Financial Development and Economic Growth: Long Run Equilibrium and Transitional Dynamics^{*}

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Abstract

We analyze the impact of financial development on economic growth. Differently from previous studies that focus mainly on balanced growth path outcomes, we also analyze the transitional dynamics of our model economy by using a finance-extended Uzawa-Lucas framework where financial intermediation affects both human and physical capital accumulation. We show that, under certain rather general conditions, economic growth may turn out to be non-monotonically related to financial development (as suggested by the most recent empirical evidence) and that too much finance may be detrimental to growth. We also show that the degree of financial development may affect the speed of convergence, suggesting thus that finance may play a crucial role in determining the length of the recovery process associated with exogenous shocks. Moreover, in a special case of the model, we observe that, under a realistic set of parameters, social welfare decreases with financial development, meaning that even when finance positively affects economic growth the short term costs associated with financial activities more than compensate their long run benefits.

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1 Introduction

Finance affects the real economy in several ways, hence understanding the mechanisms through which it impacts on economic growth is essential in order to derive policy recommendations (Levine, 2005). Numerous works emphasize that there may be a nonlinear (Deidda and Fattouha, 2002), and even a non-monotonic (Allen et al., 2014; Law and Singh, 2014) relationship between the degree of financial development and economic performance (long run economic growth). Specifically, according to some studies, too much finance might be ultimately harmful for economic growth, while at the same time too little finance might be suboptimal. Most of the works in the field adopt an empirical approach, and much more limited in number are those relying on a theoretical methodology. Pagano (1993) was among the first to emphasize the existence of several channels through which finance might affect economic growth in a simple Solow-type AK growth model. The main mechanisms discussed in his work are related to three fundamental activities generally run by financial intermediaries, namely funneling savings to firms, improving the allocation of capital, and affecting an economy's whole saving rate. After this seminal work, over the last two decades several studies have focused on the effects of financial intermediaries on human (De Gregorio, 1996; De Gregorio and Kim, 2000) and technological (Morales, 2003; Trew, 2008) capital accumulation, while only recently some step

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further has been made by considering also the role of financial intermediaries in channeling savings to the most efficient uses¹ (Trew, 2014). Most of the extant theoretical works on finance and growth suffer from two major limitations: they are unable to capture the apparently nonlinear, and possibly non-monotonic, relationship between finance and economic growth and, moreover, they do not analyze the transitional effects associated with financial development, as it mainly focuses on balanced growth path (BGP) outcomes. In order to fill these gaps in the literature, we develop a simple extension of the two-sector Uzawa (1965)-Lucas (1988) growth model to account for the role and the effects of financial intermediation. We focus on the Lucas-Uzawa model because this is among the most celebrated and studied (Boucekkine and Ruiz-Tamarit, 2008; Boucekkine et al., 2013) endogenous growth models, because it has been frequently extended along several different directions² and, more importantly, because it is the simplest two-sector endogenous growth model capable of yielding transitional dynamics in a very intuitive way. Its versatility allows us to obtain some insightful views on the potential implications that different degrees of financial development can have on economic performance in the short and long run, via both physical and human capital accumulation.

Specifically, in our finance-extended Uzawa-Lucas model, financial development affects physical capital by altering the amount of resources that can be potentially allocated to investment purposes (savingsfunneling channel), while it also affects human capital via both a productivity and a depreciation channel. We first analyze the steady-state of our model economy in the ratio-variables (hence, we characterize its balanced growth in terms of the level-variables), and we discuss the transitional dynamic effects associated with changes in the degree of the economy's financial development. We show that along a BGP there may exist, consistently with the latest empirical evidence, a non-monotonic (possibly inverted-U) relationship between finance and economic growth: indeed, under some rather general conditions, such a link can be positive only up to a level of financial development, after which the relation turns negative. We also analyze how financial development affects the transitional dynamics of the model by impacting on the economy's speed of convergence. This analysis allows us to better understand why, after the same global financial crises, the US seems to display a different speed of recovery in comparison with, for instance, some EU countries. Finally, following Xie (1994), we focus on a special case of our model with the aim of obtaining an analytical expression for the evolution of consumption over time and, therefore, of further investigating how financial development may affect the behavior of the economy during the transition toward its balanced growth path. In this regard, we observe that, for a realistic set of parameters, welfare is a monotonically decreasing function of financial development, which suggests that even if finance may be growth-enhancing at low levels of financial development, its short term (negative) impact on physical capital accumulation more than offsets its long run (positive) effect on human capital formation.

This paper proceeds as follows. Section 2 briefly reviews the huge body of empirical works on the finance-growth nexus in order to emphasize the variety of results obtained in applied works over the last decades. Section 3 introduces our finance-extended Lucas-Uzawa model, in which financial development influences both the investment in physical and human capital. The steady-state of our model, namely its BGP equilibrium, together with its transitional dynamics are fully characterized in section 4. In this section we show that along the BGP there may exist a non-monotonic relationship between economic growth and financial development (consistently with the latest available empirical evidence) and that the speed of convergence is crucially dependent on the degree of financial development. In section 5 a special case of the model is developed with the purpose of assessing the impact of financial development on social

¹The literature on financial development and economic growth is now quite extensive (see Trew, 2006, for a survey), and relatively varied in terms of aims pursued and methodological approaches employed (see also Khan, 2001; Horii et al., 2013). In Section 2 we review, as much compactly as possible, the main conclusions reached by the latest available empirical literature, along with some of the possible explanations that the existing theoretical works on the topic have put forward in order to explain the sign of the effects that finance may have on economic growth in the real world.

 $^{^{2}}$ For example, Bucci and Segre (2011), and Marsiglio and La Torre (2012) analyze, respectively, the growth effects of culture and demographic change within a two sector Lucas-Uzawa model; La Torre and Marsiglio (2010) propose a three-sector extension of such a model to allow for endogenous technological progress.

welfare. Section 6 briefly discusses to what extent our model can be useful in explaining some of the similarities and differences between the Great Depression following the 1929 stock-market crash and the recent global financial crises of 2007–2008. As usual section 7 concludes and proposes some possible directions for future research. Finally appendix A presents an alternative assessment of welfare effects under a different specification of some relevant functional forms.

2 Empirical Evidence on Finance and Growth

Before presenting our theoretical model, in this section we present a brief review of the main conclusions reached by the available empirical literature in order to grasp which kind of theoretical predictions our model should be able to yield to be consistent with real life experiences. In doing so, we also emphasize the main theoretical channels put forward up to now to explain the sign of the effect that finance is found to have on economic growth in the data. First of all, note that in empirical studies, the degree of financial development (or depth) of a country is mostly proxied by the amount of credit delivered to the private sector of the economy as a share of aggregate GDP³. This is done (Levine, 2005; Arcand et al., 2015) primarily because the amount of credit allotted to the private sector is likely to be positively associated with the five most important functions of a financial system, namely: (i) producing ex-ante information about possible investment opportunities; (ii) improving ex-post monitoring of investment, and exerting corporate governance; (iii) facilitating risk management and diversification; (iv) mobilizing and pooling savings; (v) easing the exchange of goods and services.

In an attempt of anticipating the main results of this section, the most recent empirical literature on the effects of finance on economic growth comes to two fundamental conclusions, namely that: (1) financial systems play a fundamental role in determining variations in economic growth; (2) financial development has a positive effect on economic growth at adequate levels of financial depth, but this effect sooner or later vanishes, and even becomes negative when finance turns out to be excessive⁴. The positive relationship between financial development and economic growth (at least until a point) has been explained by two possible (although non-mutually exclusive) channels⁵. According to the first view (Boyd and Prescott, 1986; Allen, 1990; Greenwood and Jovanovic, 1990), financial intermediaries positively contribute to economic growth because they improve aggregate resource allocation by reducing the cost of acquiring information. All these studies agree that the positive relationship between financial development and economic growth is driven by financial intermediation increasing the efficiency, as opposed to the volume, of investment (see also Goldsmith, 1969; Bencivenga and Smith, 1991). In the same strand of literature, King and Levine (1993a) convincingly show that financial intermediaries boost the rate of technological progress by identifying those entrepreneurs with the highest probability of innovating in new goods and production processes. A second view (Greenwood and Jovanovic, 1990; King and Levine, 1993b; Devereux and Smith, 1994; Obstfeld, 1994) suggests instead that financial intermediaries positively contribute to economic growth because they allow agents to create diversified portfolios with higher expected returns, while keeping economic risks reasonably low. The negative effect that financial development may have on economic growth (for example when finance becomes excessive) is, instead, ultimately explained by the fact that too much finance incubates economic booms and asset prices bubbles that result in financial crises leading to low rates of economic growth for rather long periods of time (see Allen et al., 2009, and Allen et al., 2014). According to this view, those economies that in 2006 showed particularly high levels of credit to the private sector as a share of GDP

 $^{^{3}}$ This proxy is usually considered the best measure of financial development, especially when compared to other available alternatives (Beck et al., 2007). Samargandi et al. (2015) offer a short, but comprehensive, survey on the main measures of financial development generally employed in empirical analyses.

