Promotion signaling, discrimination, and positive discrimination policies*

Marc Gürtler**, *Braunschweig Institute of Technology*

Oliver Gürtler***, *University of Cologne*

Abstract

The current paper studies discrimination in a model in which promotions are used as signals of worker ability. The model can account for both statistical and taste-based discrimination. A positive discrimination policy lowers the promotion standard for the workers who are discriminated against. This is beneficial for the workers in the middle of the ability distribution because these workers are promoted if and only if the policy is in place. Instead, workers of either high or low ability generally suffer from the policy because the policy does not change their promotion probability but weakens the positive signal of being promoted and strengthens the negative signal of not being promoted. We also show that the policy may increase or decrease efficiency and that it may aggravate wage inequality.

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**Marc Gürtler, Department of Economics, Braunschweig Institute of Technology, Abt-Jerusalem-Str. 7, D-38106 Braunschweig, Germany. Phone: +49-531-3912895; E-mail: marc.guertler@tu-bs.de

***Oliver Gürtler, Department of Economics, University of Cologne, Albertus Magnus Platz, D-50923 Cologne, Germany. Phone: +49-221-4701450; E-mail: oliver.guertler@uni-koeln.de

1 Introduction

Positive discrimination policies are implemented all over the world with the aim of promoting the opportunities of people who (are perceived to) suffer from discrimination. These policies can differ in their nature: some are intended to facilitate obtainment of higher education, while others forbid firms to discriminate against people based on their race, religion, sex and so on when making hiring or promotion decisions. In this paper, we consider positive discrimination policies that are aimed at improving the career prospects of people who are discriminated against by ordering firms to make it easier for those people to be promoted. A recent example of such a policy is the European Commission's proposal to impose a 40 percent quota for female directors on the supervisory boards of publicly listed companies.[1] We show that these policies may be counterproductive in that they can actually hurt the people they are intended to benefit. We further find that these policies can increase the difference in wages for the highest and lowest earners, thereby aggravating inequality.

In the economic literature, it has long been recognized that promotions can serve as signals about worker ability.[2] The intuition is simple. A worker's current employer is likely to receive more comprehensive information about the worker's ability than external firms, i.e., learning is asymmetric. When performance is more sensitive to ability in high-level jobs rather than in low-level jobs, the current employer promotes the worker to a high-level job if and only if the employer believes that the worker's ability is sufficiently high. As a consequence, when external firms observe a worker's promotion, they upgrade their assessment of the worker's ability. In turn, they have a greater interest in hiring that worker, with the result that the worker receives more generous wage offers. Finally, because firms must pay higher wages to retain promoted workers, firms decide to promote inefficiently few workers; that is, promotion standards are inefficiently high. Recent empirical studies find results that are in line with predictions derived from the promotion-signaling model.[3]

We consider a promotion-signaling model with two periods. In the first period, a firm hires a worker and assigns the worker to a low-level job. The firm observes the worker's performance at the end of the first period and then decides whether to promote the worker

[1] http://ec.europa.eu/justice/newsroom/gender-equality/news/121114_en.htm

[2] See, e.g., Waldman (1984), Bernhardt (1995), Zábojník and Bernhardt (2001), Ghosh and Waldman (2010), Zábojník (2012), Waldman (2013), and Gürtler and Gürtler (2015).

[3] See Cassidy et al. (2012), DeVaro and Waldman (2012), and Bognanno and Melero (2015).

to a high-level job or to reassign him to the low-level one. External firms observe whether the worker was promoted, and thereafter make wage offers to the worker to hire him. In a first step, we show that the model can be used to explain why some workers are discriminated against. In particular, the model can capture both (endogenous and exogenous) statistical discrimination and taste-based discrimination. Take endogenous statistical discrimination as an example. It is optimal for the first-period employer to promote the worker when first-period performance exceeds a certain performance standard. This performance standard need not be uniquely defined. The intuition for this result is the following: if external firms expect a worker's first-period employer to set a rather high promotion standard, they conclude that a promoted worker must be of exceptionally high ability. Therefore, they make very generous wage offers to promoted workers, which in turn, makes it optimal for the first-period employer to reassign the worker to the low-level job unless his performance is exceptionally high. In other words, the external firms' expectation of a very high promotion standard may become self-fulfilling in the current model. The same is true for a relatively low promotion standard. It is then conceivable that there are two identical workers playing different equilibria with different promotion standards. In turn, only one of them (the worker with the lower promotion standard) may be promoted, whereas the other worker is reassigned to the low-level job and thus discriminated against.

In a second step, we introduce a positive discrimination policy aimed at improving the career prospects of people who are discriminated by assuming that the first-period employer is ordered to lower the promotion standard compared to the promotion standard he would usually set. This is a natural assumption. Given that performance is more responsive to ability in high than in low-level jobs, the employer wants to promote the workers who he believes have the highest ability. If he is ordered to promote more workers from a specific group of workers than he would voluntarily do (e.g., because a quota has been introduced), it is optimal for the employer to promote the next best workers, i.e., the best workers who would have been reassigned to the low-level job were the positive discrimination policy not in place. This is equivalent to lowering the promotion standard.

A positive discrimination policy affects workers' payoffs in different ways. Consider first a worker in the first half of his career (i.e., the first period) who has already been hired when the policy is introduced. For such a worker, the first-period wage is not affected by the positive discrimination policy; the policy only affects the worker's second-period payoff.

There are two effects. First, the worker is more likely to be promoted and to obtain a wage increase, which obviously benefits the worker. Second, the positive signal of promotion to the high-level job becomes weaker, whereas the negative signal of being reassigned to the low-level job becomes stronger. If the worker ss promoted, external firms may question the worker's ability and may believe that the worker was promoted only because of the positive discrimination program. If, instead, the worker is not promoted, external firms believe that the worker's ability must be extremely low because he did not manage to become promoted in spite of the positive discrimination program. Because external firms interpret the job-assignment signal differently when a positive discrimination policy is in place, their wage offers differ as well. In particular, both the wage for a promoted worker and for a worker reassigned to the low-level job decreases, which clearly hurts workers. Summing up, the introduction of the positive discrimination policy leaves those workers worse off who have either very high ability such that they would have been promoted even without the policy, or who have such low ability that they are not considered for promotion even when the policy is in place. In contrast, workers in the middle of the ability distribution benefit from the policy because those workers are promoted if and only if the firm is bound to the positive discrimination policy. It is possible that the negative effects on the workers of either low or high ability outweigh the positive effects on the workers of middle ability so that a worker's expected payoff may actually decrease.

Positive discrimination policies are sometimes criticized for devaluing the achievements of people who are intended to benefit from the policies, possibly leading to feelings of inferiority, self-doubt, and incompetence.[4] This argument is reminiscent of our finding that the positive signal of promotion is weaker when a positive discrimination policy is in place than when there is no such policy. As we show, the problem may be less that promoted workers question their own ability, but rather that external firms do, leading to less generous wage offers.

Consider now a worker who begins his working career only after a positive discrimination policy has been introduced. As indicated above, the difference from the preceding argumentation is that the worker's first-period wage is no longer fixed so that the policy affects both the worker's first-period and second-period payoff. The change in the expected second-period payoff discussed above is exactly offset by a change in the first-period wage. This is intuitive. Suppose that the expected second-period wage decreases. Then, firms are more interested

[4]See, for example, Andre et al. (1992).

in hiring the worker in the first period because it is more profitable to retain him in the second period. Firms are therefore willing to increase the first period wage, and in the case of a perfectly competitive labor market, this increase in first-period wage simply offsets the decrease in expected second-period compensation. The effect of the positive discrimination policy on the worker's expected payoff thus depends only on whether the policy makes the firm's promotion decision more or less efficient. Because the promotion standard is inefficiently high when no positive promotion policy is in place, lowering the standard may lead to a more efficient promotion rule, thereby increasing the worker's payoff. It is also conceivable, however, that the promotion standard becomes inefficiently low and the worker's expected payoff is reduced.

In practice, whether a positive discrimination policy leads to a more or less efficient promotion rule depends on the demographic composition of the firm's workforce. Suppose that a policy requires a firm to promote at least five members of a specific group of people to the high-level job. If there are only five members of this group currently employed in the low-level job, the firm must promote all of them, rendering it likely that the promotion standard becomes inefficiently low. Instead, if the firm can select from a much larger pool of people, the effect of the policy is more muted, probably resulting in a more efficient promotion rule.

Positive discrimination policies are often argued to lead to mismatching, thereby reducing productivity and efficiency.[5] Our results demonstrate that this is not necessarily true. As shown in Waldman (1984) and as explained previously, promotion standards are expected to be set inefficiently high because firms are not willing to reveal their information regarding workers' abilities. When a positive promotion policy forces firms to lower promotion standards, it is conceivable that the selected promotion standard is closer to the efficient level and that it results in a better assignment of workers to jobs.

We also investigate the effect of the policy on workers who are disadvantaged by the policy and therefore less likely to be promoted. Extremely able workers who are promoted in spite of the rule benefit from the policy because the positive signal of being promoted grows stronger. Therefore, the wage of the highest earners (i.e., the most able members of the group that is disadvantaged by the policy) is expected to increase. As indicated above, because the wage of the lowest earners (i.e., the least able members of the group that is supposed to be favored by the policy) is likely to decrease, the introduction of a positive discrimination

[5] See Andre et al. (1992).

policy may aggravate wage inequality.

Several empirical implications can be derived from the model. The most important one is that the introduction of a positive discrimination policy lowers the wages that workers who are advantaged by the policy earn later in their career and increases wages for those workers who are disadvantaged by the policy when controlling for job level. For workers who began their working careers before the policy was put in place, their starting wage is unaffected by the policy. This in turn implies that the wage increase upon promotion for these workers is altered by the positive promotion policy. For workers who are advantaged by the policy, the post-promotion wage decreases, while the starting wage is fixed so that the wage increase upon promotion becomes lower. For workers who are disadvantaged by the policy, the effect is opposite and the wage increase upon promotion gets higher. Bertrand et al. (2014) study the effects of the introduction of a quota on the boards of publicly traded Norwegian companies that led to an increase in the share of women elected to these boards. They show that election to a company board entails a substantial financial reward for the elected worker. After the board quota was introduced, this reward fell for women (from 9.4% percent of annual earnings to 8%), whereas it increased substantially for men (from 4.6% to 10%), in line with the predictions of our model.

