Mean versus Median Voting in Multi-Dimensional Budget Allocation Problems. A Laboratory Experiment*

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We experimentally compare two natural mechanisms for the collective choice of an allocation of a fixed budget to a number of divisible public projects: the mean rule that implements the average of all individual proposals, and a suitably normalized median rule. Theoretical results predict extreme voting behavior in equilibrium under the mean rule and frequently sincere voting under the normalized median rule. Our findings confirm equilibrium behavior under the mean rule in situations in which the equilibrium strategies are easily identifiable. The empirical results for the normalized median rule are multifaceted. While we also find that many individuals play best responses, remarkably these are rarely sincere. Nevertheless, we find that the normalized median rule enjoys significantly better welfare properties than the mean rule.

JEL classifiction: D71; D72

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1 Introduction

Many decisions on public spending involve the aggregation of individual opinions either directly in various kinds of committees, or indirectly through representatives (Mueller, 2003). An important example is the allocation of expenditure on different public goods: at the state level the cabinet has to decide on the funding of national public goods and services such as health care, social security, defense and national infra structure; at the municipal level, decisions involve local public goods such as the funding of recreational facilities and regional services. The standard economic formulation of such problems is as a *budget allocation problem:* a group of individuals with generally divergent preferences has to come to an agreement about how to spend a fixed budget (e.g. tax revenue) on a number of alternative uses ('public projects') at fixed prices. The fundamental question is then which collective choice rule to use. The aim of the present paper is to shed light on this question using the results of a laboratory experiment.

The budget allocation problem is often addressed under additional assumptions on agents' preferences. One particularly attractive solution is provided by the well-known median voter theorem in the case when there are two rival projects and individuals have single-peaked preferences over the expenditure levels of them. Indeed, if preferences are single-peaked the expenditure level most preferred by the median voter wins a pairwise majority vote against all other feasible allocations; moreover, if the use of the median rule is common knowledge among the agents in a private information setting, then it is the unique (weakly) dominant strategy to sincerely report one's true preference (Black, 1948; Moulin, 1980).

However, there are two major problems with the median rule in the context of budget allocation problems. First, while sincere voting is the unique weakly dominant strategy under single-peakedness, there are also other Nash equilibria and even inefficient ones; for instance, if *all* agents vote (for whatever reason) for an allocation outside the convex hull of their preference peaks, the outcome is inefficient but nevertheless a Nash equilibrium since no single voter can change the outcome. Secondly, and more importantly, the median rule cannot be readily generalized to multi-dimensional budget allocation problems with more than two public projects; the reason is that the coordinate-wise median, i.e. the vector of the median expenditure proposals for each public project, does in general not satisfy the budget restriction if there are three or more projects (see below for examples). But applications of collective budget allocation problems will typically involve more than two projects.

Both of these problems can be addressed by using a simple alternative to the median rule, the mean rule which implements the average of the expenditure proposals for each public project. First, it has been shown that in the one-dimensional case of two public projects the mean rule has a *unique* Nash equilibrium if preferences are single-peaked (Renault and Trannoy, 2005). Secondly, there is no difficulty in generalizing the mean to the multi-dimensional setting since the average of the individual expenditure proposals always satisfies the budget constraint. More generally, in comparison to the median rule the mean rule has the advantage of being simple and easily understandable even to people without any training in basic statistics. But clearly, *sincere* voting does not constitute an equilibrium under the mean rule. For instance, if the average of the proposed expenditure levels for a project is below an agent's most preferred level it will in general be optimal to exaggerate that preference in order to 'pull' the outcome in the preferred direction. Thus, using the mean rule as the collective allocation mechanism comes at the price of loosing the robustness of the median with respect to strategic manipulation.

In order to test the empirical implications of the fundamentally different properties of the two allocation mechanisms, the mean and the median rule, we conducted a laboratory experiment. We were particularly interested in the following questions: Do subjects recognize the possibility of manipulation by voting for the extremes under the mean rule? Do they vote sincerely under median based rules when this constitutes a best response, or when optimal manipulations are hard to detect? And finally, what are the welfare implications of the different behavior under the two rules? Subjects voted in groups of five individuals on the allocation of a fixed budget on three different public projects. We had three treatment variables: RULE, PEAK DISTRIBUTION and INFO; the first two varied between subjects, the third within subjects. The variable RULE took on the values 'mean' or 'median' while the variable PEAK DISTRIBUTION determined four different distributions of preferences. As 'median' rule we used the *normalized median rule* suggested by Nehring (2009) and studied in Lindner (2011), according to which the coordinate-wise median expenditure shares are proportionally adjusted so that the budget restriction is satisfied. We induced symmetric preferences that depend only on the distance to the most preferred allocation (the 'peak'); specifically, we used the natural resource metric as distance function, i.e. the distance of an allocation is measured by the sum of the expenditure differences to the peak for each public project. The individual loss of an allocation is then given by a (convex) transform of the distance to the peak. Finally, the within-subject variable INFO determined the state of knowledge that subjects had about the preferences of the other participants: in the 'full info' treatment preferences were common knowledge, in the 'no info' treatment subjects only knew their own preferences. The 'full info' treatment was played for three periods (fixing the values of the other two treatment variables); the 'no info' treatment was played for five periods in order to examine learning effects.

Our experimental results on the mean rule largely confirm the theoretical prediction of extreme voting behavior ('polarization'), at least in those cases in which the optimal strategy is easily identifiable. This is in line with the empirical literature on the one-dimensional case in which the optimal strategy is in fact often easy to determine (Marchese and Montefiori, 2011; Louis et al., 2019). Not surprisingly, however, we find less Nash equilibrium play if optimal behavior is more difficult to identify. This can be the case in the multi-dimensional setting because one's own peak can differ from the expected average proposal of the other players in *different* directions. This also implies that one has to be even more careful in using the mean rule in multi-dimensional settings; mistakes in computing the optimal strategy (even under complete information) may result in inferior, possibly even inefficient outcomes.

The results on the normalized median rule are remarkable in several ways. In two of the four preference distributions sincere reporting was the 'focal' Nash equilibrium. In these two cases one of the voters occupied the median position in all coordinates.

By consequence, the coordinate-wise median happened to be feasible for these distributions. While the median voter indeed mostly reported sincerely, the non-pivotal voters did not always report truthfully. Non-pivotal voters frequently played a best response to sincere voting of the others but did not always turn to the focal truthful strategy themselves.[1] In the other two of our four preference distributions, due to the necessity for normalizing the coordinate-wise median, one voter could receive her most preferred allocation by strategic misrepresentation, respectively (provided that all others vote sincerely). However, identifying the optimal strategy is difficult in these cases and in fact none of our subjects was able to exploit the manipulation possibility. Although sincere voting occurred less often than one could expect from theoretical considerations under the normalized median rule, we still observed a higher share of truthful votes under the median rule than under the mean rule.

Importantly, we find that the normalized median rule fares significantly better than the mean rule in terms of welfare. Specifically, we find that the normalized median rule yields significantly higher expected utilitarian social welfare either under the full info and the no info treatment. Moreover, under the full info treatment the number of Pareto efficient outcomes is significantly higher under the normalized median rule.[2] This is all the more remarkable since we chose our peak distributions in such a way that the 'focal' Nash equilibria under both rules result in exactly the *same* allocations, respectively. Thus, at least under full information, the differences in the welfare properties can be traced back solely to the rules themselves.

Relation to the Literature

In the standard one-dimensional case with single-peaked preferences, the median rule has been studied extensively in the literature, see e.g. the recent contribution of Núñez et al. (2020) and the references therein. The mean rule and its equilibria in the one-dimensional case have been characterized by Renault and Trannoy (2005) and further studied in Renault and Trannoy (2011). Rosar (2015) compares the median and mean mechanisms in a model with incomplete information and interdependent preferences. An experimental study on voting behavior under the mean rule in the one-dimensional case has been conducted by Marchese and Montefiori (2011). An explicit experimental comparison between the mean and the median rule in the one-dimensional case is provided in Block (2014). Since the median rule is a strategy-proof mechanism if voters' preferences are single-peaked the results of the latter study also inform the experimental literature on dominant strategy implementation (Cason et al., 2006). More recently, Louis et al. (2019) have theoretically and empirically studied general *trimmed means*, of which both the mean and the median rule are special cases.

[1] Our results therefore do not lend empirical support to the hypothesis of 'partial honesty' which has been put forward as a theoretical equilibrium refinement in the implementation literature by Dutta and Sen (2012); partial honesty requires that (some) agents report their true type whenever doing so is among the best responses.

[2] Under the no info treatment, the difference of the number of Pareto efficient outcomes resulting from the two rules is not significant.

To the best of our knowledge, the present paper offers the first experimental analysis of the mean rule and a natural generalization of the median rule in a multi-dimensional setting. As noted above, one of the advantages of the mean rule is that it readily generalizes to an arbitrary number of dimensions. The question of how to generalize the median rule to multi-dimensional allocation problems under a budget constraint has received considerable attention recently, among others in the growing literature on *participatory budgeting* (Aziz and Shah, 2020). While much of this literature addresses the difficulties that arise from the potential indivisibility of projects, important benchmark models assume that projects can also be partly funded, as done here (Goel et al., 2019; Freeman et al., 2021).

