, and risk profile. For example, as part of its securities-lending activity, AIG received cash-collateral from its counterparties. Instead of holding this collateral in safe and liquid assets, such as Treasury bonds, AIG bought risky illiquid instruments, such as Residential Mortgages Backed Securities. As the value of these securities dropped, this resulted in approximately $21 billion of losses for the company in 2008 (see McDonald and Paulson, 2015). Thus, AIG's strategy can be interpreted as a lack of downside risk-management effort.

Consistent with our assumption that lack of proper risk-management effort increases downside risk, Ellul and Yerramilli (2013) document that banks with a weaker risk-management function at the onset of the financial crisis had higher tail risk and higher nonperforming loans during the financial-crisis years.

3 First best

We begin by characterising the first-best allocation, which provides a useful reference point for the rest of the analysis.

In the first best, effort is observable, and because it is efficient, it is always requested by the planner and implemented by the agents. Hence, the protection sellers' assets always yield R. The state variables, on which decisions and consumptions are contingent, are the publicly observable realisations of the protection buyers' asset (θ) and the signal (s) (for notational simplicity we drop the reference to R).

The social planner chooses the consumptions of protection buyers ($c_B(\theta, s)$), protection sellers ($c_S(\theta, s)$) and investors ($c_I(\theta, s)$), as well as the fraction of protection sellers' assets transferred to investors ($\alpha(s)$), to maximise the expected utility of protection buyers and investors (with respective Pareto weights ω_B and ω_I):

$$\omega_B E[u(c_B(\tilde{\theta}, \tilde{s}))] + \omega_I E[v(c_I(\tilde{\theta}, \tilde{s}) - \alpha(\tilde{s})\psi_I(\alpha))], \tag{3}$$

We assume that the social planner places no weight on protection sellers, i.e., $\omega_S = 0$. Correspondingly, when analysing the market equilibrium, we will assume zero bargaining power for the protection sellers.[14]

The constraints on the planner are the participation constraint of protection buyers,

$$E[u(c_B(\tilde{\theta},\tilde{s}))] \geq E[u(\tilde{\theta})], \tag{4}$$

the participation constraint of investors,

$$E[v(c_I(\tilde{\theta},\tilde{s}) - \alpha(\tilde{s})\psi_I(\alpha))] \geq v(1), \tag{5}$$

the participation constraint of protection sellers,

$$E[c_S(\tilde{\theta},\tilde{s}) - (1-\alpha(\tilde{s}))\psi] \geq R - \psi, \tag{6}$$

the budget constraints in each state,

$$c_B(\theta,s) + c_I(\theta,s) + c_S(\theta,s) \leq \theta + 1 + R, \qquad \forall(\theta,s), \tag{7}$$

and the constraint that $\alpha(s)$ must be between 0 and 1. The participation constraints reflect the respective autarky payoffs of protection buyers ($E[u(\tilde{\theta})]$), investors ($v(1)$) and protection sellers ($R-\psi$).

The following proposition states the solution of the first-best problem.

Proposition 1 *In the first best, there is no transfer of the productive asset, $\alpha(s) = 0, \forall s$, and protection buyers and investors receive constant consumption, $c_B(\theta,s) = c_B, c_I(\theta,s) = c_I$. Their total consumption is*

$$c_B + c_I = E[\tilde{\theta}] + 1, \tag{8}$$

[14] Once we analyze the case when effort is unobservable, protection sellers are agents, while protection buyers are principals. Our assumption that protection buyers have all the bargaining power is in line with the principal-agent literature, in which the principal makes a take-it-or-leave-it offer to the agent.

while protection sellers' consumption is

$$c_S(\theta, s) = \theta - E[\tilde{\theta}] + R, \quad \forall(\theta, s). \tag{9}$$

The first best allocation achieves efficiency both in terms of production and risk-sharing. With respect to production, the productive asset is held entirely by its most efficient holders, the protection sellers, i.e., $\alpha(s) = 0$. With respect to risk-sharing, all risk is borne by protection sellers. The risk-neutral protection sellers fully insure the risk-averse agents, whose consumption is equal to the expected value of their endowment.

The marginal rates of substitution of all agents are equalized in the first best. The consumption of risk-averse protection buyers and investors is the same across all states (θ, s). Their marginal rate of substitution is equal to one, which is also the marginal rate of substitution of risk-neutral protection sellers.

The first best can be decentralised in a competitive market with forward contracts on the realisation θ of the protection buyers' asset. Protection buyers engage in a forward sale of their risky asset to protection sellers at the forward price $F = E[\tilde{\theta}]$. Protection sellers fully insure protection buyers at actuarially fair terms. Investors do not participate in the market. The market equilibrium implements the point on the first-best Pareto frontier such that $c_B = E[\tilde{\theta}]$ and $c_I = 1$. Figure 1 illustrates the market implementation of the first best.

4 Second best

In the second best, protection sellers' effort is unobservable, and there is a moral-hazard problem. The social planner still chooses consumptions and asset transfers to maximise his objective function (3) under participation constraints (4), (5), and (6), and budget constraints (7). However, because effort is unobservable, the planner must also take into account protection sellers' incentive-compatibility constraint.

In what follows, we assume that the first-best allocation is not feasible in this more

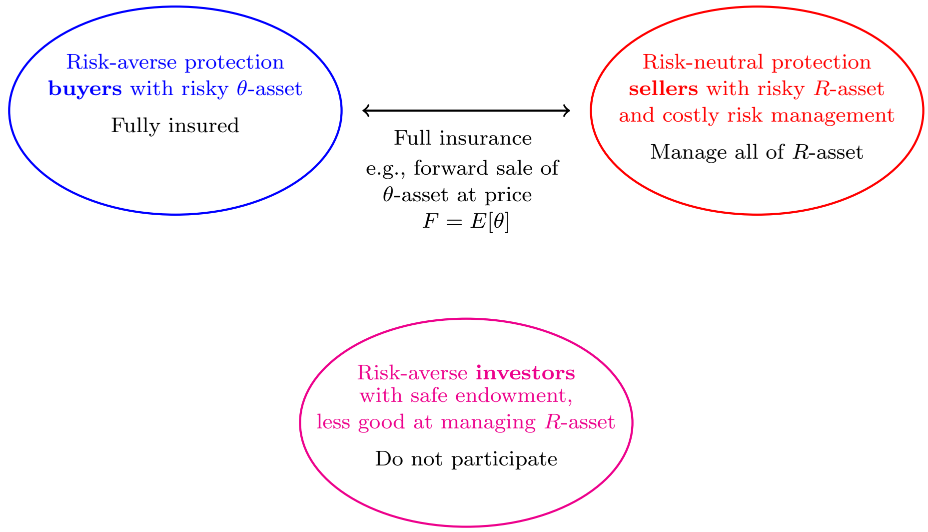

Figure 1: Market implementation of the first best

constrained, second-best problem:

$$\mathcal{P} < E[\tilde{\theta}] - E[\tilde{\theta}|\underline{s}]. \tag{10}$$

If (10) did not hold, the pledgeable return would be sufficiently large for protection sellers to credibly promise full insurance despite the moral-hazard problem.

We first derive the incentive-compatibility constraint for protection sellers. Second, we examine the second-best risk-sharing among agents for a given level of asset transfers $\alpha(s)$. Finally, we characterise the optimal level of the asset transfer.

4.1 Incentive compatibility

Protection sellers decide on their effort after the realisation of the signal. Their incentive-compatibility constraint in state s requires protection sellers to prefer effort to shirking:

$$E[c_S(\tilde{\theta}, s) - (1 - \alpha(s))\psi|s] \geq \mu E[c_S(\tilde{\theta}, s)|s].$$

The left-hand side is protection sellers' (on-equilibrium-path) expected consumption net of the cost of effort. The right-hand side is the (off-equilibrium-path) expected consumption when they shirk. Under shirking, the asset yields R only with probability μ. In this case, protection sellers still receive the same expected consumption as under effort. But with probability $1-\mu$, the asset returns zero. In order to relax the incentive-compatibility constraint, the planner optimally allocates zero consumption to limited-liability protection sellers in the (out-of-equilibrium) event of a zero asset return.

The incentive-compatibility constraint rewrites as

$$E[c_S(\tilde{\theta}, s)|s] \geq (1-\alpha(s))\frac{\psi}{1-\mu}. \tag{11}$$

The left-hand side of (11) is the expected consumption of protection sellers after observing the signal s. Giving more consumption to protection buyers means less consumption for protection sellers, which tightens the incentive-compatibility constraint. The right-hand side is the incentive-adjusted cost of managing the fraction of assets protection sellers still control after a possible transfer. Transferring more assets to investors relaxes the incentive-compatibility constraint.

4.2 Risk-sharing in the second best

Our first result in Lemma 1 is that protection buyers and investors are exposed only to the risk associated with the signal s, which we refer to as signal risk. Because protection buyers and investors bear only signal risk, we write their respective consumptions as $(c_B(\bar{s}), c_B(\underline{s}))$ and $(c_I(\bar{s}), c_I(\underline{s}))$.

Lemma 1 *The consumption of protection buyers and investors depends only on the realisation of the signal s, but not on the realisation θ of protection buyers' assets.*

According to the incentive constraint (11), only the expected consumption of protection sellers conditional on the signal matters for incentives. For a given $E[c_S(\tilde{\theta}, s)|s]$, the split between $c_S(\bar{\theta}, s)$ and $c_S(\underline{\theta}, s)$ does not affect the incentive constraint or the participation constraint of protection sellers. Hence, it is optimal to set $c_S(\bar{\theta}, s)$ and $c_S(\underline{\theta}, s)$ to fully insure protection buyers conditional on the realisation of s by equalising their marginal utility in states $(\bar{\theta}, s)$ and $(\underline{\theta}, s)$. Similarly, it is optimal to equalise investors' marginal utility in these two states.

The next lemma characterizes the second-best outcome further. No gains from trade are left unexploited and protection sellers cannot make losses after both good and bad signals.

Lemma 2 *In the second best, the resource constraint as well as the participation constraint of protection sellers bind. Moreover, one, and only one, of the two incentive-compatibility conditions (after $\bar{s}$ or $\underline{s}$) binds.*

Building on the above results, we can now state the following important characteristics on the second-best outcome:

Lemma 3 *After a good signal, the incentive-compatibility constraint of protection sellers is slack and there is no asset transfer, $\alpha(\bar{s}) = 0$. After a bad signal, the incentive-compatibility constraint of protection sellers binds. Moreover, the consumption of protection buyers is larger after a good signal than after a bad signal, $c_B(\bar{s}) > c_B(\underline{s})$.*

After a good signal, protection sellers' expected consumption is large, which relaxes their incentive constraint. As a result, there is no need to transfer protection sellers' assets to less efficient investors ($\alpha(\bar{s}) = 0$).

After a bad signal, the opposite happens. Protection sellers' expected consumption is low, which tightens their incentive constraint. Because of the binding incentive constraint, protection buyers cannot be fully insured and remain exposed to signal risk ($c_B(\bar{s}) > c_B(\underline{s})$).

In the next proposition, we use Lemmas 1, 2, and 3 to characterise the consumption of protection buyers and investors for a given level of asset transfers after a bad signal $\alpha(\underline{s})$.

Proposition 2 *After a bad signal, the total consumption of protection buyers and investors is*

$$c_B(\underline{s}) + c_I(\underline{s}) = 1 + E[\tilde{\theta}|\underline{s}] + \alpha(\underline{s})R + (1-\alpha(\underline{s}))\mathcal{P}, \tag{12}$$

while after a good signal it is

$$c_B(\bar{s}) + c_I(\bar{s}) = 1 + E[\tilde{\theta}|\bar{s}] - \frac{Pr[\underline{s}]}{Pr[\bar{s}]}[\alpha(\underline{s})(R-\psi) + (1-\alpha(\underline{s}))\mathcal{P}]. \tag{13}$$

Signal risk is perfectly shared between protection buyers and investors

$$\frac{v'(c_I(\underline{s}) - \alpha(\underline{s})\psi_I(\underline{s}))}{v'(c_I(\bar{s}))} = \frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}. \tag{14}$$

Finally, the split of consumption between protection buyers and investors reflects their Pareto weights:

$$\frac{u'(c_B(s))}{v'(c_I(s) - \alpha(s)\psi_I(\alpha(s)))} = \frac{\omega_I + \lambda_I}{\omega_B + \lambda_B}, \tag{15}$$

where λ_B and λ_I are the respective Lagrange multipliers of the participation constraint of protection buyers (4) *and investors* (5)*.*

We obtain Proposition 2 by combining the lemmas as follows. First, full risk-sharing conditional on the signal (Lemma 1) and the binding resource constraints (Lemma 2) together imply that protection sellers' consumption is given by $c_S(\theta, s) = \theta + 1 + R - (c_B(s) + c_I(s))$. Taking the expectation over θ conditional on the signal yields

$$E[c_S(\tilde{\theta}, s)|\underline{s}] = E[\tilde{\theta}|\underline{s}] + 1 + R - (c_B(\underline{s}) + c_I(\underline{s}))$$
$$E[c_S(\tilde{\theta}, s)|\bar{s}] = E[\tilde{\theta}|\bar{s}] + 1 + R - (c_B(\bar{s}) + c_I(\bar{s})).$$

Using this together with the binding participation and incentive constraint of protection sellers (Lemmas 2 and 3), we obtain the total consumption of the protection buyers and investors in (12) and (13).