The paper is organized as follows. In the next section, we present related literature. In Section 3, we turn to the basic model. The effect of positive discrimination policies on workers' wages is analyzed in Section 4. The empirical implications of the model are discussed in Section 5, while Section 6 presents the conclusions. The proofs of all lemmas and propositions are in the Appendix.

2 Related literature

The paper is related to the economic literature on discrimination. This literature can be divided into two different strands.[6] First, there is a body of work on taste-based discrimination that was originated by Becker (1957) and further developed by, e.g., Coate and Loury (1993a) and Black (1995). According to this literature, workers belonging to some group of people are discriminated against because firms incur some disutility when interacting with

[6] See Fang and Moro (2011) as well as Lang and Lehmann (2012) for surveys of the economic literature on discrimination.

these workers. Second, there is a body of literature on statistical discrimination. In this literature, it is assumed that a worker's ability is not fully known to potential employers. Firms therefore use all the available information to estimate abilities. When there are differences between groups, these differences influence the ability assessments so that it makes sense for the firms to treat two workers who belong to two different groups but are otherwise identical differently. Differences between groups can either be imposed exogenously (e.g., Phelps 1972), or can emerge endogenously (e.g., Coate and Loury 1993b, Moro and Norman 2003, Fryer 2007). The intuition for the latter possibility is as follows. When firms hold pessimistic beliefs about the abilities of a certain group of people, they are unwilling to hire these people. In turn, members of this group of people have low incentives to enhance their abilities, thus confirming firms' pessimistic beliefs.

Our paper contributes to this literature in two ways. First, we show how discrimination may arise in a promotion-signaling framework. Second, we examine the effects of positive discrimination policies on the payoffs of workers who are supposed to benefit from these policies (and on the workers who are disadvantaged by the policies). In many of the papers mentioned above, the effect of positive discrimination policies on the model outcome is analyzed. In contrast to our study, however, wages are often assumed to be exogenous. One paper that endogenizes wages is by Moro and Norman (2003), who study statistical discrimination in a general equilibrium model with two different types of jobs. As in our model, they show that positive discrimination policies can lower the wages of workers who are supposed to benefit from the policy. This happens exclusively in low-level jobs, however, and not in high-level jobs. In their model, each worker's wage schedule as a function of a signal of ability must be continuous, as firms could otherwise raid other firms' workers. If a positive discrimination policy is introduced, the threshold for the signal at which workers are assigned to the high-level job is lowered. This implies that workers of lower ability are assigned to the high-level job so that the wage of the lowest-paid worker in the high-level job decreases. In order for the wage schedule to remain continuous, the wage for the most able worker in the low-level job must also decrease, implying a uniform decrease in wages in the low-level job. In our model, the introduction of a positive discrimination policy lowers the wages of workers who are supposed to benefit from the policy in both high and low-level jobs. This is because a positive discrimination policy makes the positive signal of promotion weaker, while the negative signal of reassignment to the low-level job becomes more significant. In the model by Moro and Norman (2003), all firms obtain the same information about workers' abilities. Therefore, assignment to a specific job does not serve as a signal about worker ability, meaning that the signaling effects that drive results in the current model are not present in their model.

Promotion decisions are often modeled as a tournament in which workers exert costly effort to perform better than their coworkers and be considered for promotion (Lazear and Rosen 1981). A few papers investigate the effects of positive discrimination policies in a tournament setting. In both a theoretical model and an experimental study, Schotter and Weigelt (1992) demonstrate that not only do the workers who are intended to benefit from the policies win the tournament more often (so that their career prospects are improved) but also that efficiency is increased. The intuition for the latter result is that positive discrimination policies tend to make the tournament more equal, thus inducing workers to exert higher effort.[7] In another set of experiments, Balafoutas and Sutter (2012) show that positive discrimination policies aimed at improving the career prospects of women encourage women to participate in (promotion) tournaments more often instead of working under a piece-rate scheme. Again, it is found that the policy does not entail an efficiency loss. The most important difference between these studies and our paper is that the studies assume wages (i.e., tournament prizes) to be exogenously given, while in our model, wages are determined by firms' competition for workers' services. Wages in our model depend on what external firms learn about a worker's ability from the assignment of the worker to a specific job. As explained previously, wages can therefore be lower when a positive discrimination policy is in place than when there is no such policy, hurting the workers who are intended to benefit from the policy.

3 The basic model

3.1 Description of the model and notation

We consider a model of a competitive labor market with two periods, $\tau = 1, 2$. There are N identical firms and $n < N$ workers; all parties are risk-neutral. Each firm has two different

[7] A similar result is obtained by Fu (2006) in the context of an allpay-auction which he uses to study race-conscious preferential admissions to college.

types of job, a low-level job 1 and a high-level job 2. Jobs are indexed by $k = 1, 2$. If worker $j \in \{1, ..., n\}$ is hired by firm $i \in \{1, ..., N\}$ in period τ and assigned to job k, his output is given by

$$y_{ij\tau}^{k} = (1 + s)(c_k + d_k a_j). \tag{1}$$

Worker j's ability is denoted by a_j and is initially unknown to all firms and all workers (as, for example, in Holmström 1982). We assume that a_j is continuously, identically, and independently distributed. The probability density function (pdf) of a_j is denoted by f and has support $[\underline{a}, \overline{a}]$, with $\overline{a} > \underline{a} > 0$. The corresponding cumulative distribution function (cdf) is denoted by F. $c_k \geq 0$ and $d_k > 0$ are parameters characterizing worker productivity. Following Waldman (1984), we assume that $d_2 > d_1$ (and $c_2 < c_1$), so that output is more responsive to ability in the high-level job. We define $a^e := (c_1 - c_2)/(d_2 - d_1)$ as the ability level at which output is equalized across jobs and we assume $a^e \in (\underline{a}, \overline{a})$. Finally, $s \in \{0, S\}$ is an indicator variable capturing firm-specific human capital acquired in the first period of employment. Its realization is equal to zero ($s = 0$) if the first period is considered or if the second period is considered and worker j has moved to a different firm after the first period. The variable equals $S > 0$ if the second period is considered and the worker continues to work for the same firm as in the first period.

For the majority of the analysis, we restrict our attention to a representative firm-worker pair. We assume that the worker's expected ability, $E[a_j]$, is lower than a^e so that the firm finds it optimal to assign the worker to the low-level job 1 in $\tau = 1$. At the end of the first period, the firm observes the output of the worker and then decides which job the worker is assigned to in $\tau = 2$. Other firms (which are also referred to as the "labor market") cannot observe individual outputs, but can observe which job the worker is assigned to at the end of the first period. They use this information to update their ability assessment for the worker. We assume that $\overline{a}$ is so high that there is at least one ability level such that the firm wants to promote the worker of that ability to the high-level job at the end of the first period.

At the beginning of the second period, other firms attempt to hire the worker by making wage offers. It is assumed that all wage offers (including the one from the current employer) are made simultaneously. The worker is hired by the firm making the highest offer. Ties are broken randomly except for the case in which the current employer is among the firms offering the highest wage. In this case, the worker remains with the current employer. We assume S to be sufficiently high so that, in equilibrium, firms are never successful at hiring the worker away from the first-period employer.[8] As in Greenwald (1986) and Waldman (2013), however, there is a (small) probability γ that the worker will switch employers after the first period for exogenous reasons that are unrelated to ability and job assignment; here, the decision to switch employers is taken only after job assignments have been made. As explained in these two papers and in the next subsection, these assumptions eliminate the winner's-curse effect.

Explicit incentive schemes that link pay to performance are not feasible; nor a long-term contracts that bind workers to the firm for both periods. There is no discounting.

3.2 Model solution

As described above, the firm observes the worker's first-period output and then decides whether to promote the worker, i.e., the promotion decision depends on the observed output level. According to equation (1), there is a unique linear relationship between first-period output and the worker's ability level a_j. This implies that the firm can perfectly infer the worker's ability from the output observation. In the following, we will thus write the promotion decision as a function of the realized ability level (instead of the output level). Denote by $A_1 \subseteq [\underline{a}, \overline{a}]$ the set of ability levels for which the firm decides to reassign the worker to the low-level job in $\tau = 2$, and by $A_2 \subseteq [\underline{a}, \overline{a}]$ the set of ability levels for which the worker is promoted to the high-level job. We assume that the firm always promotes the worker to job 2 when it is indifferent between assigning the worker to either job 1 or job 2. Under this assumption, $\{A_1, A_2\}$ is a partition of the set $[\underline{a}, \overline{a}]$. Denote the external firms' belief regarding $\{A_1, A_2\}$ by $\left\{\tilde{A}_1, \tilde{A}_2\right\}$.

While the worker's first-period employer can infer the worker's ability from the observation of y^1_{ij1}, the labor market's ability assessment of the worker depends on which task the worker is assigned to at the end of the first period, as does the worker's second-period wage, w_{j2}, as indicated by the following lemma ($E\left[\cdot|\cdot\right]$ denotes the conditional expectation operator).[9]

[8] See Lemma 1 in the subsequent section.

[9] In the wage-setting subgame at the beginning of the second period, there exist equilibria in which workers receive wages that differ from those specified in Lemma 1. As these equilibria involve weakly dominated strategies (and thus do not survive equilibrium refinements such as trembling-hand perfection), we neglect these equilibria in what follows.