The particular 'normalized' version of a multi-dimensional median rule considered here lends itself naturally to experimental work because of its simplicity and intuitive appeal, but there are other possibilities. For instance, Lindner (2011) considered, among others, the orthogonal projection to the budget hyperplane of the coordinate-wise median, and a sequential rule that implements the coordinate-wise median project by project according to a fixed sequence, allocating the residual budget to the last project. Other approaches seek to directly maximize welfare under specific assumptions about individual preferences and with respect to various social welfare functions, see e.g. Laruelle (2021). The special case in which individual preferences are represented by the negative L_1–distance to the most preferred outcome is considered in Goel et al. (2019) and Freeman et al. (2021). Nehring and Puppe (2019) argue for the minimization of the sum of the L_1–distances to the individual peaks from the more general perspective of a 'frugal' approach to majoritarian preference aggregation. These approaches generally result in set-valued solutions and therefore face a selection problem that the normalized median rule avoids (for an odd number of voters).

2 The Model

A group $N = \{1, ..., n\}$ of individuals has to collectively decide how to allocate an amount Q to L public projects. We assume that the public projects are divisible, i.e. can be partly funded to any (non-negative) extent. Moreover, preferences are monotone and prices are fixed so that the set of feasible allocations is given by the budget hyperplane

$$X = \{x \in \mathbb{R}^L_{\geq 0} \mid \sum_{\ell=1}^{L} x^\ell = Q\},$$

where $x^\ell \geq 0$ is the expenditure share for project $\ell \in \{1, ..., L\}$.

Individuals have different preferences over the extent to which projects should be funded and have to find a compromise. In our specific context, not all kinds of preferences seem reasonable, and one standard assumption is convexity. In fact, to simplify the analysis we will make the even stronger assumption that each voter has a unique most preferred allocation, the *peak*, and that the utility of an allocation only depends on its distance to the peak. The relevant distance is given here by the total expenditure that has to be re-allocated. Specifically, we will assume that agent i's ordinal utility function $u_i(\cdot)$ satisfies

$$u_i(x) \;=\; f(d(x,p_i)), \tag{2.1}$$

where p_i is i's peak, $f : [0,\infty) \to \mathbb{R}$ is a strictly decreasing function and $d(\cdot,\cdot)$ is the distance function defined by $d(x,y) := \sum_{\ell=1}^{L} |x^\ell - y^\ell|$. Mathematically, the distance function d is the L_1–distance; it is the natural metric in our present context since it compares allocations in terms of their total expenditure difference. Therefore, one may refer to it in our context also as *resource metric* (Nehring and Puppe, 2019). Preferences satisfying (2.1) are referred to as *metrically single-peaked* (Nehring et al., 2008); evidently, such preferences are convex and single-peaked in each direction. Note that in the case $L = 2$, i.e. if there are only two projects, the set X is a one-dimensional budget line. In this case, the amount of expenditure for one project automatically determines the amount for the other project via the budget restriction. The assumption of metric single-peakedness means that preferences are single-peaked and the two allocations with the same distance to the peak from the left and the right are indifferent.

The case relevant for our experimental analysis is the case $L = 3$ in which the indifference curves on X are symmetric hexagons, see Fig. 1.

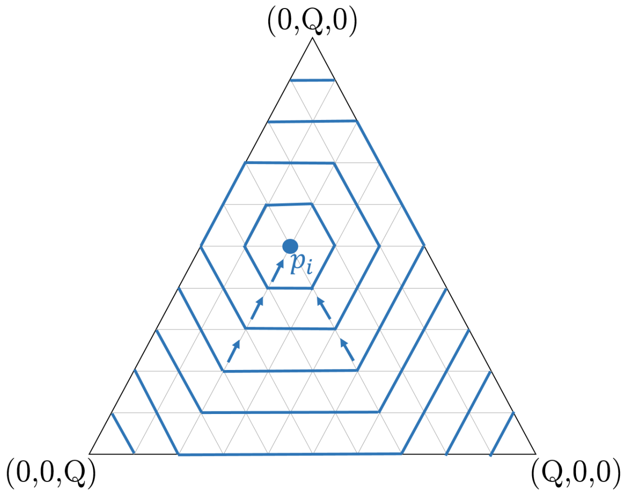

Figure 1: Indifference curves

2.1 The Mean Rule

We turn to the description of the aggregation rules. We assume that all voters simultaneously submit a proposal of how to distribute the money amount Q among the L public projects; in other words, we are considering the normal form game in which the strategy space for each individual is simply given by the set X of feasible allocations.

The *mean rule* takes the outcome to be the coordinate-wise average of the proposals. Let $q_i \in X$ be the proposal of voter i and $q = (q_1, ..., q_n)$ the vector of individual proposals. The outcome according to the mean rule is given by

$$Mean(q) := \frac{1}{n}\sum_{i=1}^{n} q_i = \left(\frac{1}{n}\sum_{i=1}^{n} q_i^1, ..., \frac{1}{n}\sum_{i=1}^{n} q_i^L\right).$$

In the one-dimensional case $L = 2$, the Nash equilibrium under the mean rule has been completely characterized by Renault and Trannoy (2005): For all peak distributions $p = (p_1, ..., p_n)$ such that $p_i \neq p_j$ for all voters $i \neq j$ there is a unique equilibrium. If the outcome of this equilibrium is x^*, then all voters with $p_i^\ell > x^{*\ell}$ propose $q_i^\ell = Q$ and all voters with $p_i < x^{*\ell}$ propose $q_i^\ell = 0$ for $\ell = 1, 2$; there is at most one voter i_0 who does *not* propose one of the extreme values, $(0, Q)$ or $(Q, 0)$, and for such voter one necessarily has $p_{i_0} = x^*$.[3] From this characterization it is immediate that the unique Nash equilibrium is also Pareto efficient.

While the mean rule is just as simple to formulate and to use in the multi-dimensional case, Nash equilibrium is no longer unique if $L > 2$; indeed, even for $L = 3$ there exist peak distributions for which there are several equilibria some of which even Pareto inefficient, see Rollmann (2020). Moreover, best responses are in general no longer unique. However, one still has the result that in any equilibrium at most one voter makes a proposal in the interior of the feasible set. The reason is that under the mean rule every voter is always pivotal. Specifically, the following result has been observed in Franken (2015). Let $int(X)$ be the set of all feasible allocations such that $x^\ell > 0$ for all $\ell = 1, ..., L$.

Proposition 1. *Suppose that the peak distribution $p = (p_1, ..., p_n)$ satisfies $p_i \neq p_j$ for $i \neq j$. Let $q^* = (q_1^*, ..., q_n^*)$ be a Nash equilibrium of proposals and $x^* = Mean(q^*)$. Then, $q_i^* \in int(X)$ for at most one voter $i = 1, ..., n$. Moreover, if $q_{i_0}^* \in int(X)$ for voter i_0, then $x^* = p_{i_0}$.*

Proof. Suppose that in a Nash equilibrium $q^* = (q_1^*, ..., q_n^*)$ with $x^* = Mean(q^*)$, one has $q_j^* \neq x^*$. Then $q_j^* \notin int(X)$, because otherwise voter j could move the outcome closer to her own peak, hence q_j^* would not be a best response. □

2.2 The Normalized Median Rule

The mean rule is simple and easily understandable also by people who do not have had any training in basic statistics. However, from an economic point of view a major drawback of the mean rule is that it does not induce sincere voting behavior with rational and strategic individuals. To overcome this, a well-studied alternative in the one-dimensional case is the *median rule*. In this case, the median rule chooses the median of the reported peaks if the number of individuals is odd and some allocation between the two middle peaks if the number of voters is even. For simplicity we will assume in the following that n is odd (in our experiments, we have $n = 5$).

[3]See Block (2014) for a simple algorithm to decide if such a voter i_0 exists, and if yes, how to identify that voter.

It is well-known that sincere voting is the unique weakly dominant strategy under the median rule (Moulin, 1980). However, the median rule has also disadvantages. First, it has been observed that there are many, and even inefficient, Nash equilibria in which voters do not follow their dominant strategy; for instance, it constitutes an equilibrium if (for whatever reason) *all* voters announce some fixed feasible allocation $x_0 \in X$ (Saijo et al., 2007). More importantly, it is not evident how to apply the median rule in multi-dimensional settings when $L > 2$. The reason is that the coordinate-wise median of feasible proposals does not necessarily satisfy the budget constraint. For example, suppose that $L = 3$ and three voters propose to split the money Q equally among two projects, respectively, in the following way: $q_1 = (Q/2, Q/2, 0)$, $q_2 = (Q/2, 0, Q/2)$, $q_3 = (0, Q/2, Q/2)$. Then, the coordinate-wise median yields the allocation $(Q/2, Q/2, Q/2)$ which is clearly not feasible. There are several ways of how one can respond to this problem but there does not seem to exist a simple 'canonical' solution, see among others Nehring et al. (2008); Lindner (2011); Goel et al. (2019); Nehring and Puppe (2019); Freeman et al. (2021). In our present study we use the *normalized* median rule suggested by Nehring (2009) and analyzed in Lindner (2011) because of its particularly simple formulation.