Second, protection buyers and investors - who are only exposed to signal risk (Lemma 1) - share this risk perfectly (equation (14)), because there is no information or incentive problem among them.

As long as the participation constraints of protection buyers and investors do not bind (so that $\lambda_I = \lambda_B = 0$), the optimal allocation (for a given level of asset transfers, $\alpha(\underline{s})$) is fully characterised by four variables: $c_B(\underline{s})$, $c_B(\bar{s})$, $c_I(\underline{s})$, and $c_I(\bar{s})$. These four variables are pinned down by the four equations in Proposition 2.

To interpret the total consumption of protection buyers and investors in the second best, it is useful to compare (12) and (13), to their counterparts in the first best:

$$\left\{c_B(\underline{s}) + c_I(\underline{s}) = 1 + E[\tilde{\theta}],\ c_B(\bar{s}) + c_I(\bar{s}) = 1 + E[\tilde{\theta}]\right\},$$

and their counterparts assuming no asset transfers

$$\left\{c_B(\underline{s}) + c_I(\underline{s}) = 1 + E[\tilde{\theta}|\underline{s}] + \mathcal{P},\ c_B(\bar{s}) + c_I(\bar{s}) = 1 + E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P}\right\}. \tag{16}$$

While in the first best, the joint consumption of protection buyers and investors is given by the unconditional expectation of their joint endowment – reflecting full insurance –, the conditional expectation of their joint endowment ($1 + E[\tilde{\theta}|s]$) appears in the second best. This reflects protection buyers' and investors' exposure to signal risk, which is mitigated by pledgeable income. After bad news the total consumption of protection buyers and investors is increased by $\mathcal{P}$, and to ensure that protection sellers break even, their total consumption after good news is decreased by $\frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P}$ (assuming no asset transfers). Note that under assumption (10), relying on pledgeable income is not sufficient to restore the first best since (10) implies that

$$1 + E[\tilde{\theta}|\underline{s}] + \mathcal{P} < 1 + E[\tilde{\theta}].$$

When asset transfers are not used, protection sellers can only pledge to pay $\mathcal{P}$ after bad news. Asking them to make larger insurance payments would jeopardise their incentives to exert effort. Transferring assets from protection sellers to investors allows to provide more insurance. Comparing (16) to (12), one can see that, with asset transfers, an additional payment $\alpha(\underline{s})(R - \mathcal{P})$ can be promised after bad news.

4.3 Optimal asset transfers after bad news

To complete the analysis of the second best, it remains to characterise the asset transfers after bad news. To do so, we build on the above analysis and obtain our next proposition:

Proposition 3 *If*

$$\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}\Big|_{\alpha(\underline{s})=0} > \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - \psi_I(0)}, \tag{17}$$

then, in the second best, the asset transfer is interior, $\alpha(\underline{s}) \in (0, 1)$, *and such that*

$$\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))} = \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - (\psi_I(\alpha(\underline{s})) + \alpha(\underline{s})\psi_I'(\alpha(\underline{s})))}, \tag{18}$$

where $c_B(\underline{s})$ *and* $c_B(\bar{s})$ *are as given in Proposition 2. Otherwise,* $\alpha(\underline{s}) = 0$.

To interpret the left-hand sides of (17) and (18), recall that Lemma 3 implies there is signal risk: $c_B(\underline{s}) < c_B(\bar{s})$, which, in turn, implies the marginal rate of substitution, $\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}$, is larger than 1. The worse is the insurance, the larger is this marginal rate of substitution. Thus, the marginal rate of substitution on the left-hand-sides of (17) and (18) reflects the marginal benefit of an increase in insurance.

While the left-hand sides of (17) and (18) reflect the preferences of protection buyers, the right-hand sides of (17) and (18) reflect the technology and incentives of those who hold the productive asset. The denominator of the right-hand side of (17) and (18) is the wedge between the productive asset's marginal pledgeable income when it is held by investors $(R - (\psi_I(\alpha(\underline{s})) + \alpha(\underline{s})\psi_I'(\alpha(\underline{s}))))$ and its counterpart when it is held by protection sellers $(\mathcal{P})$. Thus, it measures how much more income one can pledge by transferring the productive asset from protection sellers to investors. The numerator is a similar wedge between the pledgeable income in the first best $(R-\psi)$ and its counterpart under moral hazard $(\mathcal{P})$. Thus, the right-hand-sides of (17) and (18) can be interpreted as a marginal rate of transformation, reflecting the marginal cost of an increase in incentive-compatible insurance.

Condition (17) means that, at $\alpha(\underline{s}) = 0$, the marginal benefit of a small asset transfer exceeds its marginal cost. Since $\psi_I' \geq 0$ and $\psi_I'' \geq 0$, the marginal cost of effort for investors $\psi_I(\alpha(\underline{s})) + \alpha(\underline{s})\psi_I'(\alpha(\underline{s}))$ is increasing. So the right-hand-side of (18) is increasing, and takes its minimum value at $\alpha(\underline{s}) = 0$, as in the right-hand-side of (17). Furthermore, by (2), there exists threshold $\hat{\alpha} < 1$ at which the right-hand-side of (18) goes to infinity. Hence, under (17), there exists an interior value of $\alpha(\underline{s}) \in (0, \hat{\alpha})$ for which the marginal benefit of additional insurance is equal to its marginal cost. This pins down the optimal asset transfer after bad news in the second best.

Figure 2 illustrates the interaction of protection buyers, protections sellers, and investors in the second best when there are asset transfers after bad news.

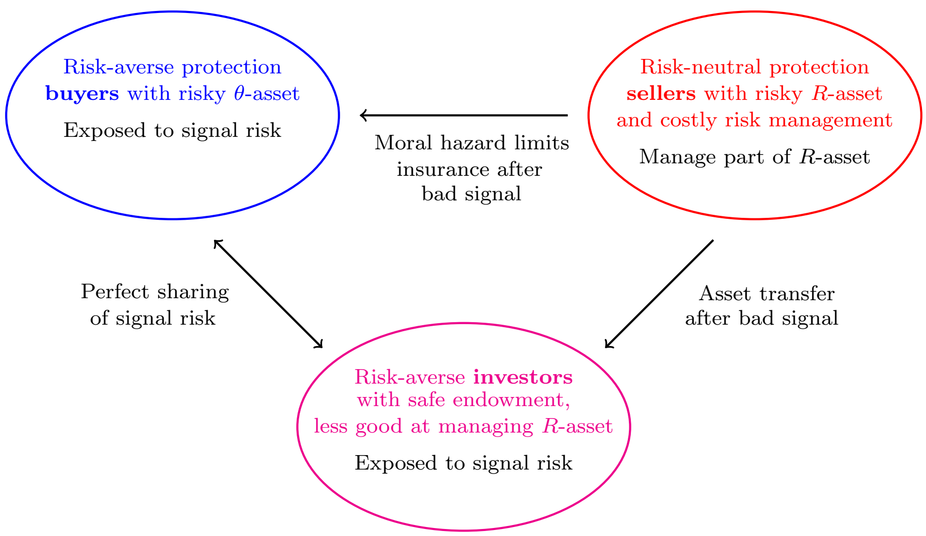

Figure 2: Second-best allocation with asset transfers after a bad signal

5 Market equilibrium

We now turn from the second best to the market equilibrium. We first consider complete markets and study whether the market equilibrium is information-constrained efficient. Then we analyse the incomplete-markets case.

The market is complete when all participants can trade contracts contingent on all the realisations of the publicly observed variables: the signal ($\bar{s}$ or $\underline{s}$ at time 1), the protection buyers' asset value ($\bar{\theta}$ or $\underline{\theta}$ at time 2), and the protection sellers' output (R or 0 at time 2). As will be clear below, the following three markets are sufficient for market completeness:

Market for insurance against the realization of $\tilde{\theta}$: Protection buyers and protection sellers participate in this market at time 0. In line with our simplifying assumption that the social planner places no weight on protection sellers, we assume that in this insurance market protection buyers have all the bargaining power (so protection sellers are held to their reservation utility). Each protection buyer is matched with one protection seller and makes him an exclusive take-it-or-leave-it offer.[15] The offer includes a set of time 2 transfers, $\tau(\theta, s, R)$ together with variation margins as we explain below. A positive transfer $\tau(\theta, s, R) > 0$ denotes a payment from the seller to the buyer and vice versa.

Market for protection sellers' assets: In the previous section we showed that the second best can involve asset transfers from protection sellers to investors after bad news. Therefore, in the market equilibrium, we allow the protection buyer to request his counterparty to sell a fraction $\alpha_S \geq 0$ of his assets after a bad signal. The asset sale occurs at time 1, after the realisation of the signal, and before effort is exerted. The price is denoted by p. While supply α_S stems from protection sellers, demand α_I stems from investors. All participants in the asset market are competitive.

The proceeds $\alpha_S p$ belong to protection sellers but are put on an escrow account, in which

[15] For an analysis of issues arising with non-exclusive contracting, see Acharya and Bisin (2014).

they are ring-fenced from moral hazard and can be used to pay protection buyers at time 2. Thus the request to sell a fraction α_S of the asset can be interpreted as a margin call, and the proceeds $\alpha_S p$ as the margin deposited in a margin account.

Market for insurance against signal risk: When effort is observable, this market is not needed, as protection sellers can fully insure protection buyers against the risk associated with their endowment $\tilde{\theta}$. With full insurance protection buyers are not exposed to signal risk. In contrast, as shown in the previous section, moral hazard limits the extent to which protection sellers can insure protection buyers, leaving them exposed to signal risk. This opens the scope for signal risk-sharing between protection buyers and investors. The corresponding market is held at time 0, and enables participants to exchange consumption after a bad signal against consumption after a good signal. Owners of one unit of the contract receive q units of consumption good after a bad signal and pay 1 unit of consumption good after a good signal. We denote protection buyers' demand by x_B and investors' supply by x_I.

Equilibrium: Equilibrium in these three markets consists of transfers $\tau(\theta, s, R)$, prices (p, q), and trades (α_S, α_I) and (x_B, x_I), such that all participants behave optimally and markets clear: $\alpha_I = \alpha_S$ and $x^d = x^s$. To solve for equilibrium, we take the following steps. First, we derive the incentive and participation constraints of protection sellers when there are these three markets. Second, we characterize investors' optimal trading decisions, α_I and x_I, for given prices. Third, we analyse contracting between protection buyers and protection sellers. Fourth, we impose market clearing in the market for insurance against signal risk and in the market for protection sellers' assets.

5.1 Protection sellers' incentive and participation constraints

Incentive compatibility: As in the second best, it can be shown that the incentive-compatibility condition of protection sellers after a good signal is slack. After a bad signal

($\underline{s}$), the incentive-compatibility condition under which protection sellers exert effort is

$$(1-\alpha_S)(R-\psi)+\alpha_S p - E[\tau(\tilde{\theta},\tilde{s},R))|\underline{s}] \geq \mu((1-\alpha_S)R+\alpha_S p - E[\tau(\tilde{\theta},\tilde{s},R))|\underline{s}]) + (1-\mu)E[\max[\alpha_S p - \tau(\tilde{\theta},\tilde{s},0),0]|\underline{s}]. \quad (19)$$

The left-hand side of (19) is the expected gain of protection sellers on the equilibrium path: They exert effort and obtain $R-\psi$ for each of the $1-\alpha_S$ units of the productive asset they keep. In addition, protection sellers own the proceeds from the asset sale, $\alpha_S p$, deposited in the margin account. Finally, the expected net payment by protection sellers to protection buyers is $E[\tau(\tilde{\theta},\tilde{s},R))|\underline{s}]$.