Lemma 1 *a) There exists a threshold value $S_1 > 0$ such that external firms are never successful at hiring worker j away from the first-period employer if $S \geq S_1$.*
b) If $S \geq S_1$, then in any equilibrium in undominated strategies, the second-period wage for worker j when assigned to job k is given by

$$w_{j2}(k) = \max\left\{c_1 + d_1 E\left[a_j \left| a_j \in \tilde{A}_k\right.\right], c_2 + d_2 E\left[a_j \left| a_j \in \tilde{A}_k\right.\right]\right\}.$$

We are now in a position to calculate the firm's second-period profit. In what follows, we impose the assumption $S \geq S_1$. When the firm assigns the worker to job k, second-period profit then can be stated as:

$$\begin{aligned}\pi_i(k) &= (1-\gamma)\cdot\left(y_{ij2}^k - w_{j2}(k)\right)\\ &= (1-\gamma)\cdot\left[(1+S)(c_k + d_k a_j) - \max\left\{c_1 + d_1 E\left[a_j \left| a_j \in \tilde{A}_k\right.\right], c_2 + d_2 E\left[a_j \left| a_j \in \tilde{A}_k\right.\right]\right\}\right].\end{aligned}$$

Obviously, the firm wants to maximize the worker's output (which depends on worker ability), while at the same time wishing to keep wage costs as low as possible. The next proposition characterizes the firm's optimal promotion rule:

Proposition 1 *a) There exists a threshold value a_{opt}^p such that the firm promotes the worker at the end of $\tau = 1$ if and only if $a_j \geq a_{opt}^p$, i.e. $A_1 = \left[\underline{a}, a_{opt}^p\right)$ and $A_2 = \left[a_{opt}^p, \overline{a}\right]$.*
b) The worker's second-period wages on the two job levels are given by $w_{j2}(1) = c_1 + d_1 E\left[a_j | a_j < a_{opt}^p\right]$ and $w_{j2}(2) = c_2 + d_2 E\left[a_j | a_j \geq a_{opt}^p\right] > w_{j2}(1)$.
c) The threshold value is implicitly defined by $(1+S)\left(c_2 - c_1 + (d_2 - d_1)\, a_{opt}^p\right) = w_{j2}(2) - w_{j2}(1)$. Any solution to this condition satisfies $a_{opt}^p > a^e$.

Proposition 1 demonstrates that the worker is promoted at the end of the first period if and only if his ability is sufficiently high. As a consequence, promotion serves as a (positive) signal of worker ability and firms offer higher wages to promoted workers rather than to workers who are reassigned to the low-level job. Because of the wage increase in response to promotion, the firm promotes inefficiently few workers, i.e., the promotion standard a_{opt}^p exceeds the efficient standard of a^e. This replicates the main finding in Waldman (1984). Note that it is possible that the optimal promotion standard a_{opt}^p is not uniquely defined, i.e., the condition $(1+S)\left(c_2 - c_1 + (d_2 - d_1)\, a_{opt}^p\right) = w_{j2}(2) - w_{j2}(1)$ may have more than one solution.[10] We return to this issue in the next subsection, when we show how our model can be used to explain worker discrimination.

To conclude this section, we turn to the beginning of the first period. Because the labor market is competitive, firms are willing to pay a worker a wage such that their total expected profit over both periods is zero. Suppose that a_{opt}^p is uniquely defined, in which case firms are perfectly able to calculate second-period profit. Because of firm-specific human capital and asymmetric learning, the firms that manage to hire a worker in the first period earn a strictly positive profit in the second period. Therefore, they are willing to incur a loss in $\tau = 1$ so that the first-period wage exceeds expected first-period output. The first-period wage is given by:

$$\begin{aligned} w_1 &= c_1 + d_1 E\left[a_j\right] \\ &\quad + (1-\gamma) F\left(a_{opt}^p\right)\left((1+S)\left(c_1 + d_1 E\left[a_j | a_j < a_{opt}^p\right]\right) - w_{j2}(1)\right) \\ &\quad + (1-\gamma)\left(1 - F\left(a_{opt}^p\right)\right)\left((1+S)\left(c_2 + d_2 E\left[a_j | a_j \geq a_{opt}^p\right]\right) - w_{j2}(2)\right). \end{aligned} \tag{2}$$

We obtain the same expression for w_1 if a_{opt}^p is not uniquely defined, but all firms still manage to correctly anticipate the equilibrium that is played at the end of the first period.

3.3 Discrimination

Thus far, we have been silent about the issue of discrimination. The model is able to capture both endogenous and exogenous statistical discrimination. Furthermore, by slightly modifying the model, we could also address the situation in which firms discriminate against some workers because of distaste for these workers. We discuss these possibilities in turn.

We begin with *statistical discrimination* that emerges *endogenously*, as in Coate and Loury (1993b). Suppose that there are two different workers α and β: denote the pdfs characterizing the distribution of ability by f_α and f_β, respectively. Suppose that the two workers are identical ex ante, that is $f_\alpha = f_\beta$. Moreover, let there be more than one solution for a_{opt}^p to the condition $(1+S)\left(c_2 - c_1 + (d_2 - d_1)\, a_{opt}^p\right) = w_{j2}(2) - w_{j2}(1)$. It is then conceivable that the two workers face different promotion standards $a_{opt}^{p,\alpha}$ and $a_{opt}^{p,\beta}$ $\left(\neq a_{opt}^{p,\alpha}\right)$, although they are identical ex ante. The intuition for this result is the following: If external firms expect a

[10] In the appendix, we provide a specific example illustrating this possibility. Note that in the original model by Waldman (1984) ability was assumed to be uniformly distributed, in which case the optimal promotion standard is always unique.

worker's first-period employer to set a rather high promotion standard, they conclude that a promoted worker must be of exceptionally high ability. Therefore, they make very generous wage offers to promoted workers, which in turn, makes it optimal for the first-period employer to reassign the worker to the low-level job unless his performance is exceptionally high. In other words, the external firms' expectation of a very high promotion standard may become self-fulfilling in the current model. The same is true for a relatively lower promotion standard, implying that the optimal promotion standard may not be uniquely defined. Assume that worker α faces a lower promotion standard than worker β, $a_{opt}^{p,\alpha} < a_{opt}^{p,\beta}$. If both workers are identical both ex ante and ex post so that they have the same ability $\hat{a}$, and if $\hat{a} \in \left[a_{opt}^{p,\alpha}, a_{opt}^{p,\beta}\right)$, worker α is promoted to the high-level job, whereas worker β is reassigned to the low-level job. In addition, because the lower promotion standard $a_{opt}^{p,\alpha}$ is already inefficiently high, worker β receives lower total income than worker α. This means that two (ex ante and ex post) identical workers are treated differently and worker β is discriminated against. The latter effect requires that firms correctly anticipate the equilibrium that is played at the end of the first period. Here, it is conceivable that firms use identifiable factors such as the race or sex of a worker to coordinate equilibrium, implying discrimination against workers who are "trapped" in the inefficient equilibrium because of these factors.

When the solution for a_{opt}^{p} to the condition $(1+S)\left(c_2 - c_1 + (d_2 - d_1)\, a_{opt}^{p}\right) = w_{j2}(2) - w_{j2}(1)$ is unique, the model can still capture *exogenous statistical discrimination*. To see this, consider again two different workers α and β, but assume that the pdfs characterizing the distribution of ability are different, $f_\alpha \neq f_\beta$. If the ability distributions are different, it is typically the case that workers differ in their value of $c_2 - c_1 + d_2 E\left[a_j | a_j \geq a_{opt}^{p}\right] - d_1 E\left[a_j | a_j < a_{opt}^{p}\right] = w_{j2}(2) - w_{j2}(1)$. In turn, it is optimal for the firm to set different promotion standards for the two workers ($a_{opt}^{p,\alpha} \neq a_{opt}^{p,\beta}$) so that there are first-period output levels at which one of the workers is promoted, whereas the other worker is reassigned to the low-level job.

Finally, the model could also be modified to account for a firm's *taste-based discrimination* against workers. If employers dislike promoting members of some specific group of workers, they are likely to increase the promotion standard for these workers above the one we have determined before. Therefore, we could simply replace the above promotion standard with $a_{opt}^{p} + \Delta$, where $\Delta \geq 0$ measures how strongly the considered group of workers is discriminated against.

In the following section, we investigate the effects of positive discrimination policies on the model outcome. If the promotion standard is not unique, even a small policy intervention may induce firms and workers to switch from one equilibrium to another, implying a substantial change in the promotion standard. We avoid such difficulties by focusing on the effects of the policies conditional on a specific equilibrium being played. The easiest way to justify this procedure is to come up with conditions that guarantee that the solution for a^p_{opt} to the condition $(1+S)\left(c_2 - c_1 + (d_2 - d_1)\, a^p_{opt}\right) = w_{j2}(2) - w_{j2}(1)$ is unique.

4 Positive discrimination policy

We now introduce a positive discrimination policy into the model that is aimed at improving the career prospects of people who are discriminated against. In particular, we assume that the first-period employer is ordered to lower the promotion standard compared to the promotion standard he would normally set, and we denote the new promotion standard by $a^p (< a^p_{opt})$. This is a natural assumption. Given that performance is more responsive to ability in high-level than low-level jobs, the employer wants to promote the workers who he believes have the highest ability. If he is ordered to promote more workers from a specific group of workers than he would voluntarily (e.g., because some quota has been introduced), it is optimal for the employer to promote the next best workers, i.e., the best workers who would have been reassigned to the low-level job were the positive discrimination policy not in place. This is equivalent to lowering the promotion standard.