⁴According to Zingales (2015), "... There is no theoretical reason or empirical evidence to support the notion that all the growth of the financial sector in the last forty years has been beneficial to society". Moreover, he also emphasizes that there is much about the financial sector in developed economies that is wasteful, crooked and socially destructive.

 $^{^{5}}$ See the extensive reviews provided by Levine (2005), Panizza (2013), and Allen et al. (2014).

tended to experience costly banking crises in 2007-2008 and sharp downturns in 2007-2009⁶. In other words, the proponents of this argument claim that the recent financial crisis episodes illustrate very well that too large and malfunctioning financial systems directly and/or indirectly contribute to increase volatility, waste resources, discourage savings and promote speculation and economic crashes that, in turn, all lead to under-investment, misallocation of scarce resources and, ultimately, to lower economic growth rates.

In the data, a positive causal effect from finance to growth, even after accounting for possible endogeneity problems, is found in a large empirical literature that includes cross-country growth regression analysis (King and Levine, 1993b; Levine and Zervos, 1998; Benhabib and Spiegel, 2000); instrumental variable analysis (Levine, 1998 and 1999; Levine et al., 2000); time series analysis (Rousseau and Wachtel, 1998; Rousseau and Sylla, 1999); regional analysis within a country (Guiso et al., 2004; Burgess and Pande, 2005); industry-level analysis (Rajan and Zingales, 1998; Beck and Levine, 2002; Wurgler, 2000); and firm-level analysis (Demirgüc-Kunt and Maksimovic, 2002). In general, all these papers converge to the conclusion that a well-developed financial system is growth enhancing, and therefore they are consistent with the claim: "more finance, more growth". Some empirical studies, however, show that the ultimate effect of financial development on economic growth is nonlinear and possibly related both to the stage or degree of financial and economic development of a country. Rioja and Valev (2004a), for example, split a panel of 72 countries from 1961 to 1995 into three regions and show that there exists an S-shaped relation between financial depth and economic growth: the relationship is strong and positive at intermediate levels of financial development, weaker but still positive and statistically significant at higher levels of financial development, but no statistically significant at low levels of financial depth.

Using a dataset covering 95 countries from 1960 to 1985, De Gregorio and Guidotti (1995) were among the first to find that long run economic growth is in general positively correlated with bank credit to the private sector (as a percentage of GDP) but that in low-income economies this effect is relatively small and not significant in the period 1970-1985. They explain this "vanishing effect" of finance with the fact that low-income economies may be at a point at which financial development no longer affects the efficiency of investment⁷. Rousseau and Wachtel (2011) also find a vanishing effect in the positive relationship between financial development and long run economic growth. They show that this relationship is positive and significant in the 1960-1989 period, but is not statistically different from zero in 1990-2004. They find evidence that this vanishing effect is associated with the incidence of financial crises since the positive impact of financial development on economic growth would remain intact for the whole period if the crisis episodes were removed from the sample. According to Arcand et al. (2015), the vanishing effect found in earlier studies is not driven by a change in the fundamental relationship between finance and economic growth (which, instead, appears to be quite stable), but rather by the fact that standard models do not allow for a non-monotonic relationship between financial development and economic growth. After allowing explicitly for this non-monotonicity, Arcand et al. (2015) find a positive marginal effect of financial depth on economic growth in economies where the level of credit to the private sector falls below a threshold of about 80-100% of aggregate GDP. Above this threshold, the relationship becomes negative. These findings are showed to be robust to different types of data and estimators, and to controlling for macroeconomic volatility, banking crises, and institutional quality.

An inverted-U shape relation between financial depth and economic growth is also found by Cecchetti and Kharroubi (2012), and Law and Singh (2014). Based on a sample of developed and emerging economies, Cecchetti and Kharroubi (2012) observe that when private credit exceeds 100% of GDP, or financial sector's

⁶Loayza and Rancière (2006) reconcile the existence of a positive long run relationship between financial development and economic growth with a negative short run relationship between the two variables. They suggest that the negative short run effect of finance may be the result of cross-country heterogeneity, in general, and higher volatility of business cycles driven by financial crises, in particular.

⁷More recently, Rioja and Valev (2004b) have confirmed that there is no significant relationship between financial development and economic growth in low-income countries, whereas the relationship is positive and significant in middle-income countries, and weakly significant in high-income countries.

share of total employment exceeds 4%, then economic growth starts deteriorating. The threshold above which private sector credit as a share of GDP negatively affects economic growth is found to be 88% in Law and Singh (2014), who conclude that their inverted-U result is robust to different measures of financial development indicators, additional explanatory variables, sub-sample countries, as well as estimation procedures. Using a panel of 52 middle-income countries over the 1980–2008 period, Samargandi et al. (2015) also find that financial development and economic growth are linked by an inverted U-shaped relationship in the long run. Finally, the conjecture that financial development may have a nonlinear effect on output growth - that is, it may promote growth only up to a point - finds further support in the sectoral analysis provided by Aizenman et al. (2015).

Overall, the findings from this latest and rather robust empirical literature support the idea that more finance is definitely not always better and it can hurt economic growth after a threshold. In terms of policy implication, this means that policymakers should focus less on increasing at all costs the size/depth of the financial sector and more on determining the optimal amount of financial resources to be channeled toward production activities, through financial intermediaries. As we shall show later, our theoretical model can explain the eventual existence of an inverted-U relationship between financial development and economic growth. Unlike the scant theoretical literature on the topic, our explanation is based, though, on the relative importance of two simultaneous effects (what we shall refer to as productivity and depreciation effects) regulating human-capital-based growth at different degrees of financial development.

3 The Model

The framework is an extended Uzawa-Lucas (1988) two-sector endogenous growth model that allows for a role of financial intermediation. We abstract from population growth and the population size is normalized to unity for the sake of simplicity; we thus state the problem directly in per capita terms. The economy produces a unique consumption good, y_t , by combining physical capital, k_t , and the amount of human capital allocated to productive activities, $u_t h_t$, where h_t is the human capital stock and $0 \le u_t \le 1$ is the share of the existing human capital devoted to production. In order to manufacture the homogeneous consumption good, the economy employs a Cobb-Douglas technology: $y_t = Ak_t^{\alpha}(u_t h_t)^{1-\alpha}$, where $0 < \alpha < 1$ is the physical capital share and A > 0 is a technological parameter. The social planner seeks to maximize the social welfare subject to the physical and human capital accumulation constraints, by choosing consumption, c_t , and the share of human capital to employ in production, u_t . Social welfare is the infinite discounted ($\rho > 0$ is the pure rate of time preference) sum of the instantaneous utilities; the utility function is assumed to be iso-elastic, $u(c_t) = \frac{c_t^{1-\sigma}-1}{1-\sigma}$, with $\sigma > 0$ representing the inverse of the intertemporal elasticity of substitution. Physical capital accumulation is given by the difference between production of the unique final good (net of financial intermediation and depreciation costs) and consumption activity: $k_t = [1 - \xi(\phi)]y_t - \delta_k k_t - c_t$ where $0 < \delta_k < 1$ is the depreciation rate of physical capital, and $0 < \xi(\phi) < 1$ is the share of output lost in the process of financial intermediation which depends upon the degree of development of the financial sector, $\phi > 0$. Human capital accumulation coincides with the net (of depreciation) production of new human capital: $\dot{h}_t = \theta(\phi)(1-u_t)h_t - \delta_h(\phi)h_t$, where $\theta(\phi) > 0$ measures the efficiency of the human capital creation process which is a function of financial development, $0 < \delta_h(\phi) < 1$ is the depreciation of human capital which depends on the degree of financial development as well, and $1 - u_t$ is the share of human capital devoted to the production of new human capital. In order not to impose a priori any limit to human capital accumulation we assume that $\theta(\phi) > \delta_h(\phi)$; indeed, if this were not the case human capital would be meant to either decrease or, at most, remain constant for any $0 \le u_t \le 1$. Given the initial conditions for physical and human capital, $k_0 > 0$ and $h_0 > 0$ respectively, the planner's problem reads as follows:

$$\max_{c_t, u_t} \quad W = \int_0^\infty \frac{c_t^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t} dt$$
(1)

s.t.
$$\dot{k}_t = [1 - \xi(\phi)]Ak_t^{\alpha}(u_t h_t)^{1-\alpha} - \delta_k k_t - c_t$$
 (2)

$$\dot{h}_t = \theta(\phi)(1 - u_t)h_t - \delta_h(\phi)h_t \tag{3}$$

From the problem (1), (2) and (3), it is clear that financial intermediation affects the economy through two different channels, namely physical and human capital accumulation. Indeed, on the one hand, financial intermediation subtracts resources to physical capital investments (Pagano, 1993), while, on the other hand, it also affects human capital formation (de Gregorio and Kim, 2000). More specifically, financial intermediaries absorb a share of income equal to $\xi(\phi)$ that otherwise would go to physical capital accumulation. This absorption of resources is primarily a reward for the financial services supply and may also reflect the X-inefficiency related to the market power of financial intermediaries. How intermediaries affect this term is not clear a priori, but it is reasonable to believe that the more the financial sector is developed (the larger is ϕ), the less resources are wasted in the process of intermediation, that is $\xi' < 0$. From our discussion in the previous section, this is consistent with the view (Goldsmith, 1969; Bencivenga and Smith, 1991, among others) that, ceteris paribus, if a positive relationship between financial development and economic growth does exist, it may well be driven by financial intermediaries increasing the productivity of gross capital investment. Concerning the human capital accumulation equation, the impact of financial intermediation is twofold. First, we assume that financial intermediaries do also affect the total factor productivity of the education sector through the term $\theta(\phi)$: by relaxing borrowing constraints, they allow the best (more productive) workers to invest more in human capital, thus generating a positive effect on the process of acquiring new skills. This means that in our framework $\theta' > 0$. Second, we also postulate that financial intermediaries can influence human capital investment via a change in the rate at which the existing stock of human capital depreciates. In our formulation this occurs through the term $\delta_h(\phi)$. In this respect, it is well-recognized that one important task of the financial sector in mature economies consists of channeling resources towards the most promising R&D projects, which increases the rate of technological progress. A faster technical change, in turn, indirectly contributes to depreciate more rapidly individual abilities embodied in human capital⁸ (for a formal example of this idea, see among many others Galor and Moav, 2002), implying that $\delta'_h > 0$.