For each vector $q = (q_1, ..., q_n)$ of proposals and each project $\ell = 1, ..., L$, denote by $Med^\ell(q)$ the median of the values $\{q_1^\ell, ..., q_n^\ell\}$ (recall that we are henceforth assuming n to be odd). The normalized median allocates to each project the median level of expenditure proposed for the project adjusted by the potential excess expenditure factor $Q/\sum_{\ell=1}^{L} Med^\ell(q)$. Specifically, for all $\ell = 1, ..., L$,

$$NMed(q) \; := \; Med(q) \cdot \frac{Q}{\sum_{\ell=1}^{L} Med^\ell(q)} \; = \; \big(Med^1(q), ..., Med^L(q)\big) \cdot \frac{Q}{\sum_{\ell=1}^{L} Med^\ell(q)}.$$

Observe that this definition requires that $\sum_{\ell=1}^{L} Med^\ell(q) > 0$; if $\sum_{\ell=1}^{L} Med^\ell(q) = 0$, we set $NMed^\ell(q) = Q/L$ by convention.

Due to the normalization, sincere voting is no longer a weakly dominant strategy, as shown by the following example.

Example 1. *Let* $Q = 100$*,* $L = 3$*, and consider five voters with peaks* $p_1 = (20, 20, 60)$*,* $p_2 = (10, 65, 25)$*,* $p_3 = (10, 8, 82)$*,* $p_4 = (70, 10, 20)$ *and* $p_5 = (5, 10, 85)$*, respectively. The outcome of sincere voting under the normalized median rule is*

$$(12.5, 12.5, 75) \; = \; (10, 10, 60) \cdot \tfrac{100}{80},$$

because the coordinate-wise median of the above distribution is $(10, 10, 60)$ *with a total expenditure of* $80 < 100 = Q$*. But if agents 2 – 5 vote sincerely, agent 1 can get her peak as the outcome by reporting, e.g.,* $q_1 = (35, 35, 30)$*. Indeed, the coordinate-wise median is then* $(10, 10, 30)$*, hence the outcome after normalization is* $(20, 20, 60)$ *which is exactly agent 1's peak. In the appendix we show that this in fact constitutes a Nash equilibrium, i.e. sincere voting is optimal for agents 2 – 5 provided that agent 1 reports* $q_1 = (35, 35, 30)$*.*

Although sincere voting is thus no longer a dominant strategy (nor in general a best response even if all others vote sincerely), we can show that sincere voting constitutes a Nash equilibrium if there is one agent i_0 whose true peak happens to occupy the median expenditure level for *every* project.[4]

Proposition 2. *Consider the peak distribution $p = (p_1, ..., p_n)$ with n odd, and assume that there exists $i_0 = 1, .., n$ such that $Med^\ell(p) = p^\ell_{i_0}$ for all $\ell = 1, ..., L$. If all voters have metrically single-peaked preferences then sincere voting, i.e. $q^*_i = p_i$ for all $i = 1, ..., n$, constitutes a Nash equilibrium under the normalized median rule with outcome p_{i_0}.*

Proof. To simplify notation, let $m = (m^1, ..., m^L)$ denote the peak of the median voter i_0 in every project and hence the outcome of the normalized median rule under sincere voting. Clearly, if all other agents vote sincerely, truthful reporting is also optimal for i_0 since she receives her peak by doing so. Consider any voter $j \neq i_0$ and suppose that all other agents vote sincerely. By misreporting, voter j can bring the median in no coordinate closer than m^ℓ is to her most preferred level in that coordinate; in other words, if $x = (x^1, ..., x^L)$ is the coordinate-wise median resulting from an arbitrary vote of agent j, we have

$$x^\ell \leq m^\ell \text{ whenever } m^\ell < p^\ell_j \quad \text{and} \quad x^\ell \geq m^\ell \text{ whenever } m^\ell > p^\ell_j. \tag{2.2}$$

In particular, voter j cannot benefit from a non-truthful vote if the resulting coordinate-wise median x is feasible. Thus, consider the case in which $R := \sum_{\ell=1}^{L} x^\ell \neq Q$, i.e. the case in which a normalization is necessary. Without loss of generality, assume that $R < Q$ (the argument in the case $R > Q$ is completely symmetric). Denote by $\hat{x}$ the normalized coordinate-wise median, i.e. $\hat{x} := x \cdot (Q/R)$, and observe that the normalization increases the value in each coordinate, i.e. $\hat{x}^\ell \geq x^\ell$ for all ℓ.

Denote by $K^+ := \{\ell : \hat{x}^\ell > m^\ell\}$ and $K^- := L \setminus K^+$, and by L^+ the set of those coordinates in which $\hat{x}$ is closer to p_j than m, i.e.,

$$L^+ := \{\ell : |\hat{x}^\ell - p^\ell_j| < |m^\ell - p^\ell_j|\};$$

finally, let $L^- := L \setminus L^+$.

First, we show that $K^- \subseteq L^-$. Indeed, $\hat{x}^\ell = m^\ell$ directly implies $\ell \in L^-$, and if $\hat{x}^\ell < m^\ell$ we obtain $x^\ell \leq \hat{x}^\ell < m^\ell$, hence $m^\ell \leq p^\ell_j$ by (2.2), and therefore $\ell \in L^-$. Since $K^- \subseteq L^-$ we also have $L^+ \subseteq K^+$ by contraposition. By feasibility of $\hat{x}$ and m, we obtain

$$0 = \sum_{\ell \in L} (\hat{x}^\ell - m^\ell) = \sum_{\ell \in L^+} (\hat{x}^\ell - m^\ell) + \sum_{\ell \in K^+ \setminus L^+} (\hat{x}^\ell - m^\ell) + \sum_{\ell \in K^-} (\hat{x}^\ell - m^\ell).$$

Since $\sum_{\ell \in K^+ \setminus L^+} (\hat{x}^\ell - m^\ell) \geq 0$, this implies

$$\sum_{\ell \in L^+} (\hat{x}^\ell - m^\ell) \leq \sum_{\ell \in K^-} (m^\ell - \hat{x}^\ell). \tag{2.3}$$

[4] We are grateful to a referee who suggested a generalization of our original argument. The current formulation of the proof of Proposition 2 also greatly benefitted from conversations with Claudio Kretz.

By the (reverse) triangle inequality, we have $|\,|m^\ell - p_j^\ell| - |\hat{x}^\ell - p_j^\ell|\,| \le |m^\ell - \hat{x}^\ell|$, hence from (2.3) and the fact that $L^+ \subseteq K^+$,

$$\sum_{\ell\in L^+} |\,|m^\ell - p_j^\ell| - |\hat{x}^\ell - p_j^\ell|\,| \;\le\; \sum_{\ell\in L^+} (\hat{x}^\ell - m^\ell) \;\le\; \sum_{\ell\in K^-} (m^\ell - \hat{x}^\ell). \tag{2.4}$$

Finally, using again the fact that $\hat{x}^\ell < m^\ell$ and $x^\ell \le \hat{x}^\ell$ jointly imply $m^\ell \le p_j^\ell$ by (2.2), we have

$$\sum_{\ell\in K^-} (m^\ell - \hat{x}^\ell) \;=\; \sum_{\ell\in K^-} |\,|m^\ell - p_j^\ell| - |\hat{x}^\ell - p_j^\ell|\,| \;\le\; \sum_{\ell\in L^-} |\,|m^\ell - p_j^\ell| - |\hat{x}^\ell - p_j^\ell|\,|\;.$$

Together with (2.4) we thus obtain

$$\begin{aligned}
\sum_{\ell\in L^+} |m^\ell - p_j^\ell| - |\hat{x}^\ell - p_j^\ell| \;&=\; \sum_{\ell\in L^+} |\,|m^\ell - p_j^\ell| - |\hat{x}^\ell - p_j^\ell|\,| \\
&\le\; \sum_{\ell\in L^-} |\,|m^\ell - p_j^\ell| - |\hat{x}^\ell - p_j^\ell|\,| \\
&=\; \sum_{\ell\in L^-} |\hat{x}^\ell - p_j^\ell| - |m^\ell - p_j^\ell|,
\end{aligned}$$

that is,

$$\sum_{\ell\in L} |m^\ell - p_j^\ell| \;\le\; \sum_{\ell\in L} |\hat{x}^\ell - p_j^\ell|.$$

In other words, agent j cannot benefit from misreporting. Note that the argument only depends on the assumption that some agent occupies the median position in every coordinate given the true peak of agent j and the reports of all other agents. □

3 The Experiment: Design and Hypotheses

3.1 Design

To analyze voting behavior of real subjects under the mean and the normalized median rule, and to assess their respective merits, we conducted a laboratory experiment. We ran eight sessions at the KD2Lab of Karlsruhe Institute of Technology in October 2015. Each session involved of three fixed groups with five participants, respectively. For each of the eight sessions, 15 participants (120 in total) were recruited via ORSEE (Greiner, 2015). A session lasted on average about 1 hour and 15 minutes, the software we used was z-Tree (Fischbacher, 2007). The average payoff of the participants in the experiment was 13.98 Euros, including a show-up fee of 5 Euros.