In the basic model, when an external firm observes that a worker was promoted, it would be optimal for that firm to also assign the worker to the high-level job. This is because the promotion standard in the basic model, a^p_{opt}, is inefficiently high, meaning that a promoted worker must be rather highly talented. If the first-period employer is ordered to lower the promotion standard to a^p, this is no longer necessarily true. In contrast, when a^p becomes too low, the promotion signal is rather weak so that an external firm would prefer to assign a promoted worker to the low-level job 1. This affects the worker's wage, as shown in Lemma 2:

Lemma 2 *Let $a^p \in (\underline{a}, \overline{a})$ be the promotion threshold and $w_{j2}(k)$ the second-period wage of a worker when assigned to job k as a function of a^p.*

a) For all $a^p \in (\underline{a}, \overline{a})$ *the second-period wage of a non-promoted worker is* $w_{j2}(1) = c_1 + d_1 E(a_j | a_j < a^p)$*.*
b) There exists a unique $\hat{a}^1 \in (\underline{a}, a^e)$ *such that*

$$E\left(a_j \left| a_j \geq \hat{a}^1\right.\right) = a^e.$$

c) For all $a^p \in (\underline{a}, \hat{a}^1)$ *the second-period wage of a promoted worker is* $w_{j2}(2) = c_1 + d_1 E(a_j | a_j \geq a^p)$ *and for all* $a^p \in [\hat{a}^1, \overline{a})$ *it is* $w_{j2}(2) = c_2 + d_2 E(a_j | a_j \geq a^p)$*.*

4.1 Workers who are already employed when the policy is introduced

We start by considering a worker who has already been hired and is in the first half (i.e., the first period) of his career when the policy is introduced. For such a worker, the first-period wage is not affected by the positive discrimination policy. In contrast, the policy affects the worker's second-period payoff, as shown in the following proposition:

Proposition 2 *Let* $a^p \in (\underline{a}, \overline{a})$ *be the promotion threshold and* $w_{j2}(k)$ *the second-period wage in job* k *as a function of* a^p*.*
a) $w_{j2}(k)$ *is strictly increasing in* a^p*.*
b) When a positive discrimination policy lowers the promotion standard from a^p_{opt} *to* a^p*, workers with ability* $a_j \in [\underline{a}, a^p) \cup \left[a^p_{opt}, \bar{a}\right]$ *receive a lower second-period wage and workers with ability* $a_j \in \left[a^p, a^p_{opt}\right)$ *receive a higher wage.*

A positive discrimination policy has two effects on the second-period payoff of a worker who is favored by the policy. First, the worker is more likely to be promoted and obtain a wage increase, which obviously benefits the worker. Second, the positive signal of promotion to the high-level job becomes weaker, whereas the negative signal of being reassigned to the low-level job becomes stronger. If the worker is promoted, external firms may question the worker's ability and may believe that the worker was promoted only because of the positive discrimination policy. If the worker is not promoted, external firms may believe that the worker's ability must be extremely low because he was not promoted in spite of the positive discrimination program. Because external firms interpret the job-assignment signal differently when a positive discrimination policy is in place, their wage offers also differ. In particular, both the wage for a promoted worker and a worker reassigned to the low-level job decrease, as shown in part a) of Proposition 2. This obviously hurts workers. The second part of the proposition demonstrates that the introduction of the positive discrimination policy leaves those workers who either have a very high ability so that they would have been promoted even without the policy, or who have such low ability that they are not considered for promotion even when the policy is in place, worse off. In contrast, workers in the middle of the ability distribution benefit from the policy because those workers are promoted if and only if the firm is bound to the positive discrimination policy.

It is possible that the negative effects on workers of either low or high ability outweigh the positive effects on the workers of middle ability so that a worker's expected payoff may actually decrease. As shown in Proposition 3, this happens only if the promotion standard becomes so low that inefficiently many workers are promoted:

Proposition 3 *Let $a^p \in (\underline{a}, \overline{a})$ be the promotion threshold and W_2 a worker's expected second-period wage as a function of a^p.*
a) Expected second-period wage corresponds to:

$$\begin{aligned} W_2(a^p) &= (1-\gamma) F(a^p)(c_1 + d_1 E(a_j | a_j < a^p)) \\ &\quad + (1-\gamma)(1-F(a^p)) \cdot \max\{c_1 + d_1 E(a_j | a_j \geq a^p), c_2 + d_2 E(a_j | a_j \geq a^p)\}. \end{aligned}$$

b) W_2 has a global maximum at a^e.
c) There exists a threshold $\hat{a}^p \in (\underline{a}, a^e)$ such that $W_2(a^p) < W_2(a^p_{opt})$ for all $a^p \in (\underline{a}, \hat{a}^p)$.

4.2 Workers who begin their working career after the policy is introduced

Consider now a worker who begins his working career only after a positive discrimination policy has been introduced. As indicated before, the difference from the argumentation in the preceding subsection is that the worker's first-period wage is no longer fixed so that the policy affects both the worker's first-period and second-period payoff. Proposition 4 discusses the effects of the policy:

Proposition 4 *Let $a^p \in (\underline{a}, \overline{a})$ be the promotion threshold and $w_{j,1+2}(k, a^p)$ the sum of first-period and second-period wage for worker j when assigned to job k in the second period, i.e.*

$w_{j,1+2} = w_1 + (1-\gamma)\, w_{j2}$.

a) The sum of first-period and second-period wage corresponds to:

$$
\begin{aligned}
& w_{j,1+2}(1, a^p) \\
&= c_1 + d_1 E(a_j) + (1-\gamma) F(a^p)\, S\left(c_1 + d_1 E\left(a_j \left| a_j < a^p \right.\right)\right) \\
&\quad + (1-\gamma)\left(1 - F(a^p)\right)(1+S)\left(c_2 + d_2 E\left(a_j \left| a_j \geq a^p \right.\right)\right) \\
&\quad - (1-\gamma)\left(1 - F(a^p)\right) \max\left\{c_1 + d_1 E\left(a_j \left| a_j \geq a^p \right.\right), c_2 + d_2 E\left(a_j \left| a_j \geq a^p \right.\right)\right\} \\
&\quad + (1-\gamma)\left(c_1 + d_1 E\left(a_j \left| a_j < a^p \right.\right)\right),
\end{aligned}
$$

$$
\begin{aligned}
& w_{j,1+2}(2, a^p) \\
&= w_{j,1+2}(1, a^p) + (1-\gamma) \max\left\{c_1 + d_1 E\left(a_j \left| a_j \geq a^p \right.\right), c_2 + d_2 E\left(a_j \left| a_j \geq a^p \right.\right)\right\} \\
&\quad - (1-\gamma)\left(c_1 + d_1 E\left(a_j \left| a_j < a^p \right.\right)\right).
\end{aligned}
$$

b) If $h_1(a^p) = \left(a^p - E\left(a_j \left| a_j < a^p \right.\right)\right) / F(a^p) + (S/d_1) \cdot (c_1 - c_2 - (d_2 - d_1) \cdot a^p)$ *has at most one root in the interval* $(a^e, \overline{a})$*, there exists a threshold* $\hat{a}_1^p \in (\underline{a}, a^p_{opt})$ *such that* $w_{j,1+2}(1, a^p) < w_{j,1+2}(1, a^p_{opt})$ *for all* $a^p \in (\underline{a}, \hat{a}_1^p)$.

c) If $h_2(a^p) = \left(E\left(a_j \left| a_j \geq a^p \right.\right) - a^p\right) / \left(1 - F(a^p)\right) + (S/d_1) \cdot (c_1 - c_2 - (d_2 - d_1) \cdot a^p)$ *has at most one root in the interval* $(a^e, \overline{a})$*, there exists a threshold* $\hat{a}_2^p \in (\underline{a}, a^p_{opt})$ *such that* $w_{j,1+2}(2, a^p) < w_{j,1+2}(2, a^p_{opt})$ *for all* $a^p \in (\underline{a}, \hat{a}_2^p)$.

As shown in Proposition 2, a worker's second-period wage conditional on the job level always decreases when the promotion standard is lowered from a^p_{opt} to a^p. In contrast, the first-period wage may well increase. Two effects are at work. First, the change in the promotion standard changes the worker's expected second-period compensation, which in turn, affects the firms' willingness to pay for the worker in the first period. Second, lowering the promotion standard changes the total output that the worker is expected to produce, which again affects firms' willingness to pay for the worker's services. Because of these (partly) opposing effects, a worker's total wage upon being assigned to a specific job in the second period decreases only under certain conditions when the promotion standard is lowered. Note in this context that the assumptions in parts b) and c) are fulfilled if, e.g., F represents a uniform distribution with $F(a^p) = (a^p - \underline{a}) / (\overline{a} - \underline{a})$. In this case $E\left(a_j \left| a_j < a^p \right.\right) = 0.5(a^p + \underline{a})$ and $E\left(a_j \left| a_j \geq a^p \right.\right) = 0.5(a^p + \overline{a})$ and both h_1 and h_2 are linear and strictly decreasing functions in a^p, which consequently have at most one root.

Whereas Proposition 4 assumed a specific task assignment in the second period, Proposition 5 determines the worker's expected total wage.

Proposition 5 *Let $a^p \in (\underline{a}, \overline{a})$ be the promotion threshold and W_{1+2} a worker's expected total wage as a function of a^p.*
a) Expected total wage corresponds to:

$$\begin{aligned} W_{1+2}(a^p) &= c_1 + d_1 E(a_j) + (1-\gamma) F(a^p)(1+S)(c_1 + d_1 E(a_j | a_j < a^p)) \\ &\quad + (1-\gamma)(1-F(a^p))(1+S)(c_2 + d_2 E(a_j | a_j \geq a^p)). \end{aligned}$$

b) W_{1+2} has a global maximum at a^e.
c) There exists a threshold $\hat{a}^p \in (\underline{a}, a^e)$ such that $W_{1+2}(a^p) < W_{1+2}(a^p_{opt})$ for all $a^p \in (\underline{a}, \hat{a}^p)$.

The change in the expected second-period payoff discussed in the preceding subsection is exactly offset by a change in the first-period wage. This is intuitive. Suppose that the expected second-period wage decreases. Then, firms are more interested in hiring the worker in the first period because it is more profitable to retain him in the second period. Firms are therefore willing to increase the first period wage, and in the case of a perfectly competitive labor market, this increase in the first-period wage simply offsets the decrease in expected second-period compensation. The effect of the positive discrimination policy on the worker's expected payoff thus depends only on whether the policy makes the firm's promotion decision more or less efficient. Because the promotion standard is inefficiently high when no positive promotion policy is in place, lowering the standard may lead to a more efficient promotion rule, thereby increasing the worker's payoff. It is also conceivable, however, that the promotion standard becomes inefficiently low and the worker's expected payoff is reduced.