Necessary and sufficient conditions for the maximization problem above yield the Euler equations for consumption and the share of human capital devoted to output production:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma} \left\{ \alpha [1 - \xi(\phi)] A k_t^{\alpha - 1} (u_t h_t)^{1 - \alpha} - \delta_k - \rho \right\}$$

$$\tag{4}$$

$$\frac{\dot{u}_t}{u_t} = \frac{1-\alpha}{\alpha} [\theta(\phi) - \delta_h(\phi) + \delta_k] + \theta(\phi)u_t - \frac{c_t}{k_t}$$
(5)

Equation (4) is the standard Ramsey-Keynes rule for consumption and states that consumption growth increases with the average product of physical capital $(\frac{y_t}{k_t})$, the physical capital share and the share of output non-lost in the process of financial intermediation, while decreases with the rate of time preference, the depreciation rate of physical capital and the inverse of the intertemporal elasticity of substitution. Equation (5) states that the growth rate of the share of human capital allocated to production activities increases with the productivity of human capital in generating new human capital, the level of the human capital share allocated to production activities and the depreciation rate of physical capital, while it decreases with the depreciation of human capital and the consumption to capital ratio $(\frac{c_t}{k_t})$.

⁸Bucci (2008) proposes a growth model with purposive R&D activity and human capital investment in which technological progress affects per capita human capital depreciation, which is therefore endogenous. In the present paper we do not model the direct effect that financial development has on R&D activity (a recent work that analyzes this issue is Trew, 2014), but we move one step further by postulating that a higher degree of financial development, by accelerating the rate of technological progress, is indirectly able to speed up the rate of obsolescence of an individual's human capital.

4 BGP Equilibrium and Transitional Dynamics Analysis

We are interested in characterizing a long run (BGP) equilibrium in which all variables grow at an exponential, constant (and possibly positive) rate. Under this definition, simple inspection of equations (2), (3), (4) and (5) reveals that along a BGP, consumption, physical capital and human capital must grow at the same rate (i.e., $\gamma_c = \gamma_k = \gamma_h$), and that the allocation of human capital between production and educational activities (u_t) must also remain constant. In light of this, the shape of the production function suggests that the output to capital ratio must be constant as well along a BGP. Hence, in the long run equilibrium, we would ultimately observe: $\gamma \equiv \gamma_c = \gamma_k = \gamma_h = \gamma_y$, along with a constant $u_t = u$. In order to study the transitional dynamics of the model it is convenient to introduce the following two intensive variables: $\chi_t = \frac{c_t}{k_t}$ and $\psi_t = \frac{y_t}{k_t} = Ak_t^{\alpha-1}(u_th_t)^{1-\alpha}$, representing the consumption to capital ratio and the average product of physical capital, respectively. By using these two variables, it is possible to represent our economy through the following three-dimensional system of differential equations:

$$\frac{\dot{\chi}_t}{\chi_t} = \chi_t - \frac{\sigma - \alpha}{\sigma} [1 - \xi(\phi)] \psi_t - \frac{\rho}{\sigma} + \frac{\sigma - 1}{\sigma} \delta_k$$
(6)

$$\frac{\psi_t}{\psi_t} = \frac{1-\alpha}{\alpha} [\theta(\phi) - \delta_h(\phi) + \delta_k] - (1-\alpha)[1-\xi(\phi)]\psi_t$$
(7)

$$\frac{\dot{u}_t}{u_t} = \frac{1-\alpha}{\alpha} [\theta(\phi) - \delta_h(\phi) + \delta_k] + \theta(\phi)u_t - \chi_t$$
(8)

The steady state of the system above, representing our BGP equilibrium, can be straightforwardly obtained by setting the previous equations equal to zero, which yields:

$$\overline{\chi} = \frac{(\sigma - \alpha)[\theta(\phi) - \delta_h(\phi)] + \alpha\rho + \sigma(1 - \alpha)\delta_k}{\alpha\sigma}$$
(9)

$$\overline{\psi} = \frac{\theta(\phi) - \delta_h(\phi) + \delta_k}{[1 - \xi(\phi)]\alpha}$$
(10)

$$\overline{u} = \frac{\rho + (\sigma - 1)[\theta(\phi) - \delta_h(\phi)]}{\sigma\theta(\phi)}.$$
(11)

Sufficient conditions for the above expressions to be strictly positive and the share of human capital employed in production activities to be also smaller than one require that $(1 - \alpha)[\theta(\phi) - \delta_h(\phi)] < \rho < \theta(\phi) - \delta_h(\phi)$ and $\sigma \geq \alpha$. By plugging (10) into (4) it is immediate to obtain the common BGP growth rate of the economy as: $\gamma = \frac{\theta(\phi) - \delta_h(\phi) - \rho}{\sigma}$, which under the above conditions turns out to be strictly positive too. In order to study the transitional dynamics of the system (6), (7) and (8), we can proceed via linearization, by obtaining the Jacobian matrix, which evaluated at steady state reads as:

$$J(\overline{\chi}, \overline{\psi}, \overline{u}) = \begin{bmatrix} \overline{\chi} & -\frac{\sigma-\alpha}{\sigma} [1-\xi(\phi)]\overline{\chi} & 0\\ 0 & -(1-\alpha)[1-\xi(\phi)]\overline{\psi} & 0\\ -\overline{u} & 0 & \theta(\phi)\overline{u} \end{bmatrix}$$

It is straightforward to note that its eigenvalues, ϑ , coincide with the elements on the main diagonal. Specifically, two of them are positive, $\vartheta_1 = \overline{\chi}$ and $\vartheta_3 = \theta(\phi)\overline{u}$, and one is negative, $\vartheta_2 = -(1-\alpha)[1-\xi(\phi)]\overline{\psi}$. This means that the steady state $(\overline{\chi}, \overline{\psi}, \overline{u})$, and thus the BGP equilibrium, is saddle-point stable. We can thus summarize these results as follows:

Proposition 1. Assume $(1-\alpha)[\theta(\phi) - \delta_h(\phi)] < \rho < \theta(\phi) - \delta_h(\phi)$ and $\sigma \ge \alpha$; then along the BGP equilibrium the common economic growth rate is given by:

$$\gamma \equiv \gamma_c = \gamma_k = \gamma_h = \gamma_y = \frac{\theta(\phi) - \delta_h(\phi) - \rho}{\sigma} > 0$$
(12)

while the consumption to physical capital ratio, $\overline{\chi} > 0$, the average product of capital, $\overline{\psi} > 0$, and the share of human capital allocated to final goods production, $\overline{u} \in (0, 1)$, are respectively given by (9), (10) and (11). Moreover, the BGP equilibrium is saddle-point stable. The results in Proposition 1 are pretty standard in a Lucas-Uzawa framework. As usual, some technical conditions, namely $(1 - \alpha)[\theta(\phi) - \delta_h(\phi)] < \rho < \theta(\phi) - \delta_h(\phi)$ and $\sigma \ge \alpha$, are needed in order to ensure that the growth rate and the other endogenous variables are well defined. These conditions along with our model's assumption that $\theta(\phi) > \delta_h(\phi)$ jointly guarantee that the economic growth rate is strictly positive. This implies that in order for the BGP to be properly characterized, the degree of financial development, ϕ , cannot take arbitrarily large or small values but it necessarily needs to be bounded (from either above or below). Our following discussion is based on the assumption that ϕ falls in the required range; note, however, that since the degree of financial development can be thought of as an index number, our conclusions will hold with no loss of generality. Provided that these conditions are met, the steady state $(\overline{\chi}, \overline{\psi}, \overline{u})$, and thus the BGP equilibrium, is saddle-point stable. This means that our economy converges towards its BGP equilibrium along a saddle-path: given the initial conditions k_0 and h_0 , a unique combination (c_0, u_0) ensures such a converging behavior. By investigating the characteristics of our BGP, we can see that financial development affects both the economic growth rate and the share of human capital employed in output production. Because our main goal in this paper is a better understanding of the growth-finance nexus, we will focus on this relationship in the remainder of the paper.