The right-hand side of (19) is the expected profit of protection sellers if they deviate and do not exert effort. In that case, with probability μ, protection sellers' productive assets still generate R, and their expected gain is the same as on the equilibrium path, except that the cost of effort, $(1-\alpha_S)\psi$, is not incurred. With probability $1-\mu$, the productive assets held by protection sellers generate no output. In that case, because of limited liability, protection sellers cannot pay more than $\alpha_S p$. Hence their gain is $\max[\alpha_S p - \tau(\theta,s,0),0]$.

It is optimal to set $\tau(\theta,s,0)=\alpha_S p$. This relaxes the incentive constraint by reducing the right-hand side of (19), and does not affect the rest of the analysis because transfers $\tau(\theta,s,0)$ only occur off the equilibrium path. Protection sellers' incentive constraint thus reduces to

$$\alpha_S p + (1-\alpha_S)\mathcal{P} \geq E[\tau(\tilde{\theta},\tilde{s})|\underline{s}] \quad (20)$$

where we write $\tau(\tilde{\theta},\tilde{s},R)=\tau(\tilde{\theta},\tilde{s})$ to simplify the notation. The right-hand side of (20) is how much protection sellers expect to pay protection buyers, which can be interpreted as the implicit debt of protection sellers. The left-hand side of (20) is how much protection sellers can credibly pledge to pay. This is equal to the sum of i) the proceeds from the asset sale deposited on the margin account ($\alpha_S p$), which are fully pledgeable, and ii) the pledgeable part ($\mathcal{P}$) of the output (R) obtained on the $1-\alpha_S$ units of the productive assets kept by protection sellers.

Participation constraint: A protection seller accepts the contract if it gives him equilibrium expected gains no smaller than his autarky payoff, i.e., if

$$\Pr[\bar{s}](R-\psi)+\Pr[\underline{s}]((1-\alpha_S)(R-\psi)+\alpha_S p)-E[\tau(\tilde{\theta},\tilde{s})]\geq R-\psi. \tag{21}$$

5.2 Investors' optimal trades

When selling x_I units of the insurance contract against signal risk and buying α_I units of protection sellers' assets, investors obtain time 2 consumption equal to $1+x_I$ after a good signal and $1-qx_I+\alpha_I(R-\psi_I(\alpha_I)-p)$ after a bad signal. Hence, investors' expected utility at $t=0$ is

$$\Pr[\bar{s}]v(1+x_I)+\Pr[\underline{s}]v(1-qx_I+\alpha_I(R-\psi_I(\alpha_I)-p)). \tag{22}$$

Investors' supply of insurance against signal risk: At time 0, investors choose x_I to maximise (22). The first-order condition is[16]

$$\Pr[\bar{s}]v'(1+x_I)=\Pr[\underline{s}]qv'(1-qx_I+\alpha_I(R-\psi_I(\alpha_I)-p)), \tag{23}$$

which implies that x_I decreases in q.[17] Equation (23) rewrites as

$$q=\frac{\Pr[\bar{s}]}{\Pr[\underline{s}]}\frac{v'(1+x_I)}{v'(1-qx_I+\alpha_I(R-\psi_I(\alpha_I)-p))}, \tag{24}$$

which states that the price of insurance against signal risk is equal to the probability-weighted marginal rate of substitution between consumption after good and bad news.

[16]The second-order condition $\Pr[\bar{s}]v''(1+x_I)+q^2\Pr[\underline{s}]v''(1-qx_I+\alpha_I(R-\psi_I(\alpha_I)-p))<0$ holds by the concavity of the utility function.

[17]The left-hand side of (23) is decreasing in x_I, while the right-hand side is increasing in x_I. Their intersection pins down the optimal supply of insurance by investors, x_I. Now, the right-hand side is increasing in q. Thus, an increase in q shifts up the right-hand side of (23), which leads to an intersection between the right- and the left-hand sides of (23) at a lower value of x_I.

Investors' demand for protection sellers' assets: At time 1, after a bad signal, investors choose α_I to maximise their utility $v(1 - qx_I + \alpha_I(R - \psi_I(\alpha_I) - p))$. When $p \geq R - \psi_I(0)$, the price of the asset is so high that investors' demand is 0. Otherwise, their demand is pinned down by the first-order condition:

$$p = R - \left[\psi_I(\alpha_I) + \alpha_I \psi_I'(\alpha_I)\right], \tag{25}$$

which states that the price is equal to the marginal valuation of the investor for the asset. Because the marginal cost $\psi_I(\alpha_I) + \alpha_I\psi_I'(\alpha_I)$ is increasing, (25) implies that investors' demand for the asset is decreasing in p.[18]

5.3 Contracting between protection buyers and sellers

Protection buyers choose a privately-optimal contract specifying transfers $\tau(\theta, s)$ and a sale of the productive asset α_S, and demands x_B units of the insurance against signal risk. The latter generates positive transfers to the protection buyer after bad news, qx_B, and negative transfers after good news, $-x_B$. Correspondingly, the consumption of protection buyers at time 2 is $\theta + \tau(\theta, \bar{s}) - x_B$ after a good signal and $\theta + \tau(\theta, \underline{s}) + qx_B$ after a bad signal. The program of protection buyers is to choose x_B, $\tau(\theta, s)$ (for all $\theta \in \{\underline{\theta}, \bar{\theta}\}$ and $s \in \{\underline{s}, \bar{s}\}$), as well as $\alpha_S \in [0, 1]$ to maximise

$$Pr[\bar{s}]E[u(\tilde{\theta} + \tau(\tilde{\theta}, \tilde{s}) - x_B)|\bar{s}] + Pr[\underline{s}]E[u(\tilde{\theta} + \tau(\tilde{\theta}, \tilde{s}) + qx_B)|\underline{s}], \tag{26}$$

subject to the protection seller's incentive and participation constraints, (20) and (21). The next lemma states protection buyers' consumption under optimal transfers $\tau(\theta, s)$ as a function of the asset sale α_S.

Lemma 4 *In equilibrium, in the privately-optimal contract between protection buyers and*

[18]Increasing marginal cost also implies the second-order condition holds.

protection sellers, protection sellers' participation and incentive constraints bind. Moreover, protection buyers receive full insurance conditional on the signal, i.e., for a given realisation of the signal, their consumption does not depend on the realisation of $\tilde{\theta}$:

$$c_B(\bar{\theta}, \bar{s}) = c_B(\underline{\theta}, \bar{s}) = E[\tilde{\theta}|\bar{s}] - \frac{Pr[\underline{s}]}{Pr[\bar{s}]}[\alpha_S(R-\psi) + (1-\alpha_S)\mathcal{P}] - x_B, \tag{27}$$

$$c_B(\bar{\theta}, \underline{s}) = c_B(\underline{\theta}, \underline{s}) = E[\tilde{\theta}|\underline{s}] + \alpha_S p + (1-\alpha_S)\mathcal{P} + qx_B. \tag{28}$$

Lemma 4 is similar to Lemma 1. Both in the second best and in the market equilibrium, protection buyers are fully insured conditional on the signal, and the economic intuition is the same in the two cases.

The next lemma states what fraction of their assets protection sellers are required to sell after bad news.

Lemma 5 *When*

$$p \leq \mathcal{P} + (R - \psi - \mathcal{P})\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{Pr[\underline{s}]}{Pr[\bar{s}]}\mathcal{P} - x_B\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx_B)} \tag{29}$$

then $\alpha_S = 0$, otherwise α_S is strictly positive and such that

$$\frac{u'(E[\tilde{\theta}|\underline{s}] + \alpha_S p + (1-\alpha_S)\mathcal{P} + qx_B)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{Pr[\underline{s}]}{Pr[\bar{s}]}[\alpha_S(R-\psi) + (1-\alpha_S)\mathcal{P}] - x_B\right)} = \frac{\lambda_1}{(p-\mathcal{P})Pr[\underline{s}]\lambda_S} + \frac{\frac{\psi}{1-\mu} - \psi}{p - \mathcal{P}}, \tag{30}$$

where λ_1 is the Lagrange multiplier of the constraint $\alpha_S \leq 1$.

Equation (30) is similar to (18). In both cases, the left-hand side is protection buyers' marginal rate of substitution between consumption after a bad signal and after a good signal, and therefore reflects the marginal benefit of an increase in insurance. In (18), the right-hand side involves the ratio

$$\frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - (\psi_I(\alpha(\underline{s})) + \alpha(\underline{s})\psi_I'(\alpha(\underline{s})))},$$

while in (30) the corresponding ratio is

$$\frac{\frac{\psi}{1-\mu} - \psi}{p - \mathcal{P}}.$$

In both ratios the numerator is the same, and the denominator reflects how much more income can pledged for insurance by transferring the productive asset from protection sellers to investors.

5.4 Equilibrium

Equilibrium in the market for insurance against signal risk: Taking the first-order condition with respect to x_B in (26), protection buyers' trade in that market is x_B such that

$$q = \frac{\Pr[\bar{s}]}{\Pr[\underline{s}]} \frac{u'(\theta + \tau(\theta, \bar{s}) - x_B)}{u'(\theta + \tau(\theta, \underline{s}) + qx_B)}. \tag{31}$$

Since the right-hand side of (31) is increasing in x_B, (31) implies x_B is increasing in q, while (24) implies that x_I decreases in q. At equilibrium, q is such that $x_B = x_I$. Combining (24) and (31), we obtain our next proposition:

Proposition 4 *Equilibrium in the market for insurance against signal risk involves price q^* and trading volume x^* such that*

$$q^* = \frac{Pr[\bar{s}]}{Pr[\underline{s}]} \frac{v'(1+x^*)}{v'(1 - q^*x^* + \alpha_I(R - \psi_I(\alpha_I) - p))} = \frac{Pr[\bar{s}]}{Pr[\underline{s}]} \frac{u'(\theta + \tau(\theta, \bar{s}) - x^*)}{u'(\theta + \tau(\theta, \underline{s}) + q^*x^*)}. \tag{32}$$

Equation (32) states that in equilibrium, the marginal rates of substitution between consumption after a bad signal and after a good signal is equated among protection buyers and investors, i.e., they share risk optimally, as in the second best (see Proposition 2). Moreover, this marginal rate of substitution (weighted by the probabilities of a good and a bad signal) is equal to the price of insurance against signal risk.

As long as protection buyers are exposed to signal risk, we have

$$\frac{u'(\theta + \tau(\theta, \bar{s}) - x^*)}{u'(\theta + \tau(\theta, \underline{s}) + qx^*)} < 1,$$

which, combined with (32), implies that insurance against signal risk is not actuarially fair. Investors who supply protection buyers with insurance against a bad signal earn profits on average. This, in turn, means that investors' equilibrium supply is strictly positive. Thus, the market for insurance against signal risk is active, i.e., $x^* > 0$. Protection buyers - who cannot get full insurance from protection sellers because of moral hazard - demand strictly positive amount of additional insurance from investors.

Equilibrium in the market for protection sellers' assets: Given equilibrium (q^*, x^*) in the market for insurance against signal risk, equilibrium in the market for protection sellers' assets is defined by a price p^* and a trading volume α^*, such that the market clears, i.e., $\alpha_S(p^*) = \alpha_I(p^*) = \alpha^*$. The next proposition characterises equilibrium in the market for protection sellers' assets.

Proposition 5 *If*

$$\frac{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^*)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{Pr[\underline{s}]}{Pr[\bar{s}]}\mathcal{P} - x^*\right)} > \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - \psi_I(0)}, \tag{33}$$

the equilibrium level of asset sales α^* is strictly positive and such that

$$\frac{u'(E[\tilde{\theta}|\underline{s}] + \alpha^* p + (1-\alpha^*)\mathcal{P} + qx^*)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{Pr[\underline{s}]}{Pr[\bar{s}]}[\alpha^*(R-\psi) + (1-\alpha^*)\mathcal{P}] - x^*\right)} = \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - (\psi_I(\alpha^*) + \alpha^*\psi_I'(\alpha^*))}, \tag{34}$$

while the market price of protection sellers' assets is:

$$p^* = R - [\psi_I(\alpha^*) + \alpha_I \psi_I'(\alpha^*)] \,. \tag{35}$$

Otherwise, if (33) does not hold, there are no asset sales in equilibrium, i.e., $\alpha^* = 0$.