Subjects were informed that they would participate in the collective determination of the allocation of 100 monetary units to three public projects in a group with four other anonymous subjects. Subjects' per period payoff function in the unit *ECU* was the following:[5]

$$u_i(x) \;=\; 10 + \frac{760}{4 + d(x, p_i)}\;, \tag{3.1}$$

[5] Experimental Currency Unit; 100 *ECU* corresponded to 1.00 Euro.

where p_i denotes the peak of individual i and x the social outcome.[6] With this specification, the maximal payoff per period (when the outcome coincides with one's peak) is 200 *ECU* and the minimum payoff is about 13.73 *ECU* (the highest possible distance of the outcome to one's own peak is 200). Fig. 2 depicts the payoff function which was also shown to subjects during the experiment; we chose the sharp decline close to the maximum to incentivize optimal behavior.

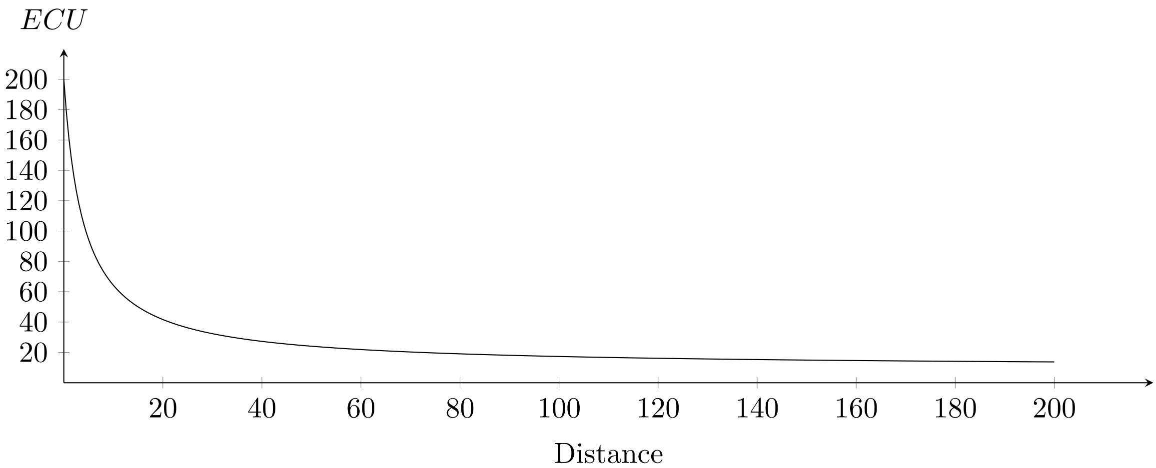

Figure 2: The payoff function

Subjects were informed about their own peaks (and hence the entire payoff function) under the 'no info' treatment and also about the peaks of the four other participants of their group under the 'full info' treatment. Once all group members had cast their vote, the outcome was made public and subjects were informed about their respective payoffs. Information on the individual votes of previous rounds was not disclosed. The aggregation of the votes was either done by the mean (the first four sessions) or the normalized median rule (the last four sessions). Under the 'no info' treatment each peak distribution was played for five periods and under the 'full info' treatment for three periods.[7] The 'no info' treatment was played first followed by the 'full info' treatment. The peak distributions, information and number of periods were identical for the two voting rules so that we can directly compare them.

[6] This function arises by taking the function $f(t) = 10 + 760/(4 + t)$ in (2.1).

[7] We chose the larger number of periods for the 'no info' treatment to give subjects the possibility to learn relevant qualitative aspects of the peak distribution.

3.2 Peak Distributions and their Focal Nash Equilibria

We used the following four peak distributions as between-subject variables.[8] Peak distribution I is the same as in Example 1. The numbers in the top block in Table 1 describe the peaks of the five participants, for instance participant 1 has the peak $p_1 = (20, 20, 60)$, participant 2 has the peak $p_2 = (10, 65, 25)$, etc. The second block describes the 'focal' Nash equilibrium under the mean rule in which every agent reports the vertex that is closest to her own peak. In the appendix we prove that this is indeed the unique Nash equilibrium in which all agents report one of the three extreme points in which all the money is allocated to one project; moreover, in this equilibrium best responses are unique (i.e. the strategy combination constitutes a *strict* Nash equilibrium). Observe that participant 1 receives her peak in this equilibrium. The third block describes the 'focal' Nash equilibrium under the normalized median rule. As already noted in Example 1 above, participant 1 receives her peak in this equilibrium by misrepresenting her peak. In the appendix, we show that this is the only Nash equilibrium in which at least four participants report truthfully.

	Participant					
I	**1**	**2**	**3**	**4**	**5**	$\boldsymbol{x(q^*)}$
	20	10	10	70	5	
p_i	20	65	8	10	10	
	60	25	82	20	85	
	0	0	0	100	0	20
$q_i^{*(Mean)}$	0	100	0	0	0	20
	100	0	100	0	100	60
	$\geq$10	10	10	70	5	20
$q_i^{*(NMed)}$	$\geq$10	65	8	10	10	20
	30	25	82	20	85	60

Table 1: Peak distribution I with focal Nash equilibria

The second peak distribution with 'focal' equilibria is as follows.

[8] Each group of 5 subjects was assigned three out of four distributions. The peak distribution was kept fixed within each session, but the individual peaks were re-assigned after the 'no info' treatment. E.g. in session 1, all groups played with peak distributions I,II, and III, whereas in session 2, groups played peak distributions I,II, and IV and so on. Sessions 1-4 differed from sessions 5-8 only by the treatment variable RULE.

II	Participant 1	2	3	4	5	
	40	75	70	20	12	
p_i	30	10	15	15	78	
	30	15	15	65	10	
	0	100	100	0	0	40
$q_i^{*(Mean)}$	50	0	0	0	100	30
	50	0	0	100	0	30
	20	75	70	20	12	40
$q_i^{*(NMed)}$	$\geq$15	10	15	15	78	30
	$\geq$15	15	15	65	10	30

Table 2: Peak distribution II with focal Nash equilibria

The 'focal' Nash equilibrium for the peak distribution II under the mean rule is given in the second block in Table 2. The appendix shows that this is the only (strict) Nash equilibrium in which at least four participants vote for one of the three extreme allocations under the mean rule. Participant 1 receives her peak splitting the budget evenly between projects 2 and 3. Also under the normalized median rule, participant 1 can receive her peak by misrepresentation – provided that all other report truthfully. Again, from the point of view of strategic simplicity, this is the 'focal' Nash equilibrium since it is the unique equilibrium in which at least four participants report truthfully (see appendix).

The third distribution III is qualitatively different. As can be seen from the peaks of the five participants in the top block of Table 3, participant 3 occupies the median position in every coordinate. By consequence, in this example sincere voting is indeed a Nash equilibrium under the normalized median rule by Proposition 2, and arguably the 'focal' one. On the other hand, the focal equilibrium under the mean rule, i.e. the unique equilibrium in which *all* participants vote for an extreme allocation, is more difficult to identify. The reason is that in this equilibrium – unlike in the two previous examples – some participants (4 and 5) do not vote for the extreme allocation that is closest to their own peak.

III			**Participant**			
	1	**2**	**3**	**4**	**5**	
	10	20	20	30	15	
p_i	20	10	20	15	30	
	70	70	60	55	55	
	0	0	0	100	0	20
$q_i^{*(Mean)}$	0	0	0	0	100	20
	100	100	100	0	0	60
	10	20	20	30	15	20
$q_i^{*(NMed)}$	20	10	20	15	30	20
	70	70	60	55	55	60

Table 3: Peak distribution III with focal Nash equilibria

The fourth peak distribution IV is similar to the third in that participant 3 again occupies the median position in every coordinate. Again, sincere voting is a Nash equilibrium under the normalized median rule by Proposition 2. The 'focal' Nash equilibrium under the mean rule (i.e. the unique equilibrium in which at least four participants report extreme allocations) is, however, different in character from the one in the previous distribution III. If all participants except 3 vote for the closest extreme allocation, participant 3 can receive her peak by sincere reporting; this constitutes a strict Nash equilibrium.