From (12) it is clear that the only engine of economic growth in the model is human capital investment. This result is due to the fact that our framework is an extension of the Lucas-Uzawa growth model. Unlike this model, however, finance plays a role in our setting. In particular, note that the ultimate effect that financial development has on BGP growth crucially depends on how a more developed financial system can eventually (and simultaneously) influence the two parameters governing the technology of skill acquisition. This is more formally suggested by the following derivative:

$$\frac{\partial\gamma}{\partial\phi} = \frac{\theta'(\phi) - \delta_h'(\phi)}{\sigma} \tag{13}$$

Equation (13) shows that two terms control the whole impact of financial development on economic growth. The former term, $\frac{\theta'(\phi)}{\sigma}$, which we refer to as the "productivity effect", implies that a more developed financial sector tends to increase the productivity of human capital investment, and thus economic growth. The latter term, $-\frac{\delta'_h(\phi)}{\sigma}$, which we refer to as the "depreciation effect", reveals instead that a more developed financial sector, by encouraging R&D activities and hence technical progress, may make at the same time the existing stock of human capital more subject to depreciation via faster obsolescence, which harms economic growth. Depending on which of these two opposing effects eventually prevails, in the very long run economic growth would be either positively, or negatively, or else not related at all to the degree of financial development of the economy. In this regard, it is straightforward to claim the following:

Proposition 2. Along the BGP there may be a non-monotonic relationship between economic growth and financial development. Whenever the productivity effect is larger than the depreciation effect $(\theta'(\phi) > \delta'_h(\phi))$, economic growth increases with financial development; otherwise either financial development reduces $(\theta'(\phi) < \delta'_h(\phi))$ or has no impact at all $(\theta'(\phi) = \delta'_h(\phi))$ on economic performance.

Proposition 2 deals with the sign that the finance-growth nexus can take along the BGP in our model economy. In order to make the economic implications of the Proposition more clear, consider the following simple explicit examples which help illustrating more concretely some of the possible effects that a more developed financial system may have on economic growth depending on the specific shape of the functions $\theta(\cdot)$ and $\delta_h(\cdot)$. Remember that in order for our model to make full sense we need to restrict our analysis to situations in which $\theta(\cdot) > \delta_h(\cdot)$, imposing thus some constraint on the relative size of the two functions.

Example 1. Assume that $\theta(\phi)$ and $\delta_h(\phi)$ are both linear functions of ϕ :

$$\theta(\phi) = \theta \phi$$
 and $\delta_h(\phi) = \delta_h \phi$,

with $\theta > \delta_h > 0$, consistently with our model's requirements. In this case, the productivity effect prevails over the depreciation effect. Thus, the relation between financial development and economic growth is monotonic and always positive, $\frac{\partial \gamma}{\partial \phi} > 0.9$

Example 2. Assume that $\theta(\phi)$ and $\delta_h(\phi)$ are exponential functions of ϕ , respectively:

$$\theta(\phi) = \theta e^{\phi}$$
 and $\delta_h(\phi) = \delta_h \phi e^{\phi}$,

with $\theta > \delta_h > 0$. Our model's specification in this case requires that $\phi < \frac{\theta}{\delta_h}$, which does not impose a priori any restriction on the relative size of the productivity and depreciation effects. This implies that two possibilities can arise here. If $0 < \phi < \frac{\theta - \delta_h}{\delta_h}$, then the productivity effect prevails over the depreciation effect and $\frac{\partial \gamma}{\partial \phi} > 0$; otherwise, whenever $\frac{\theta - \delta_h}{\delta_h} < \phi < \frac{\theta}{\delta_h}$, the depreciation effect prevails over the productivity effect and $\frac{\partial \gamma}{\partial \phi} < 0$. As a whole, the relationship between financial development and economic growth is inverted-U-shaped, with a threshold of the degree of financial development at $\phi = \frac{\theta - \delta_h}{\delta_h} > 0$.

Example 3. Assume that $\theta(\phi)$ and $\delta_h(\phi)$ are linear and quadratic functions of ϕ , respectively:

$$\theta(\phi) = \theta \phi \quad \text{and} \quad \delta_h(\phi) = \delta_h \phi^2$$

with $\theta > 0$ and $\delta_h > 0$. Our model's specification also in this case requires that $\phi < \frac{\theta}{\delta_h}$, which again does not impose a priori any restriction on the relative size of the productivity and depreciation effects. This implies that two possibilities can occur. If $0 < \phi < \frac{\theta}{2\delta_h}$, then the productivity effect prevails over the depreciation effect and $\frac{\partial \gamma}{\partial \phi} > 0$; otherwise, whenever $\frac{\theta}{2\delta_h} < \phi < \frac{\theta}{\delta_h}$, the depreciation effect prevails over the productivity effect and $\frac{\partial \gamma}{\partial \phi} < 0$. As in Example 2, the relationship between financial development and economic growth is inverted-U shaped, with a threshold of the degree of financial development at $\phi = \frac{\theta}{2\delta_h} > 0$ in this case.

As the examples above illustrate, the relationship between financial development and economic growth crucially depends on the shape of the productivity and depreciation functions, which determine whether this link is ultimately monotonic or non-monotonic in sign. Example 1 shows what happens when the degree of financial development affects linearly both the productivity and the depreciation terms in the human capital accumulation equation. In this case, since the productivity effect prevails, the relationship between financial development and economic growth is monotonically positive. Example 2 and 3, instead, exemplify what happens when financial development affects nonlinearly $\theta(\phi)$, $\delta_h(\phi)$, or both. While in Example 2 the nonlinear effect of ϕ concerns both terms, Example 3 suggests that in order to have a non-monotonic relationship between finance and growth it is not necessary to postulate a nonlinear impact of ϕ on both $\theta(\phi)$ and $\delta_h(\phi)$ simultaneously. The presence of an inverted-U relation between the degree of financial development and long run economic growth that we observe in Examples 2 and 3 is consistent with the most recent empirical evidence (Arcand et al., 2015), and can be explained by our model as follows. Ceteris paribus, for sufficiently low levels of financial development, a marginal increase in ϕ , by loosening agents' borrowing constraints, makes their investment in skill acquisition easier, while keeping the economy's human capital stock and the rate of technical change reasonably small. Thus, when ϕ is low enough, increases in financial development are likely to raise the productivity of the human capital accumulation process more rapidly than the depreciation costs related to the available amount of skills, and this leads to a higher economic growth rate. Above a given threshold, however, more financial development, by entailing a larger

⁹The following trivial examples show that in case of a monotonic relationship between growth and finance, this does not necessarily need to be positive. Indeed, if $\theta(\phi) = \frac{\theta}{\phi}$ and $\delta_h(\phi) = \delta_h \phi$, with $\theta > \delta_h > 0$, our model requires that $\phi \leq 1$; it is straightforward to verify that over this range the relation between financial development and economic growth is always negative, $\frac{\partial \gamma}{\partial \phi} < 0$. If, instead, $\theta(\phi) = \theta + \phi$ and $\delta_h(\phi) = \delta_h + \phi$, with $\theta > \delta_h > 0$, the relation between financial development and economic growth is completely absent, that is $\frac{\partial \gamma}{\partial \phi} = 0$.

available stock of human capital in the economy (and, consequently, a faster rate of technological progress, too), ultimately contributes to increase the depreciation-costs of the existing amount of skills faster than the corresponding productivity-benefits related to further human capital acquisition, and this is at the heart of the emergence of a negative relation between financial development and economic growth after a point.

One of the main results attained thus far by our model suggests that, depending on the relative intensity of two effects governing the production of new human capital (the productivity and the depreciation effect, respectively), the long run (BGP) relationship between financial development and economic growth is potentially ambiguous (i.e., it can be non-monotonic/inverted-U; monotonically positive/negative; non-existent at all). Before discussing in more depth the short run transitional effects associated with financial development (in section 5 we will look at the behavior of the model also during the transition towards the BGP), we present another interesting implication of our model which deals with the speed of convergence. Indeed, the absolute value of the (unique, see Proposition 1) negative eigenvalue, ϑ_2 , represents in this context the speed of convergence of the economy towards its BGP equilibrium. It is straightforward to show that this depends (possibly) non-monotonically on the degree of financial development, as clearly shown in the following derivative:

$$\frac{\partial |\vartheta_2|}{\partial \phi} = \frac{(1-\alpha)[\theta'(\phi) - \delta_h'(\phi)]}{\alpha} \tag{14}$$

According to whether the productivity or the depreciation effect dominates, the speed of convergence will rise or fall with financial development.

Proposition 3. The speed of convergence may non-monotonically depend on the degree of financial development. It will increase when the relationship between finance and growth is positive (productivity effect larger than depreciation effect), and will decrease when the relationship between finance and growth is negative (productivity effect smaller than depreciation effect). In the absence of any long run relationship between finance and growth (productivity effect equal to depreciation effect), the speed of convergence is unaffected by changes in the degree of financial development.