5.5 Implementation

In this subsection, we discuss realistic trading strategies that implement the optimal contract between protection buyers and protection sellers. We discuss how they affect the assets and liabilities of protection sellers and how variation margins operate in this context.

Trades: As in the first best, protection sellers buy the asset of protection buyers ($\tilde{\theta}$) forward, at an actuarially fair price ($F = E[\tilde{\theta}]$). With moral hazard, this simple transaction needs to be complemented with a more complex one. Protection sellers and protection buyers engage in a derivative contract, whose payoffs are contingent on the realisation of the signal (s). After bad news ($\underline{s}$), protection buyers pay protection sellers

$$E[\tilde{\theta}] - E[\tilde{\theta}|\underline{s}] - (\alpha p + (1-\alpha)\mathcal{P}) .$$

After good news ($\bar{s}$), protection sellers pay protection buyers

$$E[\tilde{\theta}|\bar{s}] - E[\tilde{\theta}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]} (\alpha(R-\psi) + (1-\alpha)\mathcal{P}) .$$

Variation margins: After good news, the expected value of the total trading position of protection sellers (including the simple forward and the more complex derivative) is positive, and there are no margin calls. After bad news, the expected value of the total trading position of protection sellers is negative and variation margins are called. That the margin call reflects the value of the total derivative position can be interpreted as portfolio margining. The variation-margin call requests that protection sellers deposit safe assets in a margin account. Since at this point protection sellers do not hold safe assets, they must liquidate a fraction α of their productive assets (e.g., a portfolio of loans) at unit price p, and transfer the proceeds αp to the margin account. Although deposited in the margin account, these safe assets are still owned by protection sellers. Therefore the margin deposit still shows up on the asset side of their balance sheet. This is in line with the remark of McDonald and

Paulson (2015, page 92):

> "this transfer of funds based on a market value change is classified as a change in collateral and not as payment."

Economic balance sheet: Based on this discussion, we can draw the balance sheet of protection sellers at time 1 (Figure 3). This is not an accounting balance sheet, but an economic one, showing the value of all the assets and liabilities of protection sellers, including those corresponding to derivative positions.

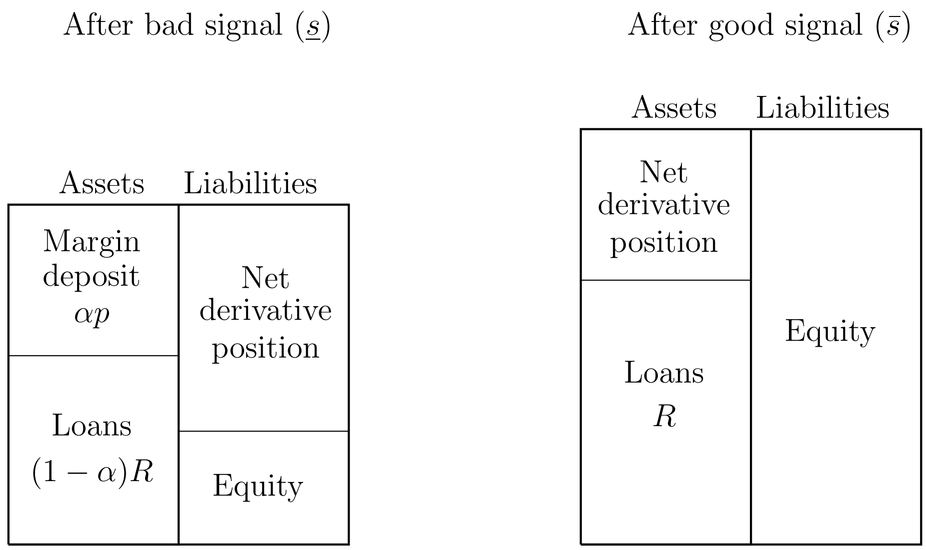

Figure 3: Balance sheet of protection sellers at $t = 1$

After good news, the total derivative position of protection sellers is expected to make a positive profit equal to $\frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\left(\alpha(R-\psi)+(1-\alpha)\mathcal{P}\right)$. This "marked-to-market" net position value is on the asset side of the balance sheet of protection sellers along with the portfolio of loans under management (which will, on the equilibrium path, return R at time 2 since the protection seller will exert effort). On the other side of the balance sheet, there is just the equity of protection sellers. After bad news, the total derivative position of protection sellers is expected to be loss-making, which triggers the variation-margin call. Correspondingly, on the asset side of the balance sheet, there is the margin deposit (αp) and the downsized portfolio of loans, $(1-\alpha)R$. On the other side of the balance sheet, there is the liability corresponding to the net value of the derivative position $\alpha p + (1-\alpha)\mathcal{P}$ and the lower equity of protection sellers.

Thus, after bad news, the assets in the margin account, showing up on the asset side of the balance sheet, serve as collateral for the liability generated by the net derivative position. In that sense we offer an *asset-side* view of variation margins, which contrasts with previous analyses of initial margins, which are a *liability-side* phenomenon (initial margins, or collateral, determine the size of loans). Our approach gives rise to new empirical and policy implications, which we discuss in section 6.

5.6 Equilibrium constrained efficiency

Comparing Lemma 4 and Propositions 4 and 5 to Propositions 2 and 3, we analyse the efficiency of market equilibria.

Proposition 6 *Market equilibrium is information-constrained Pareto efficient.*

It is striking that, in spite of moral hazard, equilibrium is constrained efficient, all the more so given that the price in the incentive constraint (20) induces pecuniary externalities.

Proposition 6 reflects the presence of two countervailing pecuniary externalities. When one protection buyer demands larger margins, this depresses the price, which tightens the incentive constraint of all protection sellers. This negative pecuniary externality tends to increase the amount of signal risk protection buyers must bear.

There is, however, a countervailing, stabilising effect. The decline in the price increases the profits of investors after a bad signal. Thus, while a negative signal is a negative shock for protection buyers, it is a positive shock for investors. This creates scope for risk-sharing gains from trade between investors and protection buyers. When the market is complete, investors and protection buyers fully exploit this risk-sharing opportunity until their marginal rates of substitution are equalised, exactly as in the second best.

5.7 Incomplete markets

In the above complete market analysis investors participate in the market at time 1, buying in the fire sale triggered by a bad signal, and also at time 0, providing insurance against bad news at time 1. It is possible, however, that in practice investors are not fully aware at time 0 of the possible occurrence of fire sales at time 1. In that case, they would not provide insurance against signal risk at time 0, which constitutes a form of market incompleteness.

To analyse that case, we study the market equilibrium with the constraint $x_B = x_I = 0$. Proceeding along similar lines as with complete markets, one can show that equilibrium in this incomplete-market case is as follows. In the privately-optimal contract between protection buyers and protection sellers, protection sellers' participation and incentive constraints bind, and protection buyers receive full insurance conditional on the signal. The consumption of protection buyers after bad news and after good news is as in Lemma 4, except that x_B is set to 0. Similarly, the condition for asset sales is the same as (33), with $x^* = 0$. When condition (33) does not hold, there are no asset sales in the incomplete-market case, $\alpha^{IM} = 0$. When condition (33) holds, α^{IM} is strictly positive and pinned down by the same equation as (34) with x^* set to 0. Finally, the equilibrium price is as in (35) and investors' consumption net of cost is equal to $1 + \alpha^{IM}(R - \psi_I(\alpha^{IM}) - p^{IM})$ after a bad signal and equation to 1 after a good signal.

The main difference between this incomplete-market setting and the complete-market setting is that protection buyers and investors cannot share risk and end up with different marginal rates of substitution. Correspondingly, equilibrium with incomplete markets is Pareto dominated by equilibrium with complete markets.

What are the consequences of market participants' inability to trade insurance against signal risk at time 0? In that case, protection buyers cannot purchase insurance against signal risk from investors. To make up for that risk exposure, protection buyers request larger variation-margin calls from protection sellers in order to increase the amount of insurance they can obtain from them. This leads to our next result.

Proposition 7 *Either $\alpha^{IM} = \alpha^* = 0$, so that there are no variation-margin calls irrespective of whether the market is complete or not, or $\alpha^{IM} > 0$, in which case margin calls are larger and the price of protection sellers' assets is lower when markets are incomplete than when they are complete.*

Proposition 7 implies that, because of market incompleteness, variation-margin calls are inefficiently high and the price for protection sellers' assets inefficiently low. That is, market incompleteness leads to inefficient fire sales.

In our simple general equilibrium context, there are interactions between markets. The ability of protection buyers and investors to trade insurance against signal risk in one market reduces the need to sell protection sellers' assets in another market. When the insurance market does not exist, this depresses prices in the asset market.

While fire sales are a symptom of the inefficiency induced by market incompleteness, they are not a necessary condition for inefficiency. Even when $\alpha^{IM} = 0$, market incompleteness prevents the equalisation of the marginal rates of substitution of protection buyers and investors, leading to an information-constrained inefficient allocation of risks.

6 Implications

6.1 Positive implications

Variation margins: In practice, variation margins are called when a position's expected losses increase. The debt-overhang effect identified by our model provides a rationale for this observation. In our model, bad news increase the expected loss for protection sellers and create a debt-overhang problem. In the optimal contract, variation margins are called to mitigate that problem. Note also that the equity of protection sellers is lower after bad news than after good news. Variation margins are called when the equity of protection sellers drops, which is in line with stylised facts.

To further illustrate the empirical implications of our model for variation-margin calls, it is useful to consider the case in which protection buyers and investors have power utility functions and investors' unit cost ψ_I is linear. As shown in Appendix B, the optimal (interior) margin call α then solves

$$\frac{1 + E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[-(1-\alpha)\left(\frac{\psi}{1-\mu} - \psi\right) + R - \psi]}{1 + E[\tilde{\theta}|\underline{s}] + (1-\alpha)\frac{\psi}{1-\mu} + R} = \left(\frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - (\psi + \delta_0 + 2\delta_1\alpha)}\right)^{\frac{1}{\gamma}}.$$

The above equation implies that variation-margin calls become larger as the volatility of the underlying asset increases, leading to a larger wedge between $E[\tilde{\theta}|\bar{s}]$ and $E[\tilde{\theta}|\underline{s}]$. This is also in line with stylised facts.

The above equation also implies margin calls increase when the protection seller's return R decreases. Empirically, this means that margin calls should increase following negative shocks to financial intermediaries' profitability. The evidence reported by McDonald and Paulson (2015) confirms this implication. In the recent financial crisis, credit ratings downgrades of protection sellers increased their variation-margins requirements:

> "AIG would make collateral payments only if the decline in the value of the insured assets exceeded some predefined threshold. These thresholds often depended on AIG's credit ratings... Goldman Sachs had 44 transactions with AIG, with a total notional value of \$17.09 billion. The threshold (level of market value change required to trigger a payment) was "4% as long as AIGFP is rated in the AA/Aa category"." McDonald and Paulson (2015, p. 93)

Contagion between asset classes: When bad news ($\underline{s}$) are publicly observed, this lowers the conditional expectation of the final value of protection buyers' assets (to $E[\tilde{\theta}|\underline{s}]$). In the first best, there is no simultaneous change in the valuation of protection sellers' assets. These assets remain in the hands of their most efficient holders, who exert effort, and value each unit of asset at $R - \psi$. The situation is different with moral hazard. The optimal contract between protection buyers and protection sellers requests variation-margin calls after bad news. The corresponding asset sales α then lower the price of protection sellers' assets to

$$p = R - (\psi_I(\alpha) + \alpha\psi_I'(\alpha) < R - \psi.$$

Thus, moral hazard generates an endogenous correlation between protection buyers' and protection sellers' assets. This can be interpreted as contagion after bad news, and is in line with the empirically observed increase in correlation during bear markets (see, e.g., Ang and Chen, 2002).

Price of protection: An increase in variation margin calls α generates productive inefficiencies by reducing the return on protection sellers' assets. The break-even constraint of protection sellers then requires them to make lower average transfers to protection buyers. Empirically, this means that the price of protection, proxied for example by CDS spreads, should increase when protection sellers face larger margin calls, e.g., after a negative shock to their balance sheets. This matches the recent finding by Siriwardane (2018) that negative shocks to the capital of protection sellers in the CDS market increase the cost of insurance they provide. Our model further predicts that the effect Siriwardane (2018) documents should be more pronounced when the asset-price impact of margin calls is large.