IV			**Participant**			
	1	**2**	**3**	**4**	**5**	
	10	50	25	15	25	
p_i	20	30	50	60	70	
	70	20	25	25	5	
	0	100	25	0	0	25
$q_i^{*(Mean)}$	0	0	50	100	100	50
	100	0	25	0	0	25
	10	50	25	15	25	25
$q_i^{*(NMed)}$	20	30	50	60	70	50
	70	20	25	25	5	25

Table 4: Peak distribution IV with focal Nash equilibria

3.3 Social Welfare Optima

For each of the four peak distributions, there is a unique optimum in terms of utilitarian social welfare, i.e. a unique maximizer of $\sum_{i=1}^{5} u_i(\cdot)$ where the $u_i(\cdot)$ are given as in (3.1); for peak distribution I it is the allocation $(10, 8, 82)$, for peak distribution II it is $(70, 15, 15)$, for peak distribution III it is $(20, 20, 60)$, and for peak distribution IV it is $(25, 50, 25)$. Note that the social optimum is always a peak of a voter but it does not always coincide with the allocation that minimizes the total raw distance to the peaks. This is due to the particular convex shape of the transformation function $f(\cdot)$ in (3.1). For peak distribution I, the minima of $\sum_{i=1}^{5} d(p_i, \cdot)$ are the feasible allocations in the range of $(10 - 20, 10 - 20, 60 - 80)$ (including voter 1's peak), and for peak distribution II these are the feasible allocations in the range $(40 - 55, 30, 15 - 30)$ (again including voter 1's peak); for peak distributions III and IV the distance minimizing and welfare maximizing allocations coincide and are given by the coordinate-wise median (i.e. voter 3's peak), respectively. In particular, the 'focal' Nash equilibria coincide with the social welfare optimum only for peak distributions III and IV.

3.4 Summary of the Design

Summarizing the design of the lab experiment, we ran four sessions for each voting rule. In each session three groups of five subjects voted on the allocation of 100 monetary units. Every group was randomly assigned three peak distributions, each of which was played for 5 periods under no information and for 3 periods with full information about the other voters' peaks. We thus observed a total of 2.880 individual decisions and 576 social outcomes. Since these observations are not independent, we clustered them in the statistical analysis, see Footnote 10 below. We have 12 independent observations per treatment variable RULE; for the treatment variables PEAK DISTRIBUTION and INFO, we have 24 observation per treatment and 12 per voting rule.

3.5 Hypotheses

With the data from our laboratory experiment, we tested the following hypotheses.

3.5.1 Mean Rule

Hypothesis (H1.1). ***Under the mean rule, sincere reporting occurs less frequently with full information than without information about the other participants' peaks.***

The rationale for this hypothesis is that without any information about the other participants' peaks the optimal strategy is ambiguous, and hence participants may be tempted to fall back to the cognitively simplest strategy available, which is sincere reporting. On the other hand, under full information the computation of the optimal (in general, non-truthful) strategy in the focal Nash equilibrium is frequently not difficult.

Hypothesis (H1.2). ***Under the mean rule, the focal Nash equilibrium strategy will be played more often with full information than with no information.***

We expect that participants understand the possibility of strategic voting under the mean rule in particular under full information. More specifically, we hypothesize that participants will turn to the 'focal' Nash strategy, at least in those cases in which this means voting for the extreme allocation closest to one's own peak. This is true for all participants under the distribution I, for participants 2-5 under the distribution II, for participants 1-3 under the distribution III, and for participants 1,2,4,5 under the distribution IV.

Hypothesis (H1.3). ***Nash play increases over time under the mean rule.***

We hypothesize that subjects adapt voting behavior over time both under no info and under full info and learn to play the focal Nash equilibrium strategy. We examine learning effects in particular by testing if individuals play a best response to the result of the previous round.

3.5.2 Normalized Median Rule

Since truth-telling is the focal Nash strategy for almost all participants independently of the information that they receive, our first hypothesis is that the share of sincere votes is the same in both information treatments.

Hypothesis (H2.1). ***Under the normalized median rule, the share of sincere votes is the same under no information and under full information.***

A possible argument against this hypothesis is that strategic voting is sometimes possible under the normalized median rule (for participant 1 under the peak distributions I and II). But the beneficial manipulation is difficult to detect and part of Hypothesis H2.1 is that also subjects in the role of participant 1 under peak distributions I and II will resort to the cognitively simplest strategy, i.e. sincere voting. One may also wonder if a Bayesian player would respond differently under the no information as compared to the full information condition. However, even for a Bayesian player sincere voting would be an optimal strategy given, say, uniform beliefs about the peak distribution of the others.

Hypothesis (H2.2). ***Under the normalized median rule, a best response to truth-telling of all other participants is played more frequently under full information than under no information.***

A weaker hypothesis than focal Nash play (i.e. sincere voting in most cases) is that subjects only play a best response to truth-telling of the other participants, without necessarily being sincere themselves whenever they are not pivotal. On the other hand, for a 'pivotal' participant (i.e. participant 1 under distributions I and II, and participant 3 under distributions III and IV, respectively) the focal Nash strategy and the best response to truth-telling of the others coincide. The rationale for Hypothesis H2.2 is that only under full information each participant *knows* if she is pivotal or not. Thus, under no info truth-telling is the 'safe' strategy while under full info also non-truthful best responses are easily identifiable.

3.5.3 Mean versus Normalized Median Rule

Concerning the comparison between the two rules, we have the following hypotheses.

Hypothesis (H3.1). ***The normalized median rule leads to a higher share of sincere votes as compared to the mean rule.***

Since truth-telling is a focal strategy for most of the subjects under the normalized median rule and non-truthful strategic voting is focal for at least four participants in each peak distribution under the mean rule, we hypothesize that the shares of sincere votes are higher under the normalized median rule as compared to the mean rule.

Hypothesis (H3.2). ***The distance of the votes from the true peak is higher under the mean than under the normalized median rule.***

This is a closely related and arguably weaker version of Hypothesis 3.1. The expectation that participants deviate *more* from the sincere vote under the mean rule is derived from the fact that the focal strategy there is to vote for an extreme allocation. Even if subjects do not vote sincerely under the normalized median rule, there is no apparent reason why they would choose to vote for extreme allocations.

4 Experimental Results

This section presents the results from our experiments. First, we look at the aggregated group outcomes and then at individual decisions.[9]

4.1 Social Outcomes

We classify the social outcomes according to the categories truth-telling, focal Nash, peak boundedness, Pareto efficiency and welfare optimality. We call an allocation *peak bounded* if it is within the range of the lowest and highest peak in each coordinate. Clearly, the peak bounded allocations contain the Pareto efficient allocations which in turn contain the welfare optimal allocations. In total, we observed 576 social outcomes, 288 for each rule; 360 outcomes were derived under no information and 216 under full information.[10]

Whereas in this section, we only consider aggregated results we go into more detail of the individual decisions in the subsequent sections. The difference is important since, e.g., an outcome might be classified as Nash outcome even though no individual subject

[9] More details and additional analyses are provided in Rollmann (2020).

[10] For the statistical analysis, we cluster the data on a group level. Each of the 24 groups provides 24 outcomes, derived from 3 different peaks and a total of 8 periods. Thus, we have 12 independent observations for each voting rule.

played a Nash strategy; conversely, even if most of the subjects played the focal Nash strategy, one single deviation may significantly distort the social outcome. The most important findings are summarized in Table 5.

4.1.1 Truth-telling Outcomes

A social outcome is classified as 'truth-telling outcome' if it corresponds to the outcome that would result from sincere voting. The truth-telling outcome is therefore either the mean or the normalized median of the true peaks. Our data exhibit 39 out of 576 (6.77%) truth-telling outcomes. Remarkably, all of these outcomes occur under the normalized median rule, and the difference between the rules is significant (Wilcoxon rank-sum test, $Z = -4.164$, $p < 0.001$). If we consider only the normalized median rule, we have 13.54% truth-telling outcomes that divide into 7.22% of all outcomes under the no information and 24.07% under the full info treatment. In line with the theoretical prediction, we observed only two truth-telling outcomes in total under peak distributions I and II, but 37 in total under distributions III and IV.

4.1.2 Focal Nash Outcomes

By 'focal Nash outcome' we mean the outcomes specified in Tables 1–4 above. Recall that the focal Nash *outcomes* are the same under both rules for all considered peak distributions. We observed a total of 47 focal Nash outcomes. More specifically, under the mean rule 2.78% of all outcomes were focal Nash, and 13.54% under the normalized median. The difference is significant according to a Wilcoxon rank-sum test ($Z = -3.189$, $p = 0.0014$). The focal Nash outcomes we observed under the mean rule were all under peak distribution II (8 in total). Recall that the 37 truth-telling outcomes under the normalized median rule and peak distributions III and IV coincide with the focal Nash outcome. This means that we observed only two focal Nash outcomes under the normalized median rule and peak distributions I and II. This already hints at the fact that participant 1 was not able to exploit the possibility of receiving her most preferred outcome by a clever strategic manipulation. The focal Nash outcomes divide into 13.89% of all outcomes under full information and 4.72% under no information. This seems reasonable since without information the optimal response is much harder to identify.