This proposition says that in our framework the degree of financial development can affect not only the BGP growth rate of the economy, but also its speed of convergence to a new BGP equilibrium following an economic or financial shock. To the best of our knowledge, this result is new in literature as in general it is found (see Aghion et al. 2005 for a notable example) that financial development can speed up the convergence to the steady-state but has ultimately a very limited effect on long run economic growth.¹⁰ According to our results, in the wake of a shock, the economies that recover faster in terms of greater speed of convergence to the new BGP are also those in which financial development plays a positive role on long term economic growth. In other words, the degree of financial development affects simultaneously long run (economic growth) and short run (transitional dynamics) outcomes. This finding is consistent with the recent analysis presented by Reinhart and Rogoff (2014), according to which out of the 12 countries that have experienced the financial crisis started in 2007–2008, only two (Germany and the US) have already reached their pre-crisis peak in per capita GDP. For all other countries, using IMF (2013) estimates, their projections (see Reinhart and Rogoff, 2014, p. 54) suggest that even by 2018 the full recovery to pre-crisis GDP will not be completed yet. In terms of the ongoing debate on how to improve the contribution of the

¹⁰Aghion et al. (2005) develop a model of technological change that predicts that countries with levels of financial development above a critical threshold will converge in growth rates. Among these countries, financial development positively influences the rate of convergence, so financial development exerts a positive but diminishing influence on steady-state levels of real per capita output. The authors find empirical support for their model's predictions, as financial development seems to explain in their empirical analysis: (i) whether there is convergence or not, and (ii) the rate of convergence (when there is convergence). Unlike our model, however, in Aghion et al. (2005) financial development does not exert a direct effect on steady-state (or long run) economic growth. In a more recent paper, Aghion et al. (2010) have also showed how financial development may help reducing the growth-cost of economic fluctuations.

financial sector to avoid future long-lasting financial crises, our paper suggests that policymakers should be aware of the true sign of the finance-growth relationship as well as of the way (i.e., the specific channels) in which this relation takes place before implementing any specific public policy.

5 On Financial Development and Welfare: the Special Case $\sigma = \alpha$

After showing that our theoretical findings are, under certain conditions, consistent with the most recent empirical evidence on the long run finance-growth nexus, we now turn to the analysis of the transitional dynamics of our model in order to quantify the welfare effects associated with financial development. Since (6), (7) and (8) form a simultaneous system of differential equations, analyzing its transitional behavior is evidently not possible in analytical terms. However, by focusing on a specific case it is possible to decouple some of these equations and thus fully solve the system, which will finally allow us to analyze the welfare effects of financial development. Such a decoupling is possible only when $\sigma = \alpha$, that is whenever the inverse of the intertemporal elasticity of substitution coincides with the physical capital share¹¹ (Xie, 1994). In this case it is possible to show that the following result holds.

Proposition 4. Assume $(1 - \alpha)[\theta(\phi) - \delta_h(\phi)] < \rho < \theta(\phi) - \delta_h(\phi)$ and $\sigma = \alpha$; then, the optimal paths of the control (c_t and u_t) and state (k_t and h_t) variables in the maximization problem (1), (2) and (3) are given by the following expressions for all $t = 0, ..., \infty$:

$$c_t = \frac{\rho + (1 - \alpha)\delta_k}{\alpha} k_t \tag{15}$$

$$u_t = \frac{\rho - (1 - \alpha)[\theta(\phi) - \delta_h(\phi)]}{\alpha \theta(\phi)} = \overline{u}$$
(16)

$$k_{t} = \left\{ \left(k_{0}^{1-\alpha} - \frac{[1-\xi(\phi)]\alpha A \overline{u}^{1-\alpha} h_{0}^{1-\alpha}}{\theta(\phi) - \delta_{h}(\phi) + \delta_{k}} \right) e^{-\frac{(1-\alpha)(\rho+\delta_{k})}{\alpha}t} + \frac{[1-\xi(\phi)]\alpha A \overline{u}^{1-\alpha} h_{0}^{1-\alpha}}{\theta(\phi) - \delta_{h}(\phi) + \delta_{k}} e^{(1-\alpha)\gamma t} \right\}^{\frac{1}{1-\alpha}}$$
(17)
$$h_{t} = h_{0} e^{\gamma t},$$
(18)

where the (strictly positive) economic growth rate is given by:

$$\gamma = \frac{\theta(\phi) - \delta_h(\phi) - \rho}{\alpha} \tag{19}$$

Since the restriction $\sigma = \alpha$ represents a particular case already considered in the previous section (see Proposition 1), the technical condition that ensures that the BGP equilibrium is well defined (positive growth rate and positive but smaller than unity share of human capital devoted to output production) coincides with what presented in Section 4. From Proposition 4 we can see that financial development affects the whole dynamic evolution of all the variables, and for all $t = 0, ..., \infty$, consumption is proportional to physical capital, the share of human capital devoted to output production is constant, the human capital growth rate coincides with the economic growth rate γ , while physical capital will grow at this same rate γ only asymptotically (whenever the first term inside the curly brackets vanishes). Despite the potential criticism towards the condition ($\sigma = \alpha$) required to derive Proposition 4, the above expressions, (15), (16), (17) and (18), result extremely useful in light of our final goal to assess welfare effects associated with financial development. Indeed, since the share of human capital allocated to the final sector is constant, as it will become more clear in a while, we can clearly isolate the two different channels through which finance may

¹¹This is a typical assumption in the growth literature aiming at disentangling the short run vs long run effects associated with alternative economic policies (Smith, 2006). Some related works in a Lucas-Uzawa framework include Boucekkine and Ruiz–Tamarit (2008), Bucci et al. (2011), Marsiglio and La Torre (2012). We are perfectly aware that such an assumption is barely consistent with empirical evidence, but nevertheless, as it will become more clear later, this will be extremely useful to achieve (even if in a simplified setup) our main goal in this section.

affect consumption, namely physical and human capital accumulation. The fact that human capital is from time zero growing at its long run rate (we have already extensively commented in the previous section on the effects of finance along the BGP) permits us to focus more precisely on the transitional effects of ϕ driven by physical capital accumulation, through the $\xi(\phi)$ term (on which we have not said much thus far). It should be clear from the following expression that financial development affects the evolution of consumption in a strong nonlinear way. In fact, plugging (17) into (15) yields:

$$c_t = \Omega \left\{ \left(k_0^{1-\alpha} - \frac{[1-\xi(\phi)]\alpha A\overline{u}^{1-\alpha} h_0^{1-\alpha}}{\theta(\phi) - \delta_h(\phi) + \delta_k} \right) e^{-\frac{(1-\alpha)(\rho+\delta_k)}{\alpha}t} + \frac{[1-\xi(\phi)]\alpha A\overline{u}^{1-\alpha} h_0^{1-\alpha}}{\theta(\phi) - \delta_h(\phi) + \delta_k} e^{(1-\alpha)\gamma t} \right\}^{\frac{1}{1-\alpha}}$$
(20)

where $\Omega = \frac{\rho + (1-\alpha)\delta_k}{\alpha}$. As anticipated above, this clearly shows that finance interacts with consumption activities thanks to both the human and physical capital accumulation channels. Thanks to human capital forma-tion, it affects consumption through three terms, $\theta(\phi) - \delta_h(\phi)$, $\overline{u} = \frac{\rho - (1-\alpha)[\theta(\phi) - \delta_h(\phi)]}{\alpha \theta(\phi)}$ and $\gamma = \frac{\theta(\phi) - \delta_h(\phi) - \rho}{\alpha}$, contributing to uniquely determine the initial consumption level needed to address the economy along its BGP from time zero; this makes sure that eventual adjustments during the transition towards the BGP are completely ruled out. This also implies that finance impacts on consumption through physical capital accumulation only through the $\xi(\phi)$ term, quantifying the amount of resources that the financial sector subtracts to capital investments; this does not play any role on long run growth rates but it does determine the consumption level at any point in time, and it can thus crucially affect social welfare. Even if, thanks to our assumption $\sigma = \alpha$, we can disregard the transitory adjustment effects associated with human capital accumulation (and its optimal intersectoral allocation), the above expression for consumption results to be particularly cumbersome, and assessing analytically welfare and the welfare effects associated with financial development may not be possible. However, in order to shed some light on this issue we turn to a numerical assessment. More specifically, we now run a numerical simulation with the objective of illustrating the impact of financial development on the dynamics of consumption, share of human capital allocated to production activities, physical and human capital. In order to proceed with our simulations, we need to explicitly specify the functions $\theta(\cdot), \delta_h(\cdot)$ and $\xi(\cdot)$. The former two, consistently with the findings of the most recent empirical evidence (namely, Arcand et al., 2015), are assumed to take the same form of the previous Example 3, while the latter is assumed to take the following form:

$$\xi(\phi) = \frac{\xi}{\xi + \phi},$$

with $\xi > 0$ such that $\xi(0) = 1$, $\xi(\infty) = 0$ and $\xi' < 0$. The parameter values we employ in our simulation are shown in Table 1, which (apart from the intertemporal elasticity of substitution, that in our framework is restricted to be equal to the physical capital share) are consistent with the values in Mullingan and Sala– i–Martin (1993). The value of ξ is arbitrarily set to be equal to 0.1 in order to make figures more clear. From a qualitative point of view, our arbitrary choice of functional forms does not substantially affect the results, which are robust with respect to different parametrizations. Only the choice of the functional from for $\xi(\cdot)$ is likely to sensibly affect our quantitative conclusions about welfare effects (see appendix A for further details).