Amplification due to market incompleteness: While fire sales and contagion can take place even when the market is complete, they are amplified by market incompleteness. As stated in Proposition 7, α is larger when the market is incomplete. Thus, market incompleteness increases contagion as well as the price of protection. Our model also suggests that one can test whether markets are complete or not, by checking if investors buying in fire sales have previously sold insurance against the event triggering fire sales.

6.2 Normative implications

Margin calls can trigger fire sales when they force agents to liquidate assets. This creates negative pecuniary externalities, as one agent's asset sales contribute to a drop in asset prices, which then adversely affects other market participants. Such pecuniary externalities can imply that equilibrium is constrained inefficient, as, for example, in Gromb and Vayanos (2002).

Regulators are concerned by margin-induced fire sales. For example, ESRB (2017, p.5) notes that:

> "the individual decisions of each market participant do not take into account the negative externalities associated with a system-wide change to collateral requirements, which can foster fire sales."

At the same time, when prices drop during fire sales, this benefits those market participants who buy assets at fire-sale prices, as noted by Meier and Servaes (2015). Meier and Servaes (2015) argue that a welfare analysis should weigh the benefits of asset buyers against the losses of asset sellers. Our comparison of market equilibrium and second best considers both the ex-ante as well as the ex-post consequences of fire sales. It clarifies under what conditions privately-optimal margining practices are efficient despite the occurrence of fire sales.

Ex post, during the fire sale, the profits of asset buyers are the mirror image of the losses of asset sellers. Ex ante, before the fire sale, what matters for welfare is the way in which those profits and losses are taken into account. When all market participants rationally anticipate the risk of fire sales, efficient insurance against that risk is supplied and equilibrium is constrained efficient. In contrast, if some participants fail to anticipate the risk of fire sales, then insurance is suboptimal and equilibrium is constrained inefficient.

This points to a form of complementarity between markets. To efficiently share protection buyers' risk, we need both i) insurance markets against the final value of protection buyers' assets, and ii) insurance markets against the interim risk of fire sales.

In practice, margin calls are often triggered by drops in the market valuation of the insured asset. In that case, the above mentioned final and interim risks correspond to different maturities of derivatives with the same underlying asset. Thus, our analysis points to a complementarity between derivatives contingent on the same underlying but with different maturities.

When overseeing the development of derivatives markets, regulators and market organisers should therefore ensure that the set of maturities and risks traded be comprehensive enough. They should also make sure all relevant market participants are fully aware of the risk of potential fire sales and offer the possibility to buy and sell insurance against that risk.

7 Conclusion

Our general equilibrium analysis of risk-sharing under moral hazard emphasizes interactions between markets. From a positive point of view, our model implies that moral hazard generates endogenous correlation between two markets after bad news: the market triggering variation-margin calls and another market where assets are sold to comply with the cash payment required for the variation margin. From a normative point of view, our analysis implies that this endogenous correlation is excessive when markets are incomplete, but not when they are complete. Market organisers and regulators should therefore facilitate ex-ante contracting among all relevant counterparties, in particular by creating contracts contingent on events triggering variation-margin calls. This would increase market resilience to negative shocks and reduce fire sales.

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Appendix

A Proofs

Proof of Proposition 1

The Lagrangian is:

$$
\begin{aligned}
L_{FB}(c_B(\theta,s), c_S(\theta,s), c_I(\theta,s), \alpha(s)) = {} & \omega_B E[u(c_B(\theta,s))] + \omega_I E[v(c_I(\theta,s) - \alpha(s)\psi_I(\alpha))] \\
& + \lambda_B[E[u(c_B(\theta,s))] - E[u(\theta)]] \\
& + \lambda_I[E[v(c_I(\theta,s) - \alpha(s)\psi_I(\alpha))] - v(1)] \\
& + \lambda_S[E[c_S(\theta,s) - (1-\alpha(s))\psi] - (R-\psi)] \\
& + \sum_{\theta,s} \lambda(\theta,s)[\theta + 1 + R - (c_B(\theta,s) + c_S(\theta,s) + c_I(\theta,s))] \\
& + \sum_{s} (\lambda_1(s)[1-\alpha(s)] + \lambda_0(s)[\alpha(s)])
\end{aligned}
$$

First-order conditions with respect to $c_B(\theta,s), c_I(\theta,s), c_S(\theta,s)$ and $\alpha(s)$ are

$$(\omega_B + \lambda_B)\Pr[\theta,s]u'(\theta,s) = \lambda(\theta,s), \tag{A.1}$$

$$(\omega_I + \lambda_I)\Pr[\theta,s]v'(\theta,s) = \lambda(\theta,s), \tag{A.2}$$

$$\lambda_S \Pr[\theta,s] = \lambda(\theta,s), \tag{A.3}$$

and

$$-(\omega_I+\lambda_I)\Pr[s]E[v'(c_I(\theta,s)-\alpha(s)\psi_I(\alpha))(\psi_I+\alpha(s)\psi_I')|s]+\lambda_S\Pr[s]\psi = \lambda_1(s)-\lambda_0(s), \tag{A.4}$$

respectively (where, in (A.4), we have used $\Pr[\theta,s] = \Pr[\theta|s]\Pr[s]$). The second-order conditions with respect to $c_B(\theta,s), c_I(\theta,s)$ and $c_S(\theta,s)$ hold because of decreasing marginal utilities. The second-order condition with respect to α is:

$$
\begin{aligned}
-(\omega_I + \lambda_I)\Pr[s]E[-v''(c_I(\theta,s) - \alpha(s)\psi_I(\alpha))(\psi_I + \alpha(s)\psi_I')^2 \\
+ v'(c_I(\theta,s) - \alpha(s)\psi_I(\alpha))(2\psi_I' + \alpha(s)\psi_I'')|s] \le 0,
\end{aligned} \tag{A.5}
$$

which holds since $\psi_I'' \ge 0$ and $v'' < 0$.

Equations (A.1), (A.2) and (A.3) imply

$$(\omega_B + \lambda_B)u'(\theta,s) = (\omega_I + \lambda_I)v'(\theta,s) = \lambda_S \qquad \forall(\theta,s). \tag{A.6}$$

Because neither $\omega_B, \omega_I, \lambda_B, \lambda_I$, nor λ_S depend on the state (θ,s), equation (A.6) implies that the marginal utilities of buyers and investors are constant across states. Hence, $c_B(\theta,s) = c_B$ and $c_I(\theta,s) = c_I$.

The resource constraints bind, $\lambda(\theta,s) > 0$. Suppose not. Because $v', u' > 0$, $\Pr[\theta,s] > 0$, this implies $\omega_B + \lambda_B = 0$ and $\omega_I + \lambda_I = 0$, and hence, $\omega_B = \omega_I = 0$. But because we also have $\omega_S = 0$ (by assumption), the planner's objective would then become zero.

The participation constraint of the sophisticated investors binds, $\lambda_S > 0$. Because $\Pr[\theta, s] > 0$, this is immediate once $\lambda(\theta, s) > 0$.

There is no asset transfer in any state, $\alpha(s) = 0$. Suppose there were positive asset transfers, i.e., $\alpha(s) > 0$. Using the second equality in (A.6), dividing by $\lambda_S \Pr[s]$, and rearranging, the first-order condition with respect to $\alpha(s)$ becomes

$$\psi - \psi_I(\alpha(s)) = \frac{\lambda_1}{\lambda_S \Pr[s]} + \alpha(s)\psi_I'.$$

Given that $\lambda_S > 0$, $\psi_I' \geq 0$ and $\psi < \psi_I(\alpha(s))$ when $\alpha(s) > 0$, this is a contradiction: the left-hand side is negative while the right-hand side is weakly positive.

Given constant consumption for buyers and investors, and the binding resource constraints, we have

$$c_B + c_I + c_S(\theta, s) = \theta + 1 + R \qquad \forall(\theta, s).$$

Using this to substitute for $c_S(\theta, s)$ in the binding participation constraint of investors, together with $\alpha(s) = 0$, we have

$$c_B + c_I = E[\tilde{\theta}] + 1.$$

QED

Proof of Lemma 1

The Lagrangian of the second-best maximisation problem is

$$\begin{aligned}
L_{SB}(c_B(\theta,s), c_S(\theta,s), c_I(\theta,s), \alpha(s)) = \; & \omega_B E[u(c_B(\theta,s))] + \omega_I E[v(c_I(\theta,s) - \alpha(s)\psi_I(\alpha))] \\
& + \sum_s \lambda_{IC(s)} \left[E[c_S(\theta,s)|s] - \frac{(1-\alpha(s))\psi}{1-\mu} \right] \\
& + \lambda_B [E[u(c_B(\theta,s))] - E[u(\theta)]] \\
& + \lambda_I [E[v(c_I(\theta,s) - \alpha(s)\psi_I(\alpha))] - v(1)] \\
& + \lambda_S [E[c_S(\theta,s) - (1-\alpha(s))\psi] - (R - \psi)] \\
& + \sum_{\theta,s} \lambda(\theta,s)[\theta + 1 + R - (c_B(\theta,s) + c_S(\theta,s) + c_I(\theta,s))] \\
& + \sum_s \left(\lambda_1(s)[1-\alpha(s)] + \lambda_0(s)[\alpha(s)] \right).
\end{aligned}$$

First-order conditions with respect to $c_B(\theta, s)$ and $c_I(\theta, s)$ are the same as in the first best, (A.1) and (A.2), respectively. The first-order conditions with respect to $c_S(\theta, s)$ and $\alpha(s)$ are altered, to take into acount the incentive constraint, and write

$$\lambda_{IC(s)} \Pr[\theta|s] + \lambda_S \Pr[\theta, s] = \lambda(\theta, s) \tag{A.7}$$

and

$$-(\omega_I+\lambda_I)\Pr[s]E[v'(\theta,s)|s](\psi_I(\alpha)+\alpha(s)\psi_I')+\lambda_{IC(s)}\frac{\psi}{1-\mu}+\lambda_S\Pr[s]\psi=\lambda_1(s)-\lambda_0(s), \quad \text{(A.8)}$$

respectively. The second-order conditions are as in the first best.

The first-order conditions with respect to $c_B(\theta,s)$ and $c_S(\theta,s)$, (A.1) and (A.7), respectively imply

$$u'(\theta,s)=\frac{1}{\omega_B+\lambda_B}\left(\lambda_{IC(s)}\frac{1}{\Pr[s]}+\lambda_S\right) \quad \text{(A.9)}$$

while the first-order conditions with respect to $c_I(\theta,s)$ and $c_S(\theta,s)$, (A.2) and (A.7), respectively imply

$$v'(\theta,s)=\frac{1}{\omega_I+\lambda_I}\left(\lambda_{IC(s)}\frac{1}{\Pr[s]}+\lambda_S\right). \quad \text{(A.10)}$$

Because their right-hand sides are independent of θ, (A.9) and (A.10) imply that, for a given realisation of the signal s, the marginal utility of consumption of the protection buyers and investors is the same in $(\bar{\theta},s)$ and $(\underline{\theta},s)$.

QED

Proof of Lemma 2

First, we prove that the resource constraints bind, $\lambda(\theta,s)>0$. Suppose not. Because $v',u'>0$, $\Pr[\theta,s]>0$, by (A.1) and (A.2), this implies $\omega_B+\lambda_B=0$ and $\omega_I+\lambda_B=0$, and hence, $\omega_B=\omega_I=0$, a contradiction.

Second, we prove that the participation constraint of the protection seller binds. Suppose not, $\lambda_S=0$. Then, using $\lambda(\theta,s)>0$ in (A.7) yields $\lambda_{IC(s)}>0$ for all (θ,s), i.e., both incentive constraints bind. From the binding incentive constraints, we have $E[c_S(\theta,s)|s]=\frac{1-\alpha(s)}{1-\mu}\psi$ and hence,

$$E[c_S(\theta,s)]=\Pr[\bar{s}]\frac{1-\alpha(\bar{s})}{1-\mu}\psi+\Pr[\underline{s}]\frac{1-\alpha(\underline{s})}{1-\mu}\psi=(1-E[\alpha(s)])\frac{\psi}{1-\mu}. \quad \text{(A.11)}$$

Substituting this into the slack participation constraint of the protection seller yields

$$(1-E[\alpha(s)])\frac{\psi}{1-\mu}-(1-E[\alpha(s)])\psi>R-\psi$$

and, after some rearranging,

$$-E[\alpha(s)]\psi\frac{\mu}{1-\mu}>R-\frac{\psi}{1-\mu},$$

which contradicts the assumption that $\mathcal{P}>0$.