4.1.3 Peak Bounded and Pareto Efficient Outcomes

In our experiment, a total of 86.81% of all outcomes were peak bounded and 73.26% were Pareto efficient. Note that under both rules the truth-telling outcome is Pareto efficient (hence *a forteriori* peak bounded) given our assumptions on preferences and given the specific peak distributions. The total percentages for the normalized median rule are slightly higher (88.89% peak bounded and 76.74% efficient) than for the mean rule (84.72% peak bounded and 69.79% efficient) (Wilcoxon rank-sum test, $Z = -1.319$, $p = 0.1872$ and $Z = -1.896$, $p = 0.058$). Importantly, the share of peak bounded and Pareto efficient outcomes is significantly higher for the normalized median rule *under full information* (Wilcoxon rank-sum test, $Z = -3.231$, $p = 0.0012$ and $Z = -3.262$,

$p = 0.0011$).[11] Since the two rules have exactly the *same* (focal) Nash equilibrium outcome for all four peak distributions, under the rationality assumption and complete information, the observed differences in the welfare properties can only be traced back to the very fact that two distinct rules have been employed.

In general, we find we find high percentages of Pareto efficient outcomes under both voting rules. The concern of the theoretical implementation literature about the existence of inefficient Nash equilibria under median-based aggregation rules (Saijo et al., 2007) therefore seems to be less relevant from an empirical perspective in our budget allocation context.

4.1.4 Utilitarian Social Welfare

The total percentage of welfare optimal outcomes was 6.60% and all these were attained by the normalized median rule under which they amount to 13.19% of all outcomes. Given that the focal Nash outcome is only welfare optimal under peak distributions III and IV, this appears to be a reasonable share. Due to the between-subject design, we can compare the outcomes under the mean and the normalized median rule directly, and we find a significantly higher share of welfare optimal outcomes under the normalized median rule as compared to the mean rule (Wilcoxon rank-sum test, $Z = -4.161$, $p < 0.001$). The degree of information also significantly affects the shares of welfare optimal outcomes under the normalized rule: under full information, we observed 23.15% welfare optimal outcomes in total, whereas under no info only 7.22% (Wilcoxon signed-rank test $Z = -2.358$, $p = 0.0184$).[12]

The expected utilitarian social welfare is significantly higher under the normalized median rule. Specifically, the expected per round payoff is 39.27 under the normalized median rule versus 33.33 under the mean rule (Wilcoxon rank-sum test, $Z = -3.349$, $p < 0.001$). The difference is also significant in each info treatment separately: 36.78 under the normalized median rule versus 32.80 under mean rule in the no info treatment (Wilcoxon rank-sum test, $Z = -2.598$, $p < 0.0094$), and 43.41 under the normalized median rule versus 34.22 under mean rule in the full info treatment (Wilcoxon rank-sum test, $Z = -2.368$, $p = 0.0179$).[13]

4.1.5 Summary

Table 5 summarizes the results on outcomes by rule and information treatment. We highlight the following findings. First, subjects understand well that sincere voting

[11] The difference between the two rules is not significant in the no info treatment (Wilcoxon rank-sum test, $Z = 0.447$, $p = 0.6546$ and $Z = 0.938$, $p = 0.3483$).

[12] As a robustness test, we also looked at the outcomes that minimize the sum of the raw distance to the voters' peaks. Their total share is 21.18% of all outcomes, and again the share under the normalized median rule (31.60%) is significantly higher than under the mean rule (10.76%) (Wilcoxon rank-sum test, $Z = -3.428$, $p < 0.001$). The share of distance minimizing outcomes is especially high under the normalized median rule in the full info treatment, where it amounts to 44.44% of all outcomes.

[13] As a robustness check, we also looked at the expected total distance of the outcome from the peaks and find again that the normalized median rule attains a significantly smaller value than the mean rule (Wilcoxon rank-sum test, $Z = 1.965$, $p = 0.0494$).

is not optimal under the mean rule. On the other hand, the share of truth-telling outcomes under the normalized median rule is quite low, in particular in the no info treatment. Secondly, even though the social outcome did not generally coincide with the Nash outcome, voters coordinated on Pareto efficient outcomes even under the no info treatment. In the full info treatment, the share of both the peak bounded and the Pareto efficient outcomes is significantly higher under the normalized median rule. Thirdly, while the share of welfare optimal outcomes is low in total they occur only under the normalized median rule (and are thus significantly higher under both info treatments for the normalized median rule). More importantly, the expected utilitarian social welfare is higher under the normalized median rule for both treatments. We therefore state this result explicitly:

Result. ***The normalized median rule leads to significantly higher expected utilitarian social welfare under either information treatment.***

	Total		**Mean**		**Median**	
	NI	**FI**	**NI**	**FI**	**NI**	**FI**
Truth-telling	6.77		0		13.54	
	3.61	12.04	0	0	7.22	24.07
Nash	8.61		2.78		13.54	
	4.72	13.89	2.22	3.70	7.78	23.15
Peak bounded	86.81		84.72		88.89	
	85.00	89.81	86.11	82.41	83.89	97.22
Pareto efficient	73.26		69.79		76.74	
	71.11	76.85	72.78	64.81	69.44	88.89
Welfare optimal	6.60		0		13.19	
	3.61	11.57	0	0	7.22	23.15
No. of outcomes	576		288		288	
	360	216	180	108	180	108

Table 5: Outcome results (percentages)

4.2 Individual Decisions

We turn to the analysis of individual decisions; their total number was 2.880, split evenly into 1.440 decisions under the mean and 1.440 under the normalized median rule.

4.2.1 Mean Rule

In the light of Hypotheses H1.1-3, we are interested in particular in the shares of truth-telling and Nash play in the context of the mean rule. We also investigate if individuals played myopic best responses.

Truth-telling 'Truth-telling' prevails if the vote corresponds exactly to the most preferred allocation, the peak. However, in the context of the mean rule we are mainly interested in those cases of sincere voting in which truth-telling is *not* the focal Nash strategy. Thus, for the mean rule we exclude the decisions of participant 3 in peak distribution IV. We already observed above that the truth-telling outcome never occurred under the mean rule, therefore we may also expect low shares of individual truth-telling; moreover, according to Hypothesis H1.1 we hypothesize that the share of truth-telling is even lower when information about the others' peaks is disclosed.

In total only 5.70% of all votes were sincere under the mean rule and this figure stays low for all four peak distributions. Moreover, truth-telling decreases over time, especially under the no info treatment. While 21.05% of the votes are sincere in the first period, this number declines to 2.34% in the fifth period. Thus, subjects indeed learn over time that sincere voting is not optimal under the mean rule.[14]

We ran a Wilcoxon signed-rank test in order to test for differences in the distribution of truth-telling among both information treatments. We are able to reject the null hypothesis and find support for the alternative Hypothesis H1.1: under the mean rule the share of truth-telling is significantly higher in the no info treatment as compared to the full info treatment ($Z = -2.280$, $p = 0.0226$).

Focal Nash Play We observed a high share of 35.76% of total votes that are in accordance with the focal Nash equilibrium under the mean rule. If we allow for small deviations and count all votes that are within a 10% distance from the precise Nash strategy as *Nash tendency*, the figure even rises to 48.54% of Nash play or Nash tendency.[15]

Hypothesis H1.2 states that we expected even more focal Nash play with full information than with no information. Given our data, we can indeed reject the hypothesis that the distribution of votes classified as focal Nash strategies are similar for the two information treatments. Instead, we find that the median of focal Nash strategies is higher with full information than with no information, supporting Hypothesis H1.2 (Wilcoxon signed-rank test, $Z = 2.866$, $p = 0.0042$).

Moreover, we observed learning effects over periods, as expected in Hypothesis H1.3. We find a higher share of focal Nash play in the last period (i.e. period 5) as compared to the first period under no information (Wilcoxon signed-rank test, $Z = -2.521$, $p = 0.0117$). The share under full information in the last period 3 is also higher as compared

[14] In order to get a better insight on the factors that influence truth-telling, we ran a regression of the absolute deviation of the vote from the true peak on a variety of independent variables. The results of the regressions can be found in Table 6 in the appendix.

[15] This is consistent with our above finding that only very few social outcomes correspond to the focal Nash outcome (2.78%) because a single individual deviation from the focal Nash strategy in a group is enough to prevent the Nash outcome to occur as the social outcome. This observation suggests at the same time that the relatively high frequency of focal Nash play prevails across all groups, and is not due to group specific factors such as e.g. the particular peak distribution.

to the first period (Wilcoxon signed-rank test, $Z = -1.651$, $p = 0.0987$).[16] Figures 3 and 4 summarize the truth-telling, Nash play and Nash tendency shares of all votes over the periods for each peak distribution under no and full information.