In practice, whether a positive discrimination policy leads to a more or less efficient promotion rule depends on the demographic composition of the firm's workforce. Suppose that a policy requires a firm to promote at least five members of a specific group of people to the high-level job. If there are only five members of this group currently employed in the low-level job, the firm must promote all of them, rendering it likely that the promotion standard becomes inefficiently low. Instead, if the firm can select from a much larger pool of people, the effect of the policy is more muted, probably resulting in a more efficient promotion rule.

4.3 Effect on people disadvantaged by the policy

In this subsection, we briefly address the effects of a positive discrimination policy on workers who are disadvantaged by the policy and who are therefore less likely to be promoted. For those workers, the introduction of the policy is equivalent to an increase in the promotion standard a^p above a^p_{opt}. Using our previous results, it is clear that some of the workers who are supposed to be disadvantaged by the policy may actually be better off. As shown in Proposition 2, extremely able workers who manage to become promoted in spite of the policy receive a higher second-period wage because the positive signal of being promoted grows stronger. The same holds for rather unable workers who are not promoted even if there is no positive discrimination policy that makes it more difficult for them to move up the corporate ladder. In contrast, workers who are promoted if and only if no positive discrimination policy is in place are typically worse off. Given that the first-period employer promotes inefficiently few workers, a further increase in the promotion standard reduces expected output. In expectation, workers therefore suffer and receive a lower total wage. This may explain why in practice, members of an initially advantaged group rarely lobby to have positive promotion policies enacted, although some of them may actually be better off when such policies are introduced.

The following is a corollary to Proposition 2:

Corollary 1 *Let $w_{\alpha 2}(2)$ be the second-period wage of a worker who is disadvantaged by a positive promotion policy but still manages to be promoted and $w_{\beta 2}(1)$ the wage of a worker who is advantaged by the positive promotion rule but is not promoted. The corresponding promotion standards are denoted by a^p_α and a^p_β, respectively. Then the introduction of the policy, i.e. an increase in a^p_α and a decrease in a^p_β, increases the wage difference $w_{\alpha 2}(2) - w_{\beta 2}(1)$.*

As explained above, the wages of the highest earners (i.e., the most able members of the group that is disadvantaged by the policy) are expected to increase, whereas the wages of the lowest earners (i.e., the least able members of the group that is supposed to be favored by the policy) are likely to decrease. Thus, the introduction of a positive discrimination policy may aggravate wage inequality.

5 Empirical implications of the model

Several empirical implications can be derived from the model. A first and very trivial implication is that the introduction of a positive discrimination policy changes the employment of people from an initially disadvantaged group and an initially advantaged group in high-level jobs. While more people from the former group are promoted to the high-level job, there are fewer people of the latter group in the very same job. In line with this prediction, Holzer and Neumark (2000), surveying the literature on positive discrimination programs, conclude that these "programs redistribute employment [...] from white males to minorities and women" (p. 558). Recent studies such as Kurtulus (2012) and Bertrand et al. (2014) underscore this observation. Kurtulus (2012) finds that the share of minorities and women who are employed in high-paying skilled jobs in the US grew more between 1973 and 2003 in firms that were subject to affirmative action regulations than in firms that were not. Bertrand et al. (2014) study the effects of a law that was passed in Norway in 2003 that mandates forty percent representation of each gender on the boards of publicly traded companies. They observe that many firms changed their status to private after 2003 to be exempt from the law. The remaining firms significantly increased the number of female directors in the board, but only after the introduction of severe sanctions for noncompliance.

The most important implication of the model is that the introduction of a positive discrimination policy lowers the wages that workers who are advantaged by the policy earn later in their careers and increases the wages for workers who are disadvantaged by the policy when controlling for job level. The starting wage of workers who began their working careers before the policy was implemented is unaffected by the policy. This, in turn, implies that the wage increase upon promotion for these workers is changed by the positive promotion policy. For workers who are advantaged by the policy, the post-promotion wage decreases, while the starting wage is fixed so that the wage increase upon promotion also decreases. For workers who are disadvantaged by the policy, the effect is the opposite and the wage increase upon promotion increases. Note that this is exactly what Bertrand et al. (2014) find in their analysis of Norwegian companies. They show that election to the board of a company entails a substantial financial reward for the elected worker. After the board quota was introduced, this reward fell for women (from 9.4% percent of annual earnings to 8%), whereas it increased substantially for men (from 4.6% to 10%).

For workers who begin their working careers only after a positive discrimination policy is introduced, the starting wage is also influenced by the policy. The starting wage increases whenever the policy increases efficiency and decreases the worker's expected wages later on in his career. Whether this is true depends, among other things, on how restrictive the policy is. When the policy is rather moderate, efficiency is likely to increase. When it is more restrictive, however, efficiency is likely to suffer. It would therefore be interesting to study the effects of positive discrimination policies of different magnitude on workers' wages. To our knowledge, there has been no such study to date.

6 Conclusions

In this paper, we consider a model in which promotions are used as signals of worker ability, and we examine the impact of a positive discrimination policy. The policy lowers the promotion standard for the workers who are discriminated against. This is beneficial for the workers in the middle of the ability distribution because these workers are promoted if and only if the policy is in place. In contrast, workers of either high or low ability generally suffer from the policy. This is because the policy does not change the promotion decision for these workers, but rather weakens the positive signal of being promoted and strengthens the negative signal of not being promoted. It is also found that the policy may increase or decrease efficiency and may aggravate wage inequality.

More generally, the findings imply that policies aimed at "leveling the playing field" are not always as beneficial as they may appear. If workers succeed in spite of many obstacles, the labor market learns a great deal about their characteristics, so it can reward the workers generously on this basis.[11]

[11] A related argument is advanced in Krishnamurthy and Edlin (2014). In a study of college admission rules they find that stereotypes against a disadvantaged group of students can only be eliminated if these students face higher admission standards. Formally, they assume that the ability distributions of the disadvantaged and the advantaged students satisfy the monotone likelihood ratio property. This assumption implies that the expected ability of admitted students can only be equalized across groups when the students of the disadvantaged groups have to meet a higher standard in order to be admitted to college.

Appendix

Proofs of lemmas and propositions:

Proof of Lemma 1. a) The maximum wage that firms are willing to pay a worker in the second period corresponds to the worker's expected second-period output. If the current employer believes the worker to possess ability $\tilde{a}_j$, the maximum wage he is willing to pay is not smaller than $(1+S)(c_1+d_1\tilde{a}_j)$ since he always has the option to assign the worker to job 1. In comparison, if some external firm believes the very same worker to possess the ability $\tilde{\tilde{a}}_j$, this firm is willing to pay a wage equal to $\max\left\{c_1+d_1\tilde{\tilde{a}}_j, c_2+d_2\tilde{\tilde{a}}_j\right\}$. Because $\tilde{a}_j, \tilde{\tilde{a}}_j \in [\underline{a}, \overline{a}]$, external firms never succeed in hiring worker j away from the first-period employer if

$$(1+S)(c_1+d_1\underline{a}) \geq c_2+d_2\overline{a}\,(=\max\{c_1+d_1\overline{a}, c_2+d_2\overline{a}\})$$

i.e. if they do not manage to hire worker j away even when they hold the most optimistic belief about the worker's ability, whereas the current employer holds the most pessimistic belief. The above condition simplifies to $S \geq (c_2-c_1+d_2\overline{a}-d_1\underline{a})/(c_1+d_1\underline{a})$, which proves part a) of the lemma with $S_1 := (c_2-c_1+d_2\overline{a}-d_1\underline{a})/(c_1+d_1\underline{a}) > 0$.

b) Suppose that $S \geq S_1$. Then the expected ability of workers that are actually switching firms is equal to the overall expected ability of workers (conditional on job assignment at the end of period 1). This is because workers are never successfully hired away (as shown in part a)), but a small fraction of workers leaves the first-period employer for reasons that are unrelated to ability and job assignment.

Consider some worker j. In equilibrium, the worker's first-period employer will exactly match the highest offer that the worker receives from the external firms. It therefore remains to determine this latter offer, which we denote by w_{j2}. Suppose that $w_{j2} < \max\left\{c_1+d_1E\left[a_j \middle| a_j \in \tilde{A}_k\right], c_2+d_2E\left[a_j \middle| a_j \in \tilde{A}_k\right]\right\} =: Z$. In this case, there is at least one external firm that gains by deviating and offering a wage from the interval (w_{j2}, Z). Thus, in equilibrium we never observe $w_{j2} < Z$. Note that, for any of the external firms, the offer of a wage above Z is (weakly) dominated by the offer of a wage equal to Z. Thus, in any equilibrium in undominated strategies none of the external firms offers a wage above Z.