Third, we prove that one of the two incentive constraints (or both) must bind. If not, then the first-best allocation would solve the second best problem. Now, with the seller's first-best consumption (9) and $\alpha(s) = 0$, the incentive constraint after a bad signal becomes

$$\Pr(\bar{\theta}|\underline{s})(\bar{\theta} - E[\tilde{\theta}] + R) + \Pr(\underline{\theta}|\underline{s})(\underline{\theta} - E[\tilde{\theta}] + R) \geq \frac{\psi}{1-\mu},$$

i.e.,

$$E[\tilde{\theta}|\underline{s}] - E[\tilde{\theta}] + R \geq \frac{\psi}{1-\mu},$$

which violates assumption (10).

Fourth, we prove that both ICs cannot bind at the same time. Suppose they do. Then, we have again (A.11), which after substituting the binding participation constraint of the sophisticated investor and rearranging yields

$$-E[\alpha(s)]\psi\frac{\mu}{1-\mu} = R - \frac{\psi}{1-\mu},$$

which contradicts the assumption that $\mathcal{P} > 0$.

QED

Proof of Lemma 3

First, we prove that when the incentive-compatibility condition in state s is slack, then $\alpha(s) = 0$. Suppose not, i.e., $\alpha(s) > 0$ and $\lambda_{IC(s)} = 0$. Then, using (A.2) and (A.7), (A.8) becomes

$$-\lambda_S \Pr[s](\psi_I(\alpha(s)) + \alpha(s)\psi_I'(\alpha(s))) + \lambda_S \Pr[s]\psi = \lambda_1(s).$$

Dividing by $\lambda_S \Pr[s] > 0$ and rearranging yields

$$\psi - \psi_I(\alpha(s)) = \frac{\lambda_1(s)}{\lambda_S(s)\Pr[s]} + \alpha(s)\psi_I'(\alpha(s)).$$

Given that $\psi_I' \geq 0$ and $\psi < \psi_I$ when $\alpha(s) > 0$, we obtain the desired contradiction. The left-hand side is negative while the right-hand side is weakly positive.

Second, we prove that the incentive-compatibility condition after a bad signal binds. Suppose not, $\lambda_{IC((\underline{s}))} = 0$, and only the incentive constraint after the good signal binds. Now, given that the incentive constraint after a bad signal is slack, so that $\alpha(\underline{s}) = 0$, and the incentive constraint after a good signal binds, we have

$$E[c_S(\theta, s)|\bar{s}] = \frac{(1-\alpha(\bar{s}))\psi}{1-\mu} = \frac{\psi}{1-\mu} - \frac{\alpha(\bar{s})\psi}{1-\mu}$$
$$E[c_S(\theta, s)|\underline{s}] > \frac{\psi}{1-\mu},$$

which implies that

$$E[c_S(\theta, s)|\underline{s}] - E[c_S(\theta, s)|\bar{s}] > 0. \tag{A.12}$$

Next, from the binding resource constraints and full risk-sharing conditional on the signal, we have

$$c_S(\theta, s) = \theta + 1 + R - (c_B(s) + c_I(s)) \tag{A.13}$$

and hence

$$E[c_S(\theta, s)|\bar{s}] = E[\tilde{\theta}|\bar{s}] + 1 + R - (c_B(\bar{s}) + c_I(\bar{s})) \tag{A.14}$$

$$E[c_S(\theta, s)|\underline{s}] = E[\tilde{\theta}|\underline{s}] + 1 + R - (c_B(\underline{s}) + c_I(\underline{s})) \tag{A.15}$$

so that

$$E[c_S(\theta, s)|\underline{s}] - E[c_S(\theta, s)|\bar{s}] = E[\tilde{\theta}|\underline{s}] - E[\tilde{\theta}|\bar{s}] - [c_B(\underline{s}) - c_B(\bar{s}) + c_I(\underline{s}) - c_I(\bar{s})]. \tag{A.16}$$

To obtain that expression in (A.16) is weakly negative, so that we have the contradiction to (A.12), the term in squared brackets with the consumptions must be weakly positive (because the signal is (weakly) informative, we have $E[\tilde{\theta}|\underline{s}] - E[\tilde{\theta}|\bar{s}] \leq 0$). From (A.1), (A.7) and the slack incentive constraint after a bad signal, we have

$$(\omega_B + \lambda_B)u'(\theta, \bar{s}) = \lambda_S + \frac{\lambda_{IC(\bar{s})}}{\Pr[\bar{s}]}$$

$$(\omega_B + \lambda_B)u'(\theta, \underline{s}) = \lambda_S$$

Together with full risk-sharing conditional on the signal, this implies that

$$c_B(\underline{s}) \geq c_B(\bar{s}).$$

The same type of argument also establishes that

$$c_I(\underline{s}) \geq c_I(\bar{s}).$$

Hence, the term in squared brackets in (A.16) is (weakly) positive, which yields the desired contradiction.

Third, we analyse the ranking of the consumptions of the protection buyers after bad and good signals. Combining (A.2) with (A.7), and using the fact that there is full risk-sharing conditional on the signal and that only the incentive constraint after the bad signal binds, we obtain:

$$(\omega_B + \lambda_B)\Pr[\theta, \bar{s}]u'(c_B(\bar{s})) = \lambda_S \Pr[\theta, \bar{s}]$$

$$(\omega_B + \lambda_B)\Pr[\theta, \underline{s}]u'(c_B(\underline{s})) = \lambda_{IC(\underline{s})}\Pr[\theta|\underline{s}] + \lambda_S \Pr[\theta, \underline{s}]$$

so that

$$\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))} = 1 + \frac{\lambda_{IC(\underline{s})}}{\Pr[\underline{s}]\lambda_S}. \tag{A.17}$$

Because $\lambda_{IC(\underline{s})} > 0$ and $\lambda_S > 0$, we have imperfect risk-sharing across signals with

$$c_B(\underline{s}) < c_B(\bar{s}).$$

QED

Proof of Proposition 3

First, we write down more precisely the first-order optimality condition with respect to $\alpha(\underline{s})$. Using (A.2) and Lemma 1, the derivative of the Lagrangian with respect to $\alpha(\underline{s})$ is

$$\frac{\partial L_{SB}}{\partial \alpha(\underline{s})} = -\lambda(\theta, s)\Pr[\underline{s}]\frac{1}{\Pr[\theta, \underline{s}]}(\psi_I(\alpha(\underline{s}))+\alpha(\underline{s})\psi_I'(\alpha(\underline{s})))+\lambda_{IC(\underline{s})}\frac{\psi}{1-\mu}+\lambda_S\Pr[\underline{s}]\psi-(\lambda_1(\underline{s})-\lambda_0(\underline{s}))\,.$$

Using (A.7), this rewrites as

$$\frac{\partial L_{SB}}{\partial \alpha(\underline{s})} = -(\lambda_S\Pr[\underline{s}]+\lambda_{IC(\underline{s})})(\psi_I(\alpha(\underline{s}))+\alpha(\underline{s})\psi_I'(\alpha(\underline{s})))+\lambda_{IC(\underline{s})}\frac{\psi}{1-\mu}+\lambda_S\Pr[\underline{s}]\psi-(\lambda_1(\underline{s})-\lambda_0(\underline{s}))\,.$$

Collecting terms,

$$\begin{aligned}\frac{\partial L_{SB}}{\partial \alpha(\underline{s})} =&\lambda_{IC(\underline{s})}\left[\frac{\psi}{1-\mu}-(\psi_I(\alpha(\underline{s}))+\alpha(\underline{s})\psi_I'(\alpha(\underline{s})))\right]\\ &+\lambda_S\Pr[\underline{s}]\big[\psi-(\psi_I(\alpha(\underline{s}))+\alpha(\underline{s})\psi_I'(\alpha(\underline{s})))\big]-(\lambda_1(\underline{s})-\lambda_0(\underline{s}))\,. \end{aligned}\tag{A.18}$$

Second, we show that under (17) there must be some asset transfer, i.e., $\alpha(\underline{s}) > 0$. Suppose not, i.e., suppose we have $\alpha(\underline{s}) = 0$. Then, $\lambda_1(\underline{s}) = 0$ and, by (A.18), the optimality condition such that $\alpha(\underline{s}) = 0$, $\frac{\partial L_{SB}}{\partial \alpha(\underline{s})} \leq 0$, writes as

$$\frac{\lambda_{IC(\underline{s})}}{\lambda_S\Pr[\underline{s}]}\left[\frac{\psi}{1-\mu}-\psi_I(0)\right]+\big[\psi-\psi_I(0)\big] \leq -\frac{\lambda_0(\underline{s})}{\lambda_S\Pr[\underline{s}]}.\tag{A.19}$$

Now, (A.17) yields

$$\frac{\lambda_{IC(\underline{s})}}{\Pr[\underline{s}]\lambda_S} = \frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}\Big|_{\alpha(\underline{s})=0}-1.\tag{A.20}$$

Substituting into (A.19) yields

$$\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}\Big|_{\alpha(\underline{s})=0}-\frac{\frac{\psi}{1-\mu}-\psi}{\frac{\psi}{1-\mu}-\psi_I(0)} \leq -\frac{\lambda_0(\underline{s})}{\lambda_S\Pr[\underline{s}]\left[\frac{\psi}{1-\mu}-\psi_I(0)\right]},\tag{A.21}$$

which contradicts (17), since the latter states that

$$\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}\Big|_{\alpha(\underline{s})=0} > \frac{\frac{\psi}{1-\mu}-\psi}{\frac{\psi}{1-\mu}-\psi_I(0)}.$$

Third, we characterise asset transfers when they are interior, i.e., when $\alpha(\underline{s}) \in (0,1)$. In that case, (A.18) and (A.17) imply

$$\left[\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}-1\right]+\frac{\psi-(\psi_I(\alpha(\underline{s}))+\alpha(\underline{s})\psi_I'(\alpha(\underline{s})))}{\frac{\psi}{1-\mu}-(\psi_I(\alpha(\underline{s}))+\alpha(\underline{s})\psi_I'(\alpha(\underline{s})))}=0$$

or, equivalently,

$$\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))} = \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - (\psi_I(\alpha(\underline{s})) + \alpha(\underline{s})\psi_I'(\alpha(\underline{s})))},$$

where $c_B(\underline{s})$ and $c_B(\bar{s})$ are as given in Proposition 2.

QED

Proof of Lemma 4

First, we write down the Lagrangian of the protection buyer and use it to show that the participation constraint of protection sellers bind. The Lagrangian is:

$$\begin{aligned} L(\tau(\theta,s),\alpha_S,x_B) = {} & \Pr[\bar{s}]E[u(\theta+\tau(\theta,s)-x_B)|\bar{s}] + \Pr[\underline{s}]E[u(\theta+\tau(\theta,s)+qx_B)|\underline{s}] \\ & +\lambda_{IC}\left[\alpha_S p + (1-\alpha_S)\mathcal{P} - E[\tau(\theta,s)|\underline{s}]\right] \\ & +\lambda_S\big[\Pr[\bar{s}](R-\psi) + \Pr[\underline{s}]((1-\alpha_S)(R-\psi)+\alpha_S p) - E[\tau(\theta,s)] - (R-\psi)\big] \\ & +\lambda_1[1-\alpha_S] - \lambda_0\alpha_S. \end{aligned} \tag{A.22}$$

The first-order conditions of (A.22) with respect to $\tau(\theta,\bar{s})$ and $\tau(\theta,\underline{s})$ are:

$$\begin{aligned} \Pr[\bar{s}]\Pr[\theta|\bar{s}]u'(\theta,\bar{s}) &= \lambda_S \Pr[\theta,\bar{s}] && \forall\theta \\ \Pr[\underline{s}]\Pr[\theta|\underline{s}]u'(\theta,\underline{s}) &= \lambda_S \Pr[\theta,\underline{s}] + \lambda_{IC}\Pr[\theta|\underline{s}] && \forall\theta \end{aligned}$$

which simplify to

$$u'(\theta,\bar{s}) = \lambda_S \qquad \forall\theta \tag{A.23}$$

$$u'(\theta,\underline{s}) = \lambda_S + \frac{\lambda_{IC}}{\Pr[\underline{s}]} \qquad \forall\theta. \tag{A.24}$$

(A.23) implies that $\lambda_S > 0$, i.e., the participation constraint of protection sellers binds.