$\sigma = \alpha$	ρ	A	δ_k	θ	δ_h	ξ	$k_0 = h_0$
0.33	0.04	1	0.05	0.1	0.05	0.1	1

Table 1: Parameter values employed in our simulation.

Figure 1 and 2 present the results of our simulation exercise for values of ϕ respecting the technical conditions given in Proposition 4, that is values falling in the range $\phi \in (0.6, 1.4)$. Since, given the specified



Figure 1: Evolution over time of consumption, share of human capital allocated to production, physical and human capital for values of $\phi \leq 1$.

parameter values, $\phi = 1$ represents the threshold below and above which financial development affects economic growth with a different sign (see Example 3), we illustrate the two cases separately. Specifically, Figure 1 presents the case in which $\phi \leq 1$ (that is the case in which $\theta'(\phi) \geq \delta'_h(\phi)$) while Figure 2 presents that in which $\phi \geq 1$ (that is the case in which $\theta'(\phi) \leq \delta'_h(\phi)$).

By comparing Figures 1 and 2 it is possible to understand how financial development affects the dynamic evolution of consumption, share of human capital allocated to production, physical and human capital. Whenever financial development is lower than the threshold value $\phi = 1$ (Figure 1), increases in ϕ will decrease both consumption and physical capital in the short run and will tend to increase them in the long run. Such an increase is due to the fact that a higher degree of financial development tends to decrease the share of human capital allocated to production activities, thus increasing the rate of growth of human capital. As a result, since in this case the productivity effect is larger than the depreciation effect, rises in ϕ will increase the speed of convergence towards the BGP equilibrium, so lowering the length of the short



Figure 2: Evolution over time of consumption, share of human capital allocated to production, physical and human capital for values of $\phi \ge 1$.

run transitionary period (see Proposition 3). Whenever financial development is higher than the threshold value $\phi = 1$ (Figure 2), the results are qualitatively the opposite. Indeed, increases in ϕ will increase both consumption and physical capital in the short run and will tend to decrease them in the long run, and this is due to the fact that a higher degree of financial development tends to increase the share of human capital allocated to production activities, thus decreasing the rate of growth of human capital. As a result, since in this case the productivity effect is smaller than the depreciation effect, rises in ϕ will decrease the speed of convergence towards the BGP equilibrium, raising thus the length of the short run transitionary period (see Proposition 3)

Overall, we can conclude that the behavior of consumption is exactly the opposite in the two cases and thus it crucially depends upon the sign of the relationship between growth and finance. If such a relationship is negative ($\phi > 1$) then consumption will increase in the short run but will decrease in the long run; if instead it is positive ($\phi < 1$) then consumption will decrease in the short run but will increase in the long run. This suggests that independently of the nature of the finance and growth relationship, we will always observe an intertemporal trade off between short run and long run consumption. Such a trade off clearly reflects how the degree of financial development affects the allocation of human capital between the production and education sectors: a higher allocation in the production sector allows to increase short term consumption but, by lowering human capital accumulation, this comes at the cost of reducing long term consumption. The existence of such different effects on consumption according to the sign of the growth and finance relation reinforces our previous conclusion that before implementing policies aiming to promote or discourage financial development, a clear understanding of the sign of the finance and growth nexus is needed.



Figure 3: Relationship between social welfare and degree of financial development, ϕ .

The existence of such an intertemporal trade off characterizing how consumption responds to financial development further suggests that the only possible way to compare short and long run costs and benefits associated with financial development consists of assessing social welfare. Figure 3 analyzes this more specifically by showing how social welfare changes with ϕ . It clearly reveals that, for the functional forms and the parameter values employed in our simulations, welfare is always a decreasing function of ϕ . This means that irrespective of the fact that finance may or may not positively affect growth, the overall welfare effect is negative. Moreover, we can also see that the rate at which social welfare falls with financial development is faster whenever the relationship between finance and growth is negative ($\phi > 1$). Hence, under our model's parametrization it would be socially desirable to discourage financial development at all, and to focus on other possible ultimate determinants of long run growth in order to raise welfare. Such a final result needs to be taken with some grain of salt since it may be driven by our choice of some relevant functional forms (see appendix A for an example of how a different functional form for $\xi(\cdot)$ may lead, instead, to a bell-shaped relationship between social welfare and financial development). In order to obtain more rigorous conclusions about finance-induced welfare effects, more accurate empirical studies will be needed to more clearly understand what might be the true shape of the relevant functional forms.

6 Discussion

In this paper we have developed and analyzed a stylized finance-extended model of endogenous growth which under certain conditions turns out to be consistent with recent empirical evidence on the growth and finance nexus. In this section, instead, we briefly discuss to what extent the model can help us to understand some features of the recent global financial crisis. In order to do so, it is essential first to analyze the common and specific characteristics of financial crises in general. As a matter of historical relevance we restrict our analysis to the two most important crises ever occurred, namely the Great Depression and the recent crisis. The 1929 stock-market crash and the subsequent Great Depression can be considered the largest economic crises that the world has ever experienced; as a consequence, when the global financial crisis has started hitting in the recent 2007–2008, a large concern to potentially end up in another depression on a similar scale has spread fast. For this reason, understanding the characteristics of the Great Depression and its eventual relationship with the global financial crisis can provide us with further insights on the growth and finance relation. Why has a financial downturn turned into a deep depression in 1929? Which have been the basic hallmarks of that crisis? Is there any analogy and/or dissimilarity (e.g., in terms of fundamental causes, length and depth) between the 1929 and 2007–2008 crises? Trying to answer these questions is nowadays a priority for both academics and policymakers, and this can also allow us to understand whether our model can say anything at all about the ultimate (dis)similarities between the two crashes and their economic consequences.

According to economic historians, there are two main similarities between the two crises. Specifically, both have been preceded by an extended period of sustained economic growth and, at the same time, both have been characterized by speculative bubbles arising from the flow of easy credit to households and firms, fueling thus both property-based and stock market-based excesses. During 1928, the Times Industrials (a pre-cursor of the actual Dow Jones) had gained a huge 35%, pushing many speculators, in an attempt to maximize their risky profits to finance their own purchase of stocks through borrowed money, buying on average one-thousand dollars of stock by putting down just one-hundred dollars (Canterbery, 2011). Similarly, the immediate cause of the recent global crisis has been seen in a "...rapid credit expansion and financial innovation that led to high leverage", according to Helbling (2009); as Bernanke (2010) has recently pointed out, although the most prominent reason behind the 2007–2008 economic crash was the prospect of losses on the sub-prime market induced by excessive leverage on the part of households, businesses and financial firms, other determinant factors did include the overreliance of banks on short term wholesale funding, deficiencies in private sector risk management, an overreliance on ratings agencies, and a failure of existing regulatory procedures worldwide.

Economic historians, however, believe that there are also substantial differences between the two crises, both in terms of "financialization" and speed of recovery. The most salient difference is probably represented by the fact that in the time passed by between the Great Depression and the global financial crisis the nature of the capitalist system has changed in a fundamental way moving from a model of productive industrialization towards a model of financial capitalization (Canterbery, 2011). In this regard, according to some economists (see Wade, 2008, among others), the world as a whole has undergone a massive financial liberalization starting from the early 1990s, leading to a huge shift from production to financial services. Along with its asset-base growth, the financial sector has become a much bigger part of the national economy: between 1978 and 2005 the financial sector has grown from 3.5% to 5.9% in the US economy in GDP terms. To put all this in a broader perspective, note that from the 1930s to around 1980 the growth rates of the financial sectors have been roughly the same; however, from 1980 to 2005 financial sector profits have grown by 800% while those of the non-financial sector by a more modest 250% (Canterbery, 2011). It is under this (completely new) mode of capitalism that the recent global crisis has taken place and propagated across the entire world. Moreover, in the US a side-effect of the financial liberalization can also be seen in the strong deregulation occurred in the sub-prime mortgage market which has rapidly become

the least regulated part of the whole American mortgage market: it is computed that while in 2000 the \$130 billion of sub-prime lending was backed up with \$55 billion of mortgage bonds, by 2005 those figures jumped to \$625 billion sub-prime loans backed by \$500 billion in securitized bonds (McNally, 2010). The creation of innovative financial instruments in the form of credit default swaps and other debt securities could only exacerbated the situation further: by 2006 the credit default swaps on mortgage bonds became eight times the value of the bonds themselves, allowing the associated wealth to be quickly wiped out with the crisis (McNally, 2010). In brief, the ability of financial firms to generate all kinds of conceivable financial innovations due to market-liberalization seems to have been a distinctive trait of the recent economic crisis as opposed to the 1929 crisis.¹² Another significant difference between the two crisis-episodes, no matter whether the metric is represented by global industrial production, or global trade volumes, or else global equity valuations, resides in the relative speed of recovery following the crash in the two cases (see Figures 1, 2, and 3 in Eichengreen and O'Rourke, 2012; see also Reinhart and Rogoff, 2014). Indeed, while most economists now agree that the Great Depression has lasted for over ten years (Reinhart and Rogoff, 2014), Eichengreen and O'Rourke (2012) emphasize the relatively fast recovery during the most recent global financial crisis since its peak (April 2008), although the picture is not the same everywhere.¹³

In the light of this brief comparison of the main features of the two most important financial crises ever experienced at world level, we believe that our model can capture in a stylized way the two distinctive features of the latest 2007–2008 economic crisis as opposed to the 1929 crash. In particular, the increasing financialization of capitalism in the second half of the XXI century (relying upon debt as a major means of making risky profits) has ultimately led to a rising degree of financial innovation (financial development in our model, ϕ), taking the form of a fast proliferation of new, more complex and more varied financial instruments. The increase in the degree of financial development has affected the speed of convergence, substantially reducing the length of recovery in some countries (those in which the relationship between finance and growth is positive, according to our model) but not in others (those characterized by a nonpositive relationship). When read under this light, what our model has ultimately done is to analyze, even if in a very stylized fashion, the prospects of a financial crisis in terms of long run (balanced) growth, speed of recovery, and ultimately welfare. Despite our approach is quite simplistic, to the best of our knowledge no other paper is able to provide any stylized description of (at least some of) the differences between the Great Depression and the global financial crisis. Such a lack of theoretical explanations calls for the need of additional efforts in order to better understand the recent economic history and respond with appropriate policies.