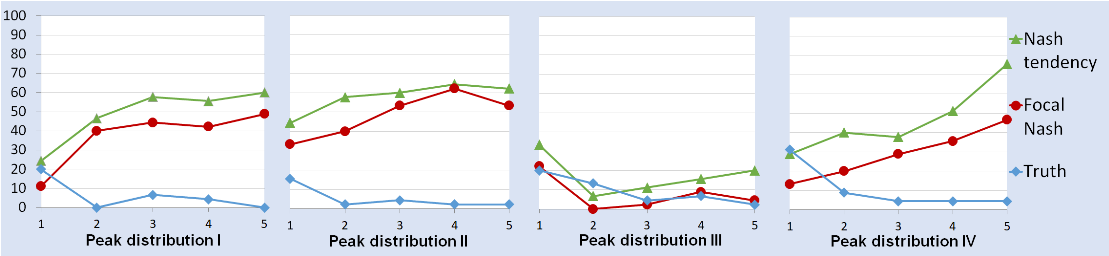

Figure 3: Results of the mean rule; no info (percentages by period)

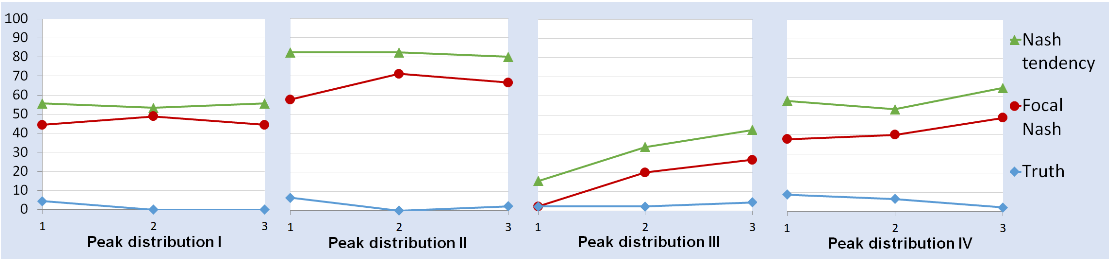

Figure 4: Results of the mean rule; full info (percentages by period)

The data in Figures 3 and 4 also clearly reflect the suspicion that the identification of the focal Nash strategy is more difficult under peak distribution III because here participants 4 and 5 must not vote for the extreme allocation that is closest to their peak.

Best Response to Previous Period A reasonable alternative hypothesis to the choice of the focal Nash strategy is the assumption that subjects act optimally vis-à-vis the observed behavior of the other participants in the previous period (*'best response to previous period,'* or, in short, *BRPP*). Under the mean rule, this is a particularly attractive behavioral rule because it only requires knowledge of the social outcome and the own vote in the previous round (and that information is available for all participants both under the no info and the full info treatment after the first period). Indeed we found a share of 46.94% of *BRPP* votes (in rounds 2-5). Of course, *BRPP* behavior is not possible in round 1. In the no info treatment, we considered the possibility that subjects behave as Bayesian players with uniform beliefs about the peak distribution of the others. In almost all cases, the best response to a uniform belief however coincides with

[16]In the appendix, we provide the regression results for the distance between the actual vote and the focal Nash strategy. We find that the period has a significant effect on the proximity between the Nash strategy and the actual vote, which indicates a learning effect over time towards Nash play.

the focal Nash strategy, which may explain why the focal Nash strategy was sometimes chosen in the no info treatment even in the first round. In those few cases in which it does not coincide with the focal Nash strategy, we do not find much empirical evidence of optimal behavior with respect to uniform beliefs.[17]

Over all peaks and degrees of information, the share of $BRPP$ is higher in the last period ($t = 5$ or $t = 3$) as compared to the second ($t = 2$). This is well in line with our analysis of Nash play and lends further support to our general hypothesis that subjects adapt their behavior and learn over time. As before, the share of $BRPP$ under peak distribution III is lower than under the other three distributions. Remarkably, it is nevertheless considerably higher than the share of focal Nash play (27.78% $BRPP$ play vs. 10.83% focal Nash play). From this, we conclude that even in those cases in which the theoretical Nash equilibrium strategy is hard to identify a considerable share of voters behaves strategically.

4.2.2 Results under the Normalized Median Rule

Under the normalized median rule, the following behavioral rules seem particularly worthwhile to look at: truth-telling, focal Nash play and best response to truth (BRT). As already noted, truth-telling is the focal Nash strategy for all participants under peak distributions III and IV. This is not true under peak distributions I and II since in both of these participant 1 can manipulate the outcome to her benefit by misreporting; for this participant, the focal Nash strategies and the set of best responses to truth-telling of the other participants coincide (note that these strategies are not unique since they prescribe a fixed level of expenditure only in one project, respectively, leaving some flexibility of how to allocate the rest to the other two projects, see Tables 1 and 2 above).

Under peak distributions I and II, we hypothesized that few of the subjects in the role of participant 1 would identify the manipulation possibility and play the focal Nash strategy. And indeed none of them did. (Under the no info treatment, it is in fact not clear how subjects in the role of participant 1 could possibly detect this manipulation opportunity.)

Truth-telling Even though truth-telling is part of the focal Nash strategy for 18 out of 20 participants only 18.96% of all votes were sincere (i.e. equal to the true peak). Moreover, we find a tendency of less truth-telling in later periods uniformly over all peak distributions and degrees of information; specifically, the share of sincere votes is lower in the last period than in the first period. Figure 5 displays the results under no info and Figure 6 under full information (recall that for peak distributions III and IV the focal Nash strategy is truth-telling, i.e. the blue and red curves coincide; only the red curve is depicted).

Remarkably, if we look at the 'pivotal' participants 1 under the peak distributions I and II, and participant 3 under distributions III and IV, we observe a higher share of sincere votes. These participants vote more often sincerely as the other non-pivotal participants

[17]The share is 8.89% in these cases, and even this figure derives mainly from one peak for which playing a best response to a uniform belief represents a *tendency* to Nash play.

(29.86% vs. 16.23%) and the difference is even greater in the full info treatment (41.67% vs. 18.98%). While this means that the pivotal participants in peak distributions I and II did not identify the manipulation possibility, it is also an indication that the pivotal participants understood well that their vote 'matters.' In the appendix, we provide regression results of the distance of the vote to the true peak which gives further insights to this issue and shows – among other things – that pivotal participants deviate less from their true peak than non-pivotal participants. Moreover, the pivotal voters for whom truth-telling is the focal Nash strategy deviate significantly less from their true peak as compared to those pivotal voters who possess a beneficial manipulation possibility.

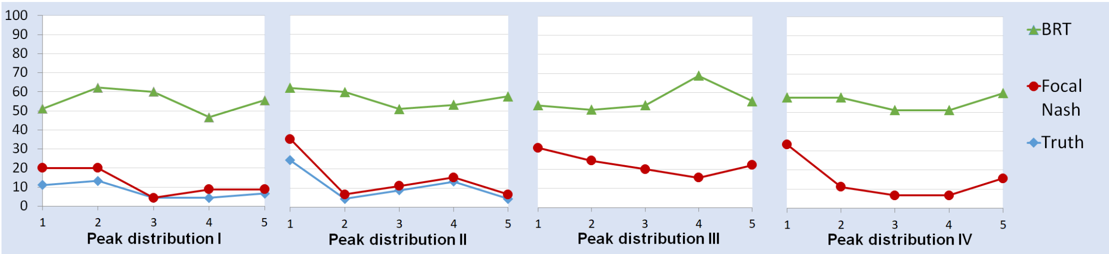

Figure 5: Results of the median rule; no info (percentages by period)

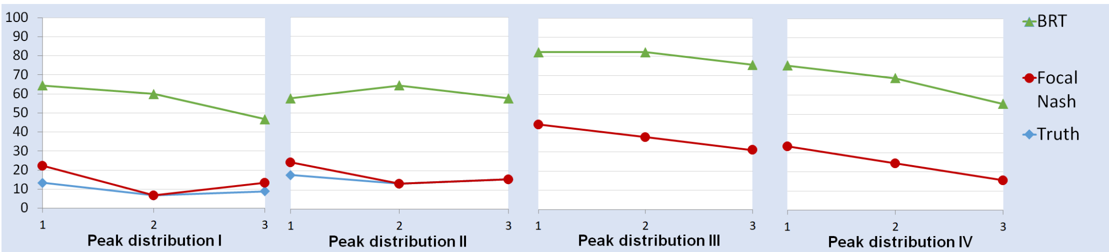

Figure 6: Results of the median rule; full info (percentages by period)

To test Hypothesis H2.1 we perform a Wilcoxon signed-rank test to compare sincere voting under the two information treatments. Contrary to H2.1 we find a higher share of truth-telling under full information over all periods and rounds (23.52% vs. 16.22%, $Z = 1.766$, $p = 0.0773$). We note again that, if at all different, we should expect to observe more truth-telling under no information because the optimal strategy of participant 1 under peak distributions I and II prescribes non-truthful voting but the precise vote can only be computed under full information. We can only speculate about the reasons why we nevertheless observe more truthful votes in the full information treatment. A possible clue may come from the answers of our questionnaire in which some subjects stated that they 'tried to irritate the others' through misrepresentation in the no info treatment.