Second, we use the first-order conditions with respect to $\tau(\theta,\bar{s})$ and $\tau(\theta,\underline{s})$ to show that the protection buyer is fully insured conditional on the signal. Because the right-hand sides of (A.23) and (A.24) do not depend on θ, we have $u'(\theta,\bar{s}) = u'(\theta,\underline{s})$, $\forall\theta$, i.e,

$$\bar{\theta} + \tau(\bar{\theta},\bar{s}) = \underline{\theta} + \tau(\underline{\theta},\bar{s}) \tag{A.25}$$

$$\bar{\theta} + \tau(\bar{\theta},\underline{s}) = \underline{\theta} + \tau(\underline{\theta},\underline{s}). \tag{A.26}$$

Thus, conditional on the realisation of the signal s,the protection buyer is fully insured against remaining θ-risk.

Third, we prove by contradiction that the incentive-compatibility condition of the protection seller binds. To do so, we proceed in two steps.

The first step is to prove that, if the incentive-compatibility condition of the protection seller was slack, there would be no asset sale in equilibrium. This first step proceeds by contradiction. Suppose $\lambda_{IC} = 0$ and $\alpha_S = \alpha_I > 0$. Consider the first-order condition of the

Lagrangian (A.22) with respect to α_S, when $\alpha_S > 0$ (and hence, $\lambda_0 = 0$) and $\lambda_{IC} = 0$:

$$-\lambda_s \Pr[\underline{s}](R - \psi - p^*) = \lambda_1, \tag{A.27}$$

where p^* is the equilibrium price in the asset market. From the investors' demand for the productive asset, we know that $\alpha_I > 0$ requires $p^* < R - \psi_I(0)$. Because $\psi_I(0) \geq \psi$ by assumption, the left-hand side of (A.27) is strictly negative, which contradicts the fact that the right-hand side is weakly positive.

The second step is to prove that slack protection seller's incentive constraint would contradict our assumption that $\mathcal{P} < E[\tilde{\theta}] - E[\tilde{\theta}|\underline{s}]$. Suppose $\lambda_{IC} = 0$. Equations (A.23) and (A.24) imply full insurance, $\tau(\theta, \bar{s}) = \tau(\theta, \underline{s}) \equiv \tau(\theta)$ for all θ, and $\bar{\theta} + \tau(\bar{\theta}) = \underline{\theta} + \tau(\underline{\theta})$. Using that $\alpha_S = 0$ and there is full insurance when $\lambda_{IC} = 0$, and substituting the binding participation constraint, we obtain $\tau(\bar{\theta}) = -(1-\pi)(\bar{\theta} - \underline{\theta})$ and $\tau(\underline{\theta}) = \pi(\bar{\theta} - \underline{\theta})$. Using this in the slack incentive constraint yields

$$\begin{aligned}\mathcal{P} >& \Pr[\bar{\theta}|\underline{s}](-1)(1-\pi)(\bar{\theta} - \underline{\theta}) + (1 - \Pr[\bar{\theta}|\underline{s}])\pi(\bar{\theta} - \underline{\theta})\\ =&(\pi - \Pr[\bar{\theta}|\underline{s}])(\bar{\theta} - \underline{\theta})\\ =&E[\tilde{\theta}] - E[\tilde{\theta}|\underline{s}],\end{aligned}$$

a contradiction.

Fourth, we compute the transfers. The binding incentive and participation constraints imply

$$E[\tau(\theta, s)|\bar{s}] = -\frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\big[\alpha_S(R - \psi) + (1 - \alpha_S)\mathcal{P}\big], \tag{A.28}$$

$$E[\tau(\theta, s)|\underline{s}] = \alpha_S p + (1 - \alpha_S)\mathcal{P}. \tag{A.29}$$

Equations (A.28) and (A.29), together with full insurance conditional on the signal, (A.25) and (A.26), yield the set of transfers, for a given α_S:

$$\begin{aligned}\tau^*(\bar{\theta}, \underline{s}) &= -\Pr[\underline{\theta}|\underline{s}](\bar{\theta} - \underline{\theta}) + \alpha_S p + (1 - \alpha_S)\mathcal{P},\\ \tau^*(\underline{\theta}, \underline{s}) &= \Pr[\bar{\theta}|\underline{s}](\bar{\theta} - \underline{\theta}) + \alpha_S p + (1 - \alpha_S)\mathcal{P},\\ \tau^*(\bar{\theta}, \bar{s}) &= -\Pr[\underline{\theta}|\underline{s}](\bar{\theta} - \underline{\theta}) - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\big[\alpha_S(R - \psi) + (1 - \alpha_S)\mathcal{P}\big],\\ \tau^*(\underline{\theta}, \bar{s}) &= \Pr[\bar{\theta}|\underline{s}](\bar{\theta} - \underline{\theta}) - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\big[\alpha_S(R - \psi) + (1 - \alpha_S)\mathcal{P}\big].\end{aligned}$$

QED

Proof of Lemma 5

The first-order condition of the Lagrangian (A.22) with respect to α_S is

$$\lambda_{IC}(p - \mathcal{P}) - \lambda_s \Pr[\underline{s}](R - \psi - p) = \lambda_1 - \lambda_0. \tag{A.30}$$

From (A.23) and (A.24) we have

$$\frac{u'(\theta,\underline{s})}{u'(\theta,\bar{s})} = 1 + \frac{\lambda_{IC}}{\Pr[\underline{s}]\lambda_S} > 1, \tag{A.31}$$

where the inequality follows from the binding incentive constraint stated in Lemma 4.

Combining (A.30) and (A.31), and using the consumptions in Lemma 4, we obtain

$$\frac{u'(E[\tilde{\theta}|\underline{s}] + \alpha_S p + (1-\alpha_S)\mathcal{P} + qx^d)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha_S(R-\psi) + (1-\alpha_S)\mathcal{P}] - x^d\right)} = \frac{\lambda_1 - \lambda_0}{(p-\mathcal{P})\Pr[\underline{s}]\lambda_S} + \frac{R-\psi-\mathcal{P}}{p-\mathcal{P}}. \tag{A.32}$$

Next, we show that when $p > \mathcal{P} + (R-\psi-\mathcal{P})\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}$ then $\alpha_S > 0$. In that case, (A.32) with $\lambda_0 = 0$ yields (30). Suppose not, $\alpha_S = 0$, so that $\lambda_0 > 0$ and $\lambda_1 = 0$. Then solving (A.32) with $\alpha_S = 0$ for p yields

$$\left[\frac{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)} + \frac{\lambda_0}{(p-\mathcal{P})\Pr[\underline{s}]\lambda_S}\right](p-\mathcal{P}) = R-\psi-\mathcal{P}$$

$$p = \mathcal{P} + \frac{R-\psi-\mathcal{P}}{\left[\frac{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)} + \frac{\lambda_0}{(p-\mathcal{P})\Pr[\underline{s}]\lambda_S}\right]}.$$

This contradicts the assumption that $p > \mathcal{P} + (R-\psi-\mathcal{P})\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}$, because

$$\frac{R-\psi-\mathcal{P}}{\left[\frac{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)} + \frac{\lambda_0}{(p-\mathcal{P})\Pr[\underline{s}]\lambda_S}\right]} < (R-\psi-\mathcal{P})\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)},$$

as

$$1 < \frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}\left[\frac{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)} + \frac{\lambda_0}{(p-\mathcal{P})\Pr[\underline{s}]\lambda_S}\right],$$

due to

$$1 < 1 + \frac{\lambda_0}{(p-\mathcal{P})\Pr[\underline{s}]\lambda_S}\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}.$$

Finally, we show that when $p \le \mathcal{P} + (R-\psi-\mathcal{P})\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}$, then $\alpha_S = 0$. To do so, we proceed in three steps, corresponding to different values of p.

First, when $p < \mathcal{P}$, $\alpha_S = 0$. Suppose not, $\alpha_S > 0$ and hence $\lambda_0 = 0$. Then, the first term on the right-hand side of (A.32) is weakly negative and the second term is strictly negative. Hence, the right-hand side is strictly negative while the left-hand side is strictly positive.

Second, when $\mathcal{P} < p \leq \mathcal{P} + (R - \psi - \mathcal{P})\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}$, then $\alpha_S = 0$. Suppose not, $\alpha_S > 0$ and hence, $\lambda_0 = 0$. Then, solving (A.32) for p yields

$$\begin{aligned} p &= \frac{\lambda_1}{\Pr[\underline{s}]\lambda_S} \frac{u'(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha_S(R-\psi) + (1-\alpha_S)\mathcal{P}] - x^d)}{u'(E[\tilde{\theta}|\underline{s}] + \alpha_S p + (1-\alpha_S)\mathcal{P} + qx^d)} \\ &\quad + \mathcal{P} + (R - \psi - \mathcal{P}) \frac{u'(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha_S(R-\psi) + (1-\alpha_S)\mathcal{P}] - x^d)}{u'(E[\tilde{\theta}|\underline{s}] + \alpha_S p + (1-\alpha_S)\mathcal{P} + qx^d)}. \end{aligned}$$

This price decreases when α_S decreases (since the ratio of marginal utilities is strictly increasing in α_S). Yet, with $\alpha_S > 0$, the price will always be larger than the largest price allowed in the starting condition

$$p = \mathcal{P} + (R - \psi - \mathcal{P}) \frac{u'(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}$$

because $\lambda_1 \geq 0$ and

$$\frac{u'(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha_S(R-\psi) + (1-\alpha_S)\mathcal{P}] - x^d)}{u'(E[\tilde{\theta}|\underline{s}] + \alpha_S p + (1-\alpha_S)\mathcal{P} + qx^d)} > \frac{u'(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}$$

when $\alpha_S > 0$.

Third, when $p = \mathcal{P}$, then $\alpha_S = 0$. Suppose not, $\alpha_S > 0$ and hence, $\lambda_0 = 0$. As $p \to \mathcal{P}$, the right-hand side of (A.32) goes to infinity, contradiction since the left-hand side is finite.

QED

Proof of Proposition 5

Lemma 5 states that if

$$p \leq \mathcal{P} + (R - \psi - \mathcal{P}) \frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x_B\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx_B)},$$

then $\alpha_S = 0$, otherwise $\alpha_S > 0$, where α_S is given by (30).

Moreover, the above analysis of investors trades showed that if $p < R - \psi_I(0)$ then $\alpha_I > 0$, while otherwise $\alpha_I = 0$. So two cases must be distinguished.

If

$$\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x_B\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx_B)} > \frac{\frac{\psi}{1-\mu} - \psi}{R - \psi - \mathcal{P}},$$

then $\alpha^* = 0$ and p^* is any price in $[R - \psi_I(0), \hat{p}(x^d, q)]$.

Otherwise, there exists (p^*, α^*) such that $\alpha_S(p^*) = \alpha_I(p^*) = \alpha^* > 0$. A sufficient condition for $\alpha^* < 1$ is provided by (2), which implies $\psi_I(1) + \psi_I' > \frac{\psi}{1-\mu}$. To see this, proceed by contradiction and suppose $\alpha^* = 1$. Then, (25) implies the price is $p^* = R - (\psi_I(1) + \psi_I')$.

Substituting into (30)

$$\frac{u'(E[\tilde{\theta}|\underline{s}] + \alpha_S p + (1-\alpha_S)\mathcal{P} + qx^d)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha_S(R-\psi) + (1-\alpha_S)\mathcal{P}] - x^d\right)} = \frac{\lambda_1}{\left(\frac{\psi}{1-\mu} - (\psi_I(1) + \psi_I')\right)\lambda_S \Pr[\underline{s}]} + \frac{R-\psi-\mathcal{P}}{\frac{\psi}{1-\mu} - (\psi_I(1)+\psi_I')}$$

The left-hand side is strictly positive but if $\psi_I(1) + \psi_I' > \frac{\psi}{1-\mu}$, the right-hand side is strictly negative, so we have a contradiction.

In the second case, the price p^* is obtained by applying (25). Substituting this price into (30) while setting $\lambda_1 = 0$ yields (34).