Finally, it is very easy to confirm the existence of an equilibrium in which the worker receives $w_{j2} = Z$. For instance, a situation in which all the firms offer such a wage represents an equilibrium. ■

Proof of Proposition 1. a) Consider arbitrary $\breve{a}_1 \in A_1$ and $\breve{a}_2 \in A_2$ which implies that the firm does not want to promote the worker when observing $\breve{a}_1$ and wants to promote the worker when observing $\breve{a}_2$, i.e.:

$$\begin{aligned}(1+S)(c_2+d_2\breve{a}_2)-w_{j2}(2) &\geq (1+S)(c_1+d_1\breve{a}_2)-w_{j2}(1)\\ (1+S)(c_2+d_2\breve{a}_1)-w_{j2}(2) &< (1+S)(c_1+d_1\breve{a}_1)-w_{j2}(1).\end{aligned}$$

Rearranging the two conditions leads to

$$\begin{aligned}(1+S)(c_2-c_1+(d_2-d_1)\breve{a}_2) &\geq w_{j2}(2)-w_{j2}(1)\\ &> (1+S)(c_2-c_1+(d_2-d_1)\breve{a}_1),\end{aligned} \tag{A1}$$

which immediately implies $\breve{a}_2 > \breve{a}_1$. Hence, because we assumed that $\overline{a}$ is so high that there is at least one ability level such that the firm wants to promote the worker of that ability, the two sets A_1 and A_2 must be of the form $A_1 = \left[\underline{a}, a^p_{opt}\right)$ and $A_2 = \left[a^p_{opt}, \overline{a}\right]$, with $a^p_{opt} \in [\underline{a}, \overline{a}]$.

b) In equilibrium the external firms correctly anticipate the firm's promotion rule as specified in a), i.e. $\tilde{A}_k = A_k$ for $k = 1, 2$. Thus, the worker's second-period wages (conditional on job assignment) can be stated as

$$w_{j2}(1) = \max\left\{c_1+d_1E\left[a_j|a_j<a^p_{opt}\right], c_2+d_2E\left[a_j|a_j<a^p_{opt}\right]\right\}$$

and

$$w_{j2}(2) = \max\left\{c_1+d_1E\left[a_j|a_j\geq a^p_{opt}\right], c_2+d_2E\left[a_j|a_j\geq a^p_{opt}\right]\right\} > w_{j2}(1).$$

Since $E\left[a_j|a_j<a^p_{opt}\right] \leq E[a_j] < a^e$, it follows that $c_1+d_1E\left[a_j|a_j<a^p_{opt}\right] > c_2 + d_2E\left[a_j|a_j<a^p_{opt}\right]$ implying

$$w_{j2}(1) = c_1+d_1E\left[a_j|a_j<a^p_{opt}\right].$$

Because $a^p_{opt} \in A_2$ and $w_{j2}(2)-w_{j2}(1) > 0$, inequality (A1) leads to

$$(1+S)\left(c_2-c_1+(d_2-d_1)a^p_{opt}\right) > 0 \quad \Rightarrow \quad c_2+d_2a_j > c_1+d_1a_j \quad \text{for all} \quad a_j \geq a^p_{opt}.$$

This immediately implies $c_2+d_2E\left[a_j|a_j\geq a^p_{opt}\right] \geq c_1+d_1E\left[a_j|a_j\geq a^p_{opt}\right]$ so that

$$w_{j2}(2) = c_2+d_2E\left[a_j|a_j\geq a^p_{opt}\right].$$

c) Analogous to (A1) the promotion threshold a^p_{opt} is implicitly defined by the condition

$$(1+S)\left(c_2-c_1+(d_2-d_1)a^p_{opt}\right) = w_{j2}(2)-w_{j2}(1) > 0.$$

If we define $g(a) := c_2 - c_1 + (d_2 - d_1)\, a$ this implies $g(a^p_{opt}) > 0$. Because

$$g(a^e) = c_2 - c_1 + (d_2 - d_1)\, a^e = c_2 - c_1 + (d_2 - d_1)\, \frac{c_1 - c_2}{d_2 - d_1} = 0$$

and g is strictly increasing it follows that $a^p_{opt} > a^e$. ■

Proof of Lemma 2. a) The statement immediately follows from Proposition 1.
b) Obviously, $E(a_j \,|a_j \geq a)$ is strictly increasing and continuous in a. Because $E(a_j \,|a_j \geq \underline{a}) = E(a_j) < a^e$ and $E(a_j \,|a_j \geq a^e) > a^e$, the statement of a) immediately results.
c) Because $c_2 - c_1 + (d_2 - d_1)E(a_j \,|a_j \geq a)$ is strictly increasing and continuous in a and $c_2 - c_1 + (d_2 - d_1)a^e = 0$ it follows

$$c_1 + d_1 E(a_j \,|a_j \geq a) < (\geq)\; c_2 + d_2 E(a_j \,|a_j \geq a) \quad \textit{for all } a > (\leq)\; \hat{a}^1.$$

Under consideration of this inequality the statement of part c) follows from the results regarding second-period wages presented in Lemma 1. ■

Proof of Proposition 2. a) The statement immediately follows from the wage formula in Lemma 2 because $c_k + d_k E(a_j \,|a_j < a)$ as well as $c_k + d_k E(a_j \,|a_j \geq a)$ are increasing functions in a for $k \in \{1, 2\}$.
b) Case 1: $a_j \in [\underline{a}, a^p)$
In this case the worker is neither promoted if the promotion standard is a^p_{opt} nor if the promotion standard is a^p. Consequently, the wage difference amounts to

$$w^p_{j2}(1) - w^p_{j2,opt}(1) = d_1 \left(E(a_j \,|a_j < a^p) - E\left(a_j \,|a_j < a^p_{opt}\right)\right) < 0.$$

The latter inequality again results from the fact that $E(a_j \,|a_j < a)$ is increasing in a.
Case 2: $a_j \in \left[a^p_{opt}, \overline{a}\right)$
In this case the worker is promoted for both promotion standards a^p_{opt} and a^p and it can be analogously shown that $w^p_{j2}(2) - w^p_{j2,opt}(2) < 0$.
Case 3: $a_j \in \left[a^p, a^p_{opt}\right)$
In this case the worker is not promoted if the promotion standard is a^p_{opt} but is promoted if the promotion standard is a^p. We first consider $a^p \geq \hat{a}^1$ where $\hat{a}^1$ is defined in Lemma 2.

The resulting wage difference corresponds to

$$\begin{aligned}
& w^p_{j2}(2) - w^p_{j2,opt}(1) \\
= \; & c_2 - c_1 + d_1 \left(E\left(a_j \left| a_j \geq a^p \right.\right) - E\left(a_j \left| a_j < a^p_{opt} \right.\right)\right) \\
& + (d_2 - d_1) E\left(a_j \left| a_j \geq a^p \right.\right) \\
> \; & c_2 - c_1 + (d_2 - d_1) E\left(a_j \left| a_j \geq \hat{a}^1 \right.\right) = 0.
\end{aligned}$$

The latter inequality results because

$$E\left(a_j \left| a_j \geq a^p \right.\right) > E(a_j) > E\left(a_j \left| a_j < a^p_{opt} \right.\right).$$

If $a^p < \hat{a}^1$ and under consideration of Lemma 2 the wage difference is equal to

$$w^P_{j2}(2) - w^P_{j2,opt}(1) = d_1 \left(E\left(a_j \left| a_j \geq a^p \right.\right) - E\left(a_j \left| a_j < a^p_{opt} \right.\right)\right) > 0.$$

■

Proof of Proposition 3. a) Obvious given the results of Lemma 2.
b) For all $a \in (\underline{a}, \hat{a}^1)$ it follows from Lemma 2 using the law of total expectation:

$$\begin{aligned}
W_2(a) &= (1-\gamma) F(a^p) \left(c_1 + d_1 E\left(a_j \left| a_j < a^p \right.\right)\right) \\
&\quad + (1-\gamma)(1 - F(a^p)) \left(c_1 + d_1 E\left(a_j \left| a_j \geq a^p \right.\right)\right) \\
&= (1-\gamma)(c_1 + d_1 E[a_j])
\end{aligned}$$

which implies $W_2'(a) = 0$. In contrast, for all $a \in (\hat{a}^1, \overline{a})$ the function W_2 corresponds to

$$\begin{aligned}
W_2(a) &= B + F(a) \cdot \left(C_1 + D_1 \cdot E\left(a_j \left| a_j < a \right.\right)\right) \\
&\quad + (1 - F(a)) \cdot \left(C_2 + D_2 \cdot E\left(a_j \left| a_j \geq a \right.\right)\right) \qquad \text{(A2)}
\end{aligned}$$

where $B := 0$,[12] $C_1 := (1-\gamma)c_1 > (1-\gamma)c_2 =: C_2$ and $D_1 := (1-\gamma)d_1 < (1-\gamma)d_2 =: D_2$. W_2 simplifies to

$$\begin{aligned}
W_2(a) &= B + C_2 + (C_1 - C_2) F(a) + D_1 E(a_j) + (D_2 - D_1)(1 - F(a)) \cdot E\left(a_j \left| a_j \geq a \right.\right) \\
&= B + C_2 + (C_1 - C_2) F(a) + D_1 E(a_j) + (D_2 - D_1) \int_a^{\overline{a}} a_j f(a_j) da_j.
\end{aligned}$$

The derivative

$$W_2'(a) = (C_1 - C_2 - (D_2 - D_1) \cdot a) \cdot f(a) \qquad \text{(A3)}$$

[12] The parameter B is not relevant in the present proof but it is helpful for reference in another proof.

leads to the following necessary condition of an extremum

$$W_2'(a^*) = 0 \quad \Leftrightarrow \quad a^* = \frac{C_1 - C_2}{D_2 - D_1} = a^e.$$

Furthermore,

$$W_2''(a) = -(D_2 - D_1) f(a) + (C_1 - C_2 - (D_2 - D_1) \cdot a) \cdot f'(a).$$

Because $(C_1 - C_2) - (D_2 - D_1)a^e = 0$ it immediately results $W_2''(a^e) < 0$ which implies a local maximum of W_2 at a^e. Because a^e is the only zero of W_2' in $(\hat{a}^1, \overline{a})$ and W_2 is constant over $[\underline{a}, \hat{a}^1)$ as well as continuous over $[\underline{a}, \overline{a}]$, a^e is the global maximum of W_2.
c) Since $W_2'(a) < 0$ for all $a \in (a^e, \overline{a}) \supset (a^p_{opt}, \overline{a})$, we have

$$W_2(\underline{a}) = C_1 + D_1 \cdot E(a_j) = W_2(\overline{a}) < W_2(a^p_{opt}).$$