7 Conclusion

Finance and financial intermediation do play an important role in modern economies. Despite the huge body of empirical research that tries to assess the nature and the sign of the finance-growth nexus, existing theoretical works on the topic are more limited in number and in general do not explain why the relation between finance and growth might be nonlinear, and possibly non-monotonic (inverted-U), as recent evidence seems mostly to suggest. Moreover, the majority of the existing theoretical works focuses mainly on BGP outcomes, so neglecting the implications of financial development for short run transitional dynamics. Our paper represents a first attempt at filling these two important gaps in literature. By analyzing the relation-

¹² "...Liquidity and funding problems have played a key role in the financial sector transmission in both episodes. Concerns about the net worth and solvency of financial intermediaries were at the root of both crises, although the specific mechanics differed given the financial systems evolution" (Helbling, 2009).

¹³ "Examining the evolution of real per capita GDP around 100 systemic banking crises reveals that a significant part of the costs of these crises lies in the protracted and halting nature of the recovery. On average it takes about eight years to reach the pre-crisis level of income; the median is about 6.5 years. Five to six years after the onset of the current crisis only Germany and the United States (out of 12 systemic crisis cases) have reached their 2007-2008 peaks in per capita income" (Reinhart and Rogoff, 2014).

ship between finance and economic growth in an extended version of the Lucas-Uzawa model, we postulate that financial development affects physical capital accumulation by altering the amount of resources that can potentially be used for investment purposes (a more development financial system wastes less resources in the process of financial intermediation), and human capital accumulation via both a productivity and a depreciation channel. Thus, while on the one hand a higher degree of financial development eases and makes human capital investment more productive via a relaxation of agents' borrowing constraints, on the other hand, by fostering technological progress, it also causes the existing stock of human capital to be more subject to depreciation via faster obsolescence. We study under which conditions, along a BGP equilibrium, there may exist a non-monotonic relationship between growth and financial development, and find that this crucially depends upon the relative intensity of the productivity and depreciation effects at different levels of financial depth. In particular, we show that finance turns out to be harmful for economic growth whenever the productivity effect is smaller than the depreciation effect, which is eventually more likely to occur at higher levels of financial development. We also analyze the impact of financial development along the transition to the BGP equilibrium, and show that the speed of convergence depends upon the degree of financial development, as well. In particular we observe that finance reduces the transitional process associated with economic growth whenever the productivity effect is larger than the depreciation effect. Thus, the different role played by finance in different economies might explain why the speed of recovery from the common, recent, and global financial crisis varies so largely from country to country. By considering a special case of our model, that is a framework in which the inverse of the intertemporal elasticity of substitution equals the physical capital share of output, we finally analyze the welfare effects associated with financial development. Under a realistic set of the model's parameters, we show that social welfare decreases with the degree of financial development, suggesting that, despite the fact that finance may be growth enhancing, in order to improve living standards it might be convenient to focus on other possible sources of long run growth.

We believe that, in its simplicity, our model is capable of capturing many of the fundamental hallmarks of the relation between finance and economic growth. However, everything comes at a cost. Indeed, while on the one hand we adopt an aggregative approach in order to maintain the model sufficiently simple and tractable and to have a neat picture of how (i.e., through which possible channels) financial intermediation can affect BGP growth and transitional dynamics, on the other hand the same aggregative approach does not allow to take into account the possible microeconomic mechanisms that may ultimately drive the impact of finance on economic growth. Extending our current analysis through a better understanding of such microeconomic mechanisms, by which financial intermediation is likely to simultaneously influence investment decisions in capital accumulation and education by individual agents along with those in research and development by innovative firms, might definitely contribute to shed a new light on the whole implications of financial development on the process of modern economic growth. We leave this challenging task to future research.

A Alternative Finance-Induced Welfare Effects

As mentioned in section 5, our discussion about the welfare effects may be largely driven by the specific functional forms used in our numerical exercise. In particular, we briefly discuss here how our results may change under a different specification of the $\xi(\phi)$ function, measuring how financial development subtracts resources to physical capital investments. Note that this affects only the transitional dynamics of consumption and eventually welfare, since this is completely irrelevant for long run growth rates. Assume that this function takes the following form:

$$\xi(\phi) = \frac{\xi}{\xi e^{\phi}},$$

with $\xi > 0$ such that $\xi(0) = 1$, $\xi(\infty) = 0$ and $\xi' < 0$. It takes thus exactly the same shape of the function discussed in the main text, and the only difference between the two specifications is related to the extent to

which finance diverts resources from physical capital accumulation. We proceed as in the main text with a numerical simulation to illustrate the effects of financial development both in the short and long run. The parameter values are assumed to take exactly the same values as in Table 1, and the results of our numerical simulations are shown in Figures 4 and 5.



Figure 4: Evolution over time of consumption for values of $\phi \leq 1$ (left) and $\phi \geq 1$ (right).

Figure 4 shows that the dynamic evolution of consumption is qualitatively identical to what discussed in section 5. An intertemporal trade off is clearly present in both the $\phi \leq 1$ and $\phi \geq 1$ cases (note that since the specification of the $\theta(\cdot)$ and $\delta_h(\cdot)$ functions have not changed, the unity still represents the threshold value below and above which the relationship between finance and growth turns out to be positive and negative, respectively).



Figure 5: Relationship between social welfare and degree of financial development, ϕ .

Even if the behavior of consumption is exactly the same, the size of the changes in consumption levels associated with financial development is not. Indeed, the welfare effects illustrated in Figure 5 are substantially different from those discussed in section 5. Under such a new specification of the $\xi(\cdot)$ function, for values of ϕ smaller than unity, welfare rises with the degree of financial development. This means that the nature of the finance and growth relationship does not determine only long run growth rates but also our conclusions about welfare effects. While financial development is not desirable if the relationship between growth and finance is negative ($\phi \ge 1$), it instead is whenever the relationship is positive ($\phi \le 1$). The eventual existence of bell-shaped welfare effects is an interesting point with potential policy implications. However, such a strong difference with the results discussed in the main text, which are exclusively due to the specific functional form of $\xi(\cdot)$, make it impossible to understand which of the two alternative scenarios is more plausible. This also suggests that more (empirical) work is needed in order to quantify the exact size of the total resources diverted from capital investments by financial intermediation, and thus which of two formulations is more consistent with real world observations.

References

- 1. Aghion, P., Angeletos, G.-M., Banerjee, A., Manova, K. (2010). Volatility and growth: credit constraints and the composition of investment, Journal of Monetary Economics 57, 246–265
- Aghion, P., Howitt, P., Mayer-Foulkes, D. (2005). The effect of financial development on convergence: theory and evidence, Quarterly Journal of Economics 120, 173–222
- 3. Aizenman, J., Jinjarak, Y., Park, D. (2015). Financial development and output growth in developing Asia and Latin America: A comparative sectoral analysis, NBER Working Paper No. 20917
- 4. Allen, F. (1990). The Market for information and the origin of financial intermediaries, Journal of Financial Intermediation 1, 3–30
- Allen, F., Babus, A., Carletti, E. (2009). Financial crises: theory and evidence, Annual Review of Financial Economics 1, 97–116
- 6. Allen, F., Carletti, E., Qian, J., Valenzuela, P. (2014). Does finance accelerate or retard growth? Theory and evidence, in (Allen, F., Behrman, J.R., Birdsall, N., Fardoust, S., Rodrik, D., Steer, A., Subramanian, A., Eds.): "Towards a better global economy: policy implications for citizens worldwide", 289-380 (Oxford: Oxford University Press)
- Arcand, J.L., Berkes, E., Panizza, U. (2015). Too much finance?, Journal of Economic Growth 20, 105–148
- Beck, T., Demirg–Kunt, A., Levine, R. (2007). Finance, inequality and the poor, Journal of Economic Growth 12, 27–49
- 9. Beck, T., Levine, R. (2002). Industry growth and capital allocation: does having a market- or bankbased system matter?, Journal of Financial Economics 64, 147–180
- Bencivenga, V., Smith, B. (1991). Financial intermediation and endogenous growth, Review of Economic Studies 58, 195–209
- 11. Benhabib, J., Spiegel, M.M. (2000). The role of financial development in growth and investment, Journal of Economic Growth 5, 341–360
- 12. Bernanke, B.S. (2010). Causes of the recent financial and economic crisis. Testimony by Ben S. Bernanke, Chairman, Board of Governors of the Federal Reserve, Before the Financial Crisis Inquiry Commission, Washington DC (September 2, 2010), available at: http://www.federalreserve.gov/newsevents/testimony/bernanke20100902a.htm
- Boyd, J.H., Prescott, E.C. (1986). Financial intermediary-coalitions, Journal of Economic Theory 38, 211–232