QED

Proof of Proposition 6

To prove Proposition 6, we first recall the equilibrium conditions, then we recall the second-best conditions, and finally we show that for any allocation that satisfies the equilbrium conditions there exists a set of Pareto weights such that this allocation satisfies the conditions for second-best optimality.

Equilibrium allocation: Substituting equilibrium prices and trades α^*, p^*, x^*, and q^* into (27) and (28), equilibrium protection buyers' consumption is

$$c_B(\bar{\theta}, \bar{s}) = c_B(\underline{\theta}, \bar{s}) = E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha^*(R-\psi) + (1-\alpha^*)\mathcal{P}] - x^*, \tag{A.33}$$

$$c_B(\bar{\theta}, \underline{s}) = c_B(\underline{\theta}, \underline{s}) = E[\tilde{\theta}|\underline{s}] + \alpha^* p^* + (1-\alpha^*)\mathcal{P} + q^* x^*. \tag{A.34}$$

Similarly, substituting α^*, p^*, x^*, and q^* into investors' consumptions

$$c_I(\bar{\theta}, \bar{s}) = c_I(\underline{\theta}, \bar{s}) = 1 + x^*, \tag{A.35}$$

$$c_I(\bar{\theta}, \underline{s}) = c_I(\underline{\theta}, \underline{s}) = 1 - q^* x^* + \alpha^*(R - p^*). \tag{A.36}$$

Substituting α^*, p^*, x^*, q^*, (A.33) and (A.34) into (32), marginal rates of substitution between consumption after good news and after bad news are equalised for protection buyers and investors.

$$\frac{v'(c_I(\theta, \underline{s}) - \alpha^* \psi_I(\alpha^*))}{v'(c_I(\theta, \bar{s}))} = \frac{u'(c_B(\theta, \underline{s}))}{u'(c_B(\theta, \bar{s}))}. \tag{A.37}$$

Substituting (A.33) and (A.34) into condition (33), the condition writes as

$$\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}\Big|_{\alpha=0} > \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - \psi_I(0)}. \tag{A.38}$$

When that condition does not hold, $\alpha^* = 0$. When it holds, substituting α^*, p^*, x^*, q^*, into (34), the marginal rate of substitution between consumption after bad news and consumption after good news is equal to what we intepreted, in the discussion of equation (18) in Proposition 3, as the marginal cost of insurance:

$$\frac{u'(c_B(\theta,\underline{s})}{u'\left(c_B(\theta,\bar{s})\right)} = \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - (\psi_I(\alpha^*) + \alpha^*\psi_I'(\alpha^*))}. \tag{A.39}$$

Second best allocation: Equations (12) and (13) state the total consumption of protection buyers and investors, after bad news and after good news, in the second best:

$$c_B(\underline{s}) + c_I(\underline{s}) = 1 + E[\tilde{\theta}|\underline{s}] + \alpha(\underline{s})R + (1-\alpha(\underline{s}))\mathcal{P}, \tag{A.40}$$

$$c_B(\bar{s}) + c_I(\bar{s}) = 1 + E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha(\underline{s})(R-\psi) + (1-\alpha(\underline{s}))\mathcal{P}]. \tag{A.41}$$

Equation (14) states that in the second best marginal rates of substitution are equalised between protection buyers and investors:

$$\frac{v'(c_I(\underline{s}) - \alpha(\underline{s})\psi_I(\underline{s}))}{v'(c_I(\bar{s}) - \alpha(\bar{s})\psi_I(\bar{s}))} = \frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}. \tag{A.42}$$

Inequality (17) states the condition under which asset transfers are strictly positive in the second best:

$$\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}\Big|_{\alpha(\underline{s})=0} > \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - \psi_I(0)}; \tag{A.43}$$

if that condition does not hold, then there are no asset transfers in the second best.

Equation (18) gives the interior asset transfer:

$$\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))} = \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - (\psi_I(\alpha(\underline{s})) + \alpha(\underline{s})\psi_I'(\alpha(\underline{s})))}. \tag{A.44}$$

Finally, equation (15) states how total consumption is split between protection buyers and investors as a function of their Pareto weights:

$$\frac{u'(c_B(s))}{v'(c_I(s) - \alpha(s)\psi_I(\alpha(s))} = \frac{\omega_I + \lambda_I}{\omega_B + \lambda_B}. \tag{A.45}$$

Investors' and protection buyers' consumptions and asset transfers such that (A.40), (A.41), (A.42), (A.43), (A.44) and (A.45) hold are second best.

Comparing second best and equilibrium allocations: Consider an equilibrium allocation

$$\mathcal{E} = \{c_I(\theta, s), c_B(\theta, s), \alpha^*\}.$$

It is such that i) (A.33) to (A.36) hold and ii) if (A.38) holds, then (A.39) holds.

Equilibrium is information-constrained Pareto efficient if $\mathcal{E}$ satisfies the second-best optimality conditions, (A.40) to (A.45). Out of these six conditions, 5 are obviously satisfied:

Adding (A.33) to (A.35), and (A.34) to (A.36), in equilibrium the total consumption of protection buyers and investors is

$$c_B(\theta, \bar{s}) + c_I(\theta, \bar{s}) = 1 + E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha^*(R - \psi) + (1 - \alpha^*)\mathcal{P}], \forall \theta, \tag{A.46}$$

after good news and

$$c_B(\theta, \underline{s}) + c_I(\theta, \underline{s}) = 1 + E[\tilde{\theta}|\underline{s}] + \alpha^* R + (1 - \alpha^*)\mathcal{P}, \forall \theta, \tag{A.47}$$

after bad news. (A.47) is equivalent to (A.40), while (A.46) is equivalent to (A.41).

Equation (A.37) shows that in equilibrium the MRS of protection buyers and investors are equalised, exactly as requested in the second best, in (A.42).

Third, (A.38) is equivalent to (A.43), and (A.39) is equivalent to (A.44).

So, it only remains to check that $\mathcal{E}$ satisfies (A.45). To do so, we need to show that there are Pareto weights ω_I and ω_B such that (A.45) holds for the consumptions in $\mathcal{E}$. Now, investors are strictly better off when participating in the market equilibrium than in autarky, since they strictly prefer to trade in the market for insurance against signal risk. Protection buyers also are strictly better off since they can, at least, extract all the surplus from contracting with protection sellers with $\alpha = 0$. Consequently, the participation constraints of protection buyers and investors are slack, implying $\lambda_I = \lambda_B = 0$. Hence, (A.45) holds for the consumptions in $\mathcal{E}$ if and only if there exist Pareto weights ω_I and ω_B such that

$$\frac{u'(c_B(s))}{v'(c_I(s) - \alpha(s)\psi_I(\alpha(s))} = \frac{\omega_I}{\omega_B}.$$

This is always the case. To see this, pick an arbitrary ω_B, then set

$$\omega_I = \omega_B \frac{u'(c_B(s))}{v'(c_I(s) - \alpha(s)\psi_I(\alpha(s))}.$$

QED

Proof of Proposition 7

First, consider the case in which that $\alpha^{IM} = 0$. In that case, we have

$$\frac{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^*)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^*\right)} < \frac{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P})}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P}\right)} < \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - \psi_I(0)},$$

where the first inequality follows from $x^* > 0$ and the fact that u' is decreasing. By Proposition 5, we have that $\alpha^* = 0$. Therefore $\alpha^* = \alpha^{IM} = 0$, and correspondingly $p^* = p^{IM}$.

Second, consider the case in which $\alpha^{IM} > 0$. Since equilibrium price decreases in α, it suffices to prove that $\alpha^{IM} > \alpha^*$. There are two possibilities: Either $\alpha^* = 0$, implying that

$\alpha^{IM} > \alpha^*$, or $\alpha^* > 0$. In the latter case, α^* is the root of

$$\frac{u'(E[\tilde{\theta}|\underline{s}] + \alpha(R - [\psi_I(\alpha) + \alpha\psi_I'(\alpha)]) + (1-\alpha)\mathcal{P} + q^* x^*)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha(R-\psi) + (1-\alpha)\mathcal{P}] - x^*\right)} = \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - (\psi_I(\alpha) + \alpha\psi_I'(\alpha))}, \tag{A.48}$$

while α^{IM} is the root of

$$\frac{u'(E[\tilde{\theta}|\underline{s}] + \alpha(R - [\psi_I(\alpha) + \alpha\psi_I'(\alpha)]) + (1-\alpha)\mathcal{P})}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha(R-\psi) + (1-\alpha)\mathcal{P}]\right)} = \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - (\psi_I(\alpha) + \alpha\psi_I'(\alpha))}. \tag{A.49}$$

The two equations are very similar. They have the same right-hand side, which is an increasing function of α. The equilibrium α is such that this right-hand side intersects the left-hand side, (A.48) for complete markets and (A.49) for incomplete markets, respectively. Note further that the left-hand side of (A.48) is lower than the left-hand side of (A.49). Consequently, the intersection of the left- and right-hand sides ocurs for lower α in (A.48) than in (A.49). Hence $\alpha^* < \alpha^{IM}$.

QED

B Power utility

To illustrate our results, assume

$$u(x) = v(x) = \frac{x^{1-\gamma}}{1-\gamma}, \tag{B.1}$$

and

$$\psi_I = \psi + \delta_0 + \delta_1 \alpha, \tag{B.2}$$

and denote the Pareto weight of investors by $\omega > 0$ and that of protection buyers by $1 - \omega$. We assume an interior solution, i.e., the participation constraints of protection buyers (4) and investors (5) are slack, and margins will be used ((17) holds).

After good news, condition (15) writes

$$\left(\frac{c_I(\bar{s})}{c_B(\bar{s})} \right)^{\gamma} = \frac{\omega}{1-\omega}$$

so that

$$c_I(\bar{s}) = \left(\frac{\omega}{1-\omega} \right)^{\frac{1}{\gamma}} c_B(\bar{s}).$$

Correspondingly,

$$c_I(\bar{s}) + c_B(\bar{s}) = \left[1 + \left(\frac{\omega}{1-\omega} \right)^{\frac{1}{\gamma}} \right] c_B(\bar{s}). \tag{B.3}$$

Similarly, after bad news, condition (15) yields

$$c_I(\underline{s}) + c_B(\underline{s}) = \left[1 + \left(\frac{\omega}{1-\omega} \right)^{\frac{1}{\gamma}} \right] c_B(\underline{s}). \tag{B.4}$$

Substituting (B.3) and (B.4) into (12) and (13), we obtain

$$c_B(\bar{s}) = \frac{1 + E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]} [\alpha(\underline{s})(R - \psi) + (1 - \alpha(\underline{s}))\mathcal{P}]}{1 + \left(\frac{\omega}{1-\omega} \right)^{\frac{1}{\gamma}}}$$

$$c_B(\underline{s}) = \frac{1 + E[\tilde{\theta}|\underline{s}] + \alpha(\underline{s})R + (1 - \alpha(\underline{s}))\mathcal{P}}{1 + \left(\frac{\omega}{1-\omega} \right)^{\frac{1}{\gamma}}}.$$

With (B.1) and (B.2), the condition on the optimal asset transfer in the second best (18) is

$$\frac{c_B(\bar{s})}{c_B(\underline{s})} = \left(\frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - (\psi + \delta_0 + 2\delta_1 \alpha(\underline{s}))} \right)^{\frac{1}{\gamma}},$$

which becomes

$$\frac{1+E[\tilde{\theta}|\bar{s}]-\frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha(\underline{s})(R-\psi)+(1-\alpha(\underline{s}))\mathcal{P}]}{1+E[\tilde{\theta}|\underline{s}]+\alpha(\underline{s})R+(1-\alpha(\underline{s}))\mathcal{P}}=\left(\frac{\frac{\psi}{1-\mu}-\psi}{\frac{\psi}{1-\mu}-(\psi+\delta_0+2\delta_1\alpha(\underline{s}))}\right)^{\frac{1}{\gamma}}, \quad \text{(B.5)}$$

after substituting the above consumptions of protections buyers and investors.

With power utility, the protection buyers' share of the total consumption of protection buyers and investors is

$$\frac{1}{1+\left(\frac{\omega}{1-\omega}\right)^{\frac{1}{\gamma}}},$$

after both signals. Moreover, asset transfers are independent from the Pareto weights, i.e., there is a separation beween production (asset transfers) and allocation decisions. The former set the level of asset transfers that maximises the sum of the protection buyers' and investors' consumptions independently of ω. The latter allocate total consumption as a function of the Pareto weight for investors, ω.