Because, moreover, W_2 is continuous, part c) is proven. ■

Proof of Proposition 4. a) The statement is a direct consequence of (2) (using a^p instead of a^p_{opt}) and Lemma 2.
b) Case 1: $a^p \leq \hat{a}^1$

$$\begin{aligned}
& w_{j,1+2}(1, a^p) \\
&= c_1 + d_1 E(a_j) + (1-\gamma) F(a^p) S (c_1 + d_1 E(a_j | a_j < a^p)) \\
&\quad + (1-\gamma)(1 - F(a^p))(1+S)(c_2 + d_2 E(a_j | a_j \geq a^p)) \\
&\quad - (1-\gamma)(1 - F(a^p))(c_1 + d_1 E(a_j | a_j \geq a^p)) \\
&\quad + (1-\gamma)(c_1 + d_1 E(a_j | a_j < a^p)) \\
&= B_1 + A_1 E(a_j | a_j < a^p) + F(a^p)(C_1 + D_1 E(a_j | a_j < a^p)) \\
&\quad + (1 - F(a^p)) \left(\hat{C}_2 + \hat{D}_2 E(a_j | a_j \geq a^p)\right)
\end{aligned}$$

with $A_1 := (1-\gamma)d_1$, $B_1 := c_1 + d_1 E(a_j) + (1-\gamma)c_1$, $C_1 := (1-\gamma)Sc_1$, $D_1 := (1-\gamma)Sd_1$, $\hat{C}_2 := (1-\gamma)((1+S)c_2 - c_1)$, and $\hat{D}_2 := (1-\gamma)((1+S)d_2 - d_1)$. Except for $A_1 E(a_j | a_j < a^p)$ the latter term obviously is of type (A2) and according to (A3) the derivative corresponds to

$$\frac{\partial w_{j,1+2}(1, a^p)}{\partial a^P} = A_1 \frac{\partial E(a_j | a_j < a^p)}{\partial a^P} + \left(C_1 - \hat{C}_2 - (\hat{D}_2 - D_1) \cdot a^p\right) \cdot f(a^p).$$

Furthermore,

$$\begin{aligned}
&\frac{\partial E\left(a_j\left|a_j<a^p\right.\right)}{\partial a^p} \\
&= \frac{\partial}{\partial a^p}\left(\frac{1}{F\left(a^p\right)}\int_{\underline{a}}^{a^p} a_j f(a_j da_j\right) = -\frac{f\left(a^p\right)}{F^2\left(a^p\right)}\int_{\underline{a}}^{a^p} a_j f(a_j)da_j + \frac{1}{F\left(a^p\right)}a^p f\left(a^p\right) \\
&= \frac{f\left(a^p\right)}{F\left(a^p\right)}\left(a^p - E\left(a_j\left|a_j<a^p\right.\right)\right).
\end{aligned}$$

Consequently,

$$\begin{aligned}
&\frac{\partial w_{j,1+2}(1,a^p)}{\partial a^P} \\
&= A_1\frac{f\left(a^p\right)}{F\left(a^p\right)}\left(a^p - E\left(a_j\left|a_j<a^p\right.\right)\right) + \left(C_1 - \hat{C}_2 - (\hat{D}_2 - D_1)\cdot a^p\right)\cdot f\left(a^p\right) \\
&= (1-\gamma)f\left(a^p\right)\left(\frac{d_1}{F\left(a^p\right)}\left(a^p - E\left(a_j\left|a_j<a^p\right.\right)\right) + (1+S)\left(c_1 - c_2 - (d_2-d_1)\cdot a^p\right)\right).
\end{aligned}$$

Obviously, both summands in parentheses are positive for all $a^p \leq \hat{a}^1 < a^e$.

Case 2: $a^p > \hat{a}^1$

$$\begin{aligned}
&w_{j,1+2}(1,a^p) \\
&= c_1 + d_1E(a_j) + (1-\gamma)F\left(a^p\right)S\left(c_1 + d_1E\left(a_j\left|a_j<a^p\right.\right)\right) \\
&\quad +(1-\gamma)\left(1-F\left(a^p\right)\right)S\left(c_2 + d_2E\left(a_j\left|a_j\geq a^p\right.\right)\right) \\
&\quad +(1-\gamma)\left(c_1 + d_1E\left(a_j\left|a_j<a^p\right.\right)\right) \\
&= B_1 + A_1E\left(a_j\left|a_j<a^p\right.\right) + F\left(a^p\right)\left(C_1 + D_1E\left(a_j\left|a_j<a^p\right.\right)\right) \\
&\quad +\left(1-F\left(a^p\right)\right)\left(C_2 + D_2E\left(a_j\left|a_j\geq a^p\right.\right)\right)
\end{aligned}$$

with $C_2 := (1-\gamma)Sc_2$, and $D_2 := (1-\gamma)Sd_2$. Analogously to the first case it follows that

$$\begin{aligned}
&\frac{\partial w_{j,1+2}(1,a^p)}{\partial a^p} \\
&= A_1\frac{f\left(a^p\right)}{F\left(a^p\right)}\left(a^p - E\left(a_j\left|a_j<a^p\right.\right)\right) + \left(C_1 - C_2 - (D_2-D_1)\cdot a^p\right)\cdot f\left(a^p\right) \\
&= (1-\gamma)f\left(a^p\right)\left(\frac{d_1}{F\left(a^p\right)}\left(a^p - E\left(a_j\left|a_j<a^p\right.\right)\right) + S\left(c_1 - c_2 - (d_2-d_1)\cdot a^p\right)\right).
\end{aligned}$$

According to the assumption of the present part the derivative of $w_{j,1+2}(1,a^p)$ has at most one root in the interval $(a^e,\overline{a})$. Consequently, under consideration of the first case $w_{j,1+2}(1,a^p)$ is strictly increasing on $(\underline{a},\overline{a})$ or attains a unique local (and thus global) maximum at $a^p \in$

$(a^e, \overline{a})$. In addition,

$$\begin{aligned}
& w_{j,1+2}(1, \underline{a}) := \lim_{\epsilon \to 0} w_{j,1+2}(1, \underline{a} + \epsilon) \\
&= B_1 + A_1 E\left(a_j \left| a_j \leq \underline{a}\right.\right) + F\left(\underline{a}\right)\left(C_1 + D_1 E\left(a_j \left| a_j \leq \underline{a}\right.\right)\right) \\
&\quad + \left(1 - F\left(\underline{a}\right)\right)\left(\hat{C}_2 + \hat{D}_2 E\left(a_j \left| a_j > \underline{a}\right.\right)\right) \\
&= B_1 + A_1 \underline{a} + \hat{C}_2 + \hat{D}_2 E\left(a_j\right) \\
&= c_1 + d_1 E(a_j) + (1-\gamma) \cdot c_1 + (1-\gamma) d_1 \cdot \underline{a} \\
&\quad + (1-\gamma)\left((1+S) \cdot c_2 - c_1 + ((1+S) \cdot d_2 - d_1) E(a_j)\right) \\
&< c_1 + d_1 E(a_j) + (1-\gamma)\left((1+S) \cdot c_2 + (1+S) \cdot d_2 \cdot E(a_j)\right)
\end{aligned}$$

and

$$\begin{aligned}
& w_{j,1+2}(1, \overline{a}) \\
&= B_1 + A_1 E\left(a_j \left| a_j < \overline{a}\right.\right) + F\left(\overline{a}\right)\left(C_1 + D_1 E\left(a_j \left| a_j < \overline{a}\right.\right)\right) \\
&\quad + \left(1 - F\left(\overline{a}\right)\right)\left(C_2 + D_2 E\left(a_j \left| a_j \geq \overline{a}\right.\right)\right) \\
&= B_1 + A_1 E(a_j) + C_1 + D_1 E\left(a_j\right) \\
&= c_1 + d_1 E(a_j) + (1-\gamma)(1+S) c_1 + (1-\gamma)(1+S) d_1 E(a_j)
\end{aligned}$$

which implies

$$w_{j,1+2}(1, \overline{a}) - w_{j,1+2}(1, \underline{a}) > (1-\gamma)(1+S)(c_1 - c_2 + (d_1 - d_2) E(a_j)).$$

The latter term is positive because $E(a_j) < a^e$ and therefore $w_{j,1+2}(1, \overline{a}) > w_{j,1+2}(1, \underline{a})$. Against the background of the above mentioned monotonicity behavior of $w_{j,1+2}(1, a^p)$ there are two possibilities. On one hand, $w_{j,1+2}(1, a^p)$ can be strictly increasing on $(\underline{a}, \overline{a})$ which immediately implies $w_{j,1+2}(1, \underline{a}) < w_{j,1+2}(1, a^p_{opt})$. On the other hand $w_{j,1+2}(1, a^p)$ can have a unique maximum $a^* \in (\underline{a}, \overline{a})$. If $a^p_{opt} \leq a^*$ the considered function again is strictly increasing on $(\underline{a}, a^p_{opt})$. If $a^p_{opt} > a^*$ the wage function is decreasing on $(a^p_{opt}, \overline{a})$ and consequently $w_{j,1+2}(1, a^p_{opt}) > w_{j,1+2}(1, \overline{a}) > w_{j,1+2}(1, \underline{a})$. To sum up, we obtain $w_{j,1+2}(1, a^p_{opt}) > w_{j,1+2}(1, \underline{a})$ in each case and the statement of part b) results from the continuity of the wage function.

c) Case 1: $a^p \leq \hat{a}^1$

$$\begin{aligned}
& w_{j,1+2}(2, a^p) \\
= \;& c_1 + d_1 E(a_j) + (1-\gamma) F(a^p) S\left(c_1 + d_1 E\left(a_j \left| a_j < a^p \right.\right)\right) \\
& +(1-\gamma)\left(1 - F(a^p)\right)\left((1+S)c_2 - c_1 + ((1+S)d_2 - d_1) E\left(a_j \left| a_j \geq a^p \right.\right)\right) \\
& +(1-\gamma)\left(c_1 + d_1 E\left(a_j \left| a_j \geq a^p \right.\right)\right) \\
= \;& B_1 + A_1 E\left(a_j \left| a_j \geq a^p \right.\right) + F(a^p)\left(C_1 + D_1 E\left(a_j \left| a_j < a^p \right.\right)\right) \\
& + \left(1 - F(a^p)\right)\left(\hat{C}_2 + \hat{D}_2 E\left(a_j \left| a_j \geq a^p \right.\right)\right).
\end{aligned}$$