- Boucekkine, R., Martinez, B., Ruiz-Tamarit, J.R. (2013). Growth vs level effect of population change on economic development: an inspection into human-capital-related mechanisms, Journal of Mathematical Economics 49, 312-334
- 15. Boucekkine, R., Ruiz-Tamarit, J.R. (2008). Special functions for the study of economic dynamics: the case of the Lucas-Uzawa model, Journal of Mathematical Economics 44, 33–54
- 16. Bucci, A. (2008). Population growth in a model of economic growth with human capital accumulation and horizontal R&D, Journal of Macroeconomics 30, 1124–1147
- 17. Bucci, A., Colapinto, C., Forster, M., La Torre, D. (2011). Stochastic technology shocks in an extended Uzawa-Lucas model: closed-form solution and long-run dynamics, Journal of Economics 103, 83–99
- Bucci, A., Segre, G. (2011). Culture and human capital in a two-sector endogenous growth model, Research in Economics 65, 279–293
- 19. Burgess, R., Pande, R. (2005). Do rural banks matter? Evidence from the Indian social banking experiment, American Economic Review 95, 780–795
- 20. Canterbery, E.R. (2011). The global great recession (Singapore: World Scientific Publishing Co.)
- Cecchetti, G., Kharroubi, E. (2012). Reassessing the impact of finance on growth. BIS (Bank for International Settlements) Working Paper No. 381
- De Gregorio, J. (1996). Borrowing constraints, human capital accumulation, and growth, Journal of Monetary Economics 37, 49–71
- 23. De Gregorio, J., Guidotti, P. (1995). Financial development and economic growth, World Development 23, 433–448
- 24. De Gregorio, J., Kim, S.-J. (2000). Credit markets with differences in abilities: education, distribution and growth, International Economic Review 41, 579–607
- Deidda, L., Fattouha, B. (2002). Non–linearity between finance and growth, Economics Letters 74, 339-345
- 26. Demirgüc-Kunt, A., Maksimovic, V. (2002). Funding growth in bank-based and market-based financial systems: evidence from firm level data, Journal of Financial Economics 65, 337–363
- 27. Devereux, M.B., Smith, G.W. (1994). International risk sharing and economic growth, International Economic Review 35, 535–550
- 28. Eichengreen, B., O'Rourke, K.H. (2012). A tale of two depressions redux, VoxEU.org, 06 March
- 29. Galor, O., Moav, O. (2002). Natural selection and the origin of economic growth, Quarterly Journal of Economics 117, 1133–1191
- 30. Goldsmith, R.W. (1969). Financial structure and development (New Haven: Yale University Press)
- Greenwood, J., Jovanovic, B. (1990). Financial development, growth, and the distribution of income, Journal of Political Economy 98, 1076-1108
- Guiso, L., Sapienza, P., Zingales, L. (2004). Does local financial development matter?, Quarterly Journal of Economics 119, 929–969
- 33. Helbling, T. (2009). How similar is the current crisis to the Great Depression?, VoxEU.org, 29 April
- Horii, R., Ohdoi, R., Yamamoto, K. (2013). Financial infrastructure, technological shift, and inequality in economic development, Macroeconomic Dynamics 17, 531–562
- 35. International Monetary Fund (2013). World economic outlook: transitions and tensions (Washington, DC: International Monetary Fund)
- 36. Khan, A. (2001). Financial development and economic growth, Macroeconomic Dynamics 5, 413-433
- 37. King, R.G., Levine, R. (1993a). Finance and growth: Schumpeter might be right, Quarterly Journal of Economics 108, 717–737
- King, R.G., Levine, R. (1993b). Finance, entrepreneurship, and growth: theory and evidence, Journal of Monetary Economics 32, 513–542

- La Torre, D., Marsiglio, S. (2010). Endogenous technological progress in a multi-sector growth model, Economic Modelling 27, 1017–1028
- Law, S.H., Singh, N. (2014). Does too much finance harm economic growth? Journal of Banking & Finance 41, 36-44
- 41. Levine, R. (1998). The legal environment, banks, and long-run economic growth, Journal of Money, Credit and Banking 30, 596–613
- 42. Levine, R. (1999). Law, finance, and economic growth, Journal of Financial Intermediation 8(1-2), 8–35
- 43. Levine, R. (2005). Finance and growth: theory and evidence, in (Aghion, P., Durlauf, S.N., Eds.): "Handbook of economic growth", 866-934 (Amsterdam: Elsevier North–Holland)
- 44. Levine, R., Loayza, N., Beck, T. (2000). Financial intermediation and growth: causality and causes, Journal of Monetary Economics 46, 31–77
- 45. Levine, R., Zervos, S. (1998). Stock markets, banks, and economic growth, American Economic Review 88, 537–558
- Loayza, N., Rancière, R. (2006). Financial development, financial fragility, and growth, Journal of Money, Credit and Banking 38, 1051–1076
- 47. Lucas, R.E. (1988). On the mechanics of economic development, Journal of Monetary Economics 22, 3–42
- 48. Marsiglio S., La Torre, D. (2012). Population growth and utilitarian criteria in the Lucas-Uzawa model, Economic Modeling 29, 1197–1204
- 49. McNally, D. (2010). Global slump: the economics and politics of crisis and resistance (Oakland, CA: PM Press)
- 50. Morales, M.F. (2003). Financial intermediation in a model of growth through creative destruction, Macroeconomic Dynamics 7, 363-393
- Mullingan, C.B., Sala–i–Martin, X. (1993). Transitional dynamics in two-sector models of endogenous growth, Quarterly Journal of Economics 108, 739–773
- 52. Obstfeld, M. (1994). Risk-taking, global diversification, and growth, American Economic Review 84, 1310–1329
- Pagano, M. (1993). Financial markets and growth: an overview, European Economic Review 37, 613–622
- 54. Panizza, U. (2013). Financial development and economic growth: Known knowns, known unknowns, and unknown unknowns, IHEID Working Paper No. 14-2013
- Rajan, R.G., Zingales, L. (1998). Financial development and growth, American Economic Review 88, 559–586
- 56. Reinhart, C.M., Rogoff, K.S. (2014). Recovery from financial crises: evidence from 100 episodes, American Economic Review: Papers & Proceedings 104, 50-55
- 57. Rioja, F., Valev, N. (2004a). Does one size fit all? A re-examination of the finance and growth relationship, Journal of Development Economics 74, 429–447
- 58. Rioja, F., Valev, N. (2004b). Finance and the sources of growth at various stages of economic development, Economic Inquiry 42, 127–140
- Rousseau, P., Sylla, R. (1999). Emerging financial markets and early U.S. growth, NBER Working Paper No. 7448
- Rousseau, P., Wachtel, P. (1998). Financial intermediation and economic performance: historical evidence from five industrial countries, Journal of Money, Credit and Banking 30, 657–678
- Rousseau, P., Wachtel, P. (2011). What is happening to the impact of financial deepening on economic growth?, Economic Inquiry 49, 276–288

- 62. Samargandi, N., Fidrmuc, J., Ghosh, S. (2015). Is the relationship between financial development and economic growth monotonic? Evidence from a sample of middle-income countries, World Development 68, 66–81
- 63. Smith, W.T. (2006). A closed form solution to the Ramsey model, Contributions to Macroeconomics 6, article 3
- 64. Trew, A. (2006). Finance and growth: a critical survey, Economic Record 82, 481-490
- 65. Trew, A. (2008). Efficiency, depth and growth: quantitative implications of finance and growth theory, Journal of Macroeconomics 30, 1550-1568
- 66. Trew, A. (2014). Finance and balanced growth, Macroeconomic Dynamics 18, 883-898
- 67. Uzawa, H. (1965). Optimum technical change in an aggregate model of economic growth, International Economic Review 6, 18–31
- 68. Wade, R. (2008). The first-world debt crisis of 2007–2010 in global perspective, Challenge 51, 23–54
- Wurgler, J. (2000). Financial markets and the allocation of capital, Journal of Financial Economics 58(1-2), 187–214
- Xie, D. (1994). Divergence in economic performance: transitional dynamics with multiple equilibria, Journal of Economic Theory 63, 97-112
- 71. Zingales, L. (2015). Does finance benefit society?, CEPR Discussion Paper No. 10350