Again, we apply (A3) and get the derivative

$$\frac{\partial w_{j,1+2}(2, a^p)}{\partial a^P} = A_1 \frac{\partial E\left(a_j \left| a_j \geq a^p \right.\right)}{\partial a^P} + \left(C_1 - \hat{C}_2 - (\hat{D}_2 - D_1) \cdot a^p\right) \cdot f(a^p).$$

Since

$$\begin{aligned}
& \frac{\partial E\left(a_j \left| a_j \geq a^p \right.\right)}{\partial a^p} \\
= \;& \frac{\partial}{\partial a^p}\left(\frac{1}{1 - F(a^p)} \int_{a^p}^{\overline{a}} a_j f(a_j) da_j\right) = \frac{f(a^p)}{\left(1 - F(a^p)\right)^2} \int_{a^p}^{\overline{a}} a_j f(a_j) da_j - \frac{1}{1 - F(a^p)} a^p f(a^p) \\
= \;& \frac{f(a^p)}{1 - F(a^p)}\left(E\left(a_j \left| a_j \geq a^p \right.\right) - a^p\right)
\end{aligned}$$

we obtain

$$\begin{aligned}
& \frac{\partial w_{j,1+2}(2, a^p)}{\partial a^P} \\
= \;& A_1 \frac{f(a^p)}{1 - F(a^p)}\left(E\left(a_j \left| a_j \geq a^p \right.\right) - a^p\right) + \left(C_1 - \hat{C}_2 - (\hat{D}_2 - D_1) \cdot a^p\right) \cdot f(a^p) \\
= \;& (1-\gamma) f(a^p)\left(\frac{d_1}{1 - F(a^p)}\left(E\left(a_j \left| a_j \geq a^p \right.\right) - a^p\right) + (1+S)\left(c_1 - c_2 - (d_2 - d_1) \cdot a^p\right)\right).
\end{aligned}$$

Again, both summands in parentheses are positive for all $a^p \leq \hat{a}^1 < a^e$.

Case 2: $a^p > \hat{a}^1$

$$\begin{aligned}
& w_{j,1+2}(2, a^p) \\
= \;& c_1 + d_1 E(a_j) + (1-\gamma) F(a^p) S\left(c_1 + d_1 E\left(a_j \left| a_j < a^p \right.\right)\right) \\
& +(1-\gamma)\left(1 - F(a^p)\right) S\left(c_2 + d_2\right) E\left(a_j \left| a_j \geq a^p \right.\right)\big) \\
& +(1-\gamma)\left(c_2 + d_2 E\left(a_j \left| a_j \geq a^p \right.\right)\right) \\
= \;& B_2 + A_2 E\left(a_j \left| a_j \geq a^p \right.\right) + F(a^p)\left(C_1 + D_1 E\left(a_j \left| a_j < a^p \right.\right)\right) \\
& + \left(1 - F(a^p)\right)\left(C_2 + D_2 E\left(a_j \left| a_j \geq a^p \right.\right)\right)
\end{aligned}$$

with $A_2 := (1-\gamma)d_2$, $B_2 := c_1 + d_1 E(a_j) + (1-\gamma)c_2$. Analogously to the first case we get the derivative

$$\begin{aligned}
&\frac{\partial w_{j,1+2}(2,a^p)}{\partial a^P} \\
&= (1-\gamma) f\left(a^p\right) \left(\frac{d_2}{1-F\left(a^p\right)} \left(E\left(a_j \left| a_j \geq a^p\right.\right) - a^p\right) + S\left(c_1 - c_2 - (d_2 - d_1) \cdot a^p\right) \right).
\end{aligned}$$

According to the assumption of part c) the derivative of $w_{j,1+2}(2,a^p)$ has at most one root in the interval $(a^e, \overline{a})$. Thus, $w_{j,1+2}(2,a^p)$ is strictly increasing on $(\underline{a}, \overline{a})$ or attains a unique local (and thus global) maximum at $a^p \in (a^e, \overline{a})$. In accordance with the proof of part b) it is sufficient to show that $w_{j,1+2}(2,\overline{a}) > w_{j,1+2}(2,\underline{a})$:

$$\begin{aligned}
& w_{j,1+2}(2,\underline{a}) \\
&= B_1 + A_1 E\left(a_j \left| a_j \geq \underline{a}\right.\right) + F\left(\underline{a}\right)\left(C_1 + D_1 E\left(a_j \left| a_j < \underline{a}\right.\right)\right) \\
&\quad + \left(1 - F\left(\underline{a}\right)\right)\left(\hat{C}_2 + \hat{D}_2 E\left(a_j \left| a_j \geq \underline{a}\right.\right)\right) \\
&= B_1 + A_1 E(a_j) + \hat{C}_2 + \hat{D}_2 E\left(a_j\right) \\
&= c_1 + d_1 E(a_j) + (1-\gamma) \cdot c_1 + (1-\gamma) d_1 E(a_j) \\
&\quad + (1-\gamma)\left((1+S) \cdot c_2 + (1+S) d_2 E(a_j)\right)
\end{aligned}$$

and

$$\begin{aligned}
& w_{j,1+2}(2,\overline{a}) \\
&= B_2 + A_2 E\left(a_j \left| a_j \geq \overline{a}\right.\right) + F\left(\overline{a}\right)\left(C_1 + D_1 E\left(a_j \left| a_j < \overline{a}\right.\right)\right) \\
&\quad + \left(1 - F\left(\overline{a}\right)\right)\left(C_2 + D_2 E\left(a_j \left| a_j \geq \overline{a}\right.\right)\right) \\
&= B_2 + A_2 \overline{a} + C_1 + D_1 E\left(a_j\right) \\
&= c_1 + d_1 E(a_j) + (1-\gamma)(S c_1 + c_2) + (1-\gamma) d_2 \overline{a} + (1-\gamma) S d_1 E(a_j)
\end{aligned}$$

leads to

$$w_{j,1+2}(2,\overline{a}) - w_{j,1+2}(2,\underline{a}) = (1-\gamma) S (c_1 - c_2 + (d_1 - d_2) E(a_j)) + (1-\gamma) d_2 (\overline{a} - E(a_j)) > 0.$$

The latter inequality applies because $E(a_j) < a^e$ leads to a positive first summand. ■

Proof of Proposition 5. a) The statement results immediately from (2).

b) and c) The proof is the same as the proof of Proposition 3 (in the case $a \in (\hat{a}^1, \overline{a})$) with

W_{1+2} replacing W_2 and $B := d_1 + c_1 E[a_j]$, $C_1 := (1-\gamma)(1+S)c_1 > (1-\gamma)(1+S)c_2 =: C_2$ and $D_1 := (1-\gamma)(1+S)d_1 < (1-\gamma)(1+S)d_2 =: D_2$. ■

Example to illustrate potential multiplicity of optimal promotion standard:

We provide an example to illustrate that there may be multiple solutions to the condition

$$(1+S)\left(c_2 - c_1 + (d_2 - d_1)\, a^p_{opt}\right) = w_{j2}(2) - w_{j2}(1),$$

which can be restated as

$$\begin{aligned} & (1+S)(d_2-d_1)\, a^p_{opt} + S(c_2 - c_1) \\ = \; & \frac{d_2}{1-F\left(a^p_{opt}\right)} \int_{a^p_{opt}}^{\bar{a}} a_j f(a_j)\, da_j - \frac{d_1}{F\left(a^p_{opt}\right)} \int_{\underline{a}}^{a^p_{opt}} a_j f(a_j)\, da_j. \end{aligned}$$

In the example, we make the following assumptions regarding the parameters:

$$\underline{a} = c_2 = 0, d_1 = 0.00001, \bar{a} = c_1 = 1, d_2 = 1.57, S = S_1 = 0.57.$$

We assume a piecewise uniform distribution for a_j, given by

$$f(a_j) = \begin{cases} 4.99993 \text{ if } a_j \in [0, 0.1], \\ 2.5 \text{ if } a_j \in [0.7, 0.85] \cup [0.95, 1], \\ 0.00001, \text{ otherwise,} \end{cases}$$

implying

$$F(a_j) = \begin{cases} 4.99993 a_j \text{ if } a_j \in [0, 0.1], \\ 0.00001 a_j + 0.499992 \text{ if } a_j \in [0.1, 0.7], \\ 2.5 a_j - 1.250001 \text{ if } a_j \in [0.7, 0.85], \\ 0.00001 a_j + 0.8749905 \text{ if } a_j \in [0.85, 0.95], \\ 2.5 a_j - 1.5 \text{ if } a_j \in [0.95, 1]. \end{cases}$$

The above condition then simplifies to

$$1.57 \cdot 1.56999 a^p_{opt} - 0.57 - \frac{1.57}{1-F\left(a^p_{opt}\right)} \int_{a^p_{opt}}^{1} a_j f(a_j)\, da_j + \frac{0.00001}{F\left(a^p_{opt}\right)} \int_0^{a^p_{opt}} a_j f(a_j)\, da_j = 0.$$

If we assume $a^p_{opt} \in [0.7, 0.85]$, the condition becomes

$$\begin{aligned} & 1.57 \cdot 1.56999 \cdot a^p_{opt} - 0.57 \\ & - \frac{1.57}{2.250001 - 2.5 a^p_{opt}} \left(\int_{a^p_{opt}}^{0.85} 2.5 a_j da_j + \int_{0.85}^{0.95} 0.00001 a_j da_j + \int_{0.95}^{1} 2.5 a_j da_j \right) \\ & + \frac{0.00001}{2.5 a^p_{opt} - 1.250001} \left(\int_0^{0.1} 4.99993 a_j da_j + \int_{0.1}^{0.7} 0.00001 a_j da_j + \int_{0.7}^{a^p_{opt}} 2.5 a_j da_j \right) = 0, \end{aligned}$$

which has the two solutions $a^{p,\alpha}_{opt} = 0.81456$ and $a^{p,\beta}_{opt} = 0.84531$.

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