We also compared the distributions of sincere voting under the peak distributions I and II versus sincere voting under peak distributions III and IV, and find higher shares of truth-telling under the latter two (Wilcoxon signed-rank test, $Z = -3.007$, $p = 0.0026$).

This is what one would expect since in peak distributions III and IV truth-telling is the focal Nash strategy for all participants.

Focal Nash Play The proportion of focal Nash votes amounts to a total of 17.01%. It is at a level of 24% in the first period under no information, but remains stable only between 10.00% and 13.33% over periods 2 to 5. Under full information, focal Nash play decreases unexpectedly from 27.22% in the first period to 17.78% in the third period. Comparing the shares of focal Nash play under the different information treatments reveals 21.85% under full information and 14.11% under no information. However, the difference is not statistically significant according to a Wilcoxon signed-rank test ($Z = 1.334$, $p = 0.1822$).

Best Response to Truth-Telling Why did subjects not vote sincerely under the median rule if they displayed quite rational behavior in general? The answer lies in the observation that non-sincere voting of non-pivotal participants is not by itself irrational. And indeed a relatively high share of votes (59.72%) are best responses to the assumption of sincere voting of the other participants (recall that for most participants truth-telling is among the best responses). The precise figure fluctuates slightly but remains high over all periods and peak distributions.

In Hypothesis H2.2, we conjectured a higher share of *BRT* under full information. And indeed, while the shares of *BRT* are high under both treatments (56.00% under no info and 65.93% under full info), we can confirm H2.2 (Wilcoxon signed-rank test, $Z = 2.275$, $p = 0.0229$). Remarkably, a closer analysis reveals that the difference is significant because of the higher share of truthful votes under full information. Indeed, if we disregard the sincere votes and compare only the *non-truthful BRT* we get 41.89% under no info and 44.07% under full info. Although the share of non-truthful *BRT* is thus still higher under full info than under no info, the difference of the distribution is no longer significant (Wilcoxon signed-rank test, $Z = 0.667$, $p = 0.5047$).

In view of the relatively low shares of truthful voting reported above, the overall conclusion of the analysis of our data under the normalized median rule is that a significant proportion of subjects play a best response to the assumption of truth-telling by the other participants without voting sincerely themselves. While this does not directly contradict individual rationality, it evidently involves inconsistent beliefs about the behavior of the group as a whole.

4.2.3 Mean versus Normalized Median Rule

We turn to the differences between the two voting rules regarding individual decisions. First, we can reject the hypothesis of equal shares of truth-telling under the two rules and find a significantly higher share of sincere votes under the normalized median rule (18.96% vs. 6.18%, Wilcoxon rank-sum test, $Z = -3.963$, $p < 0.001$), which confirms Hypothesis H3.1 and is in line with the results on truthful outcomes reported in Table 5.

Going further into detail, we consider not only truth-telling but also the 'degree of lying.' According to Hypothesis H3.2 we expect a higher deviation of votes from the true peaks under the mean rule, which is indeed what we find in our data (Wilcoxon rank-sum test, $Z = 4.157$, $p < 0.001$).

5 Conclusion

We briefly summarize our findings. First, we see a high level of rational behavior in our subjects (mainly students from the Karlsruhe Institute of Technology). We conclude this from the fact that (i) under the mean rule the focal Nash strategy is a good predictor of individual behavior, and (ii) under the normalized median rule the same holds for best response to truth-telling of the other participants. A significant proportion of subjects understood the general logic behind the possible strategic manipulations of the mean rule well and identified the optimal vote – at least in those cases in which this meant to vote for the extreme allocation closest to one's own peak. Our findings are thus well in line with the analysis of the one-dimensional case provided in Marchese and Montefiori (2011).

The behavior under the normalized median rule is multifaceted. On the one hand, a remarkably large fraction of subjects played a best response to the assumption of sincere voting by the other participants. Under full information, the pivotal participants under peak distributions III and IV understood well that they are pivotal and a significant fraction of them indeed voted sincerely.[18] However, a significant proportion of non-pivotal participants who choose a best response did not vote sincerely even when this was *among* their best responses. In this respect, our findings confirm the results of Block (2014) in the one-dimensional case.

Finally, we find significantly higher expected utilitarian social welfare under the normalized median rule as compared to the mean rule, although in all of our examples the two rules yield *exactly the same* focal Nash equilibrium outcomes. We view this as an argument for the use of the normalized median rather than the mean rule in multi-dimensional budget allocation problems. Naturally, however, conclusions concerning the empirical welfare properties of the two rules have to be drawn with some care since these depend not only on voters' preferences but also on the specific shape of the underlying peak distributions. What is comforting, though, is that the non-(Pareto)-efficient Nash equilibria that theoretically exist under the (normalized) median rule seem to play no role empirically.

[18] Not surprisingly, the pivotal participants under peak distributions I and II were not able to identify their cognitively complex best response.

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Appendix

Focal Nash equilibria under the mean rule

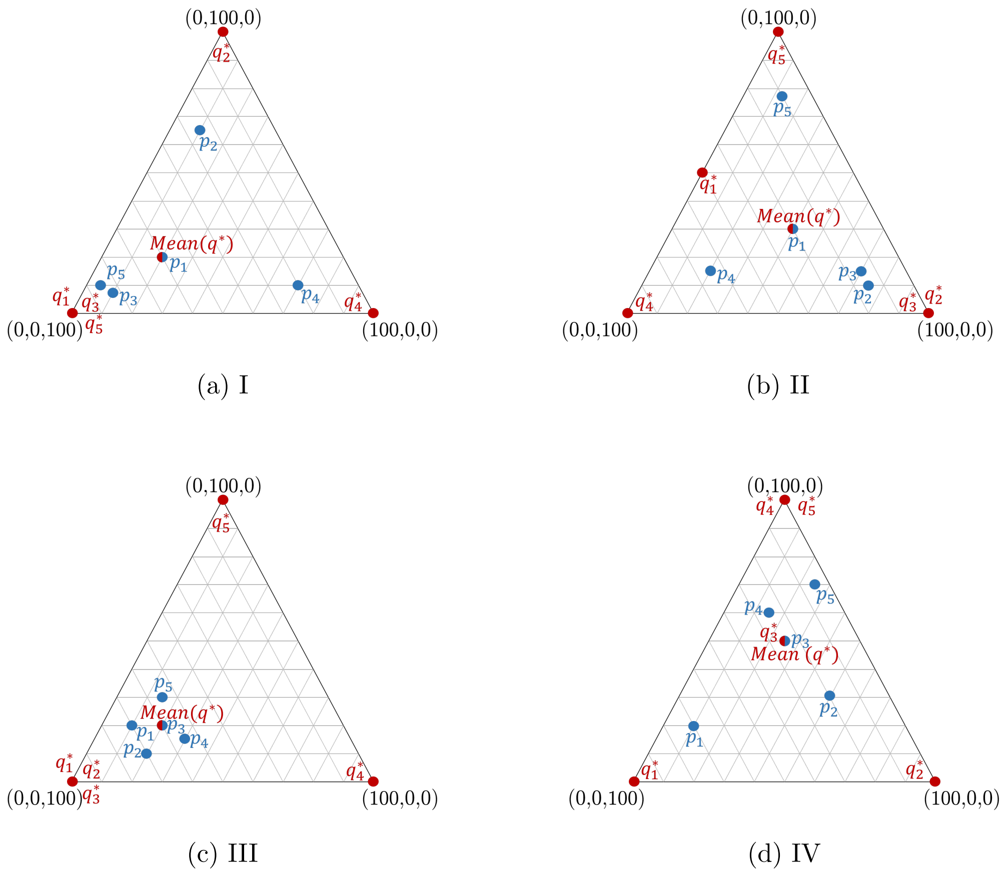
For all four peak distributions, there exists exactly one Nash equilibrium under the mean rule (the 'focal' Nash equilibrium) in which at least four agents vote for an extreme allocation, i.e. an allocation in which the total budget is used only for one project; for distributions I and III, in fact all five agents vote for an extreme allocation. That the 'focal' strategy combinations indeed constitute a Nash equilibrium is easily verified for each of the four peak distributions; moreover, it is also easily seen that the corresponding Nash equilibrium is strict (i.e. best responses are unique in equilibrium).

To verify the stated uniqueness property, one first shows that in any Nash equilibrium such that at least four agents vote for an extreme allocation all three different extreme allocations must be voted for by some agent for each of our four peak distributions. With five participants this implies that at least one extreme allocation is the vote of *exactly* one agent in equilibrium. Which one it is of course depends on the distribution. As an example, consider peak distribution I with peaks $p_1 = (20, 20, 60)$, $p_2 = (10, 65, 25)$, $p_3 = (10, 8, 82)$, $p_4 = (70, 10, 20)$ and $p_5 = (5, 10, 85)$. It is easily verified that the extreme allocation for which exactly one agent votes in equilibrium cannot be the allocation $(0, 0, 100)$ (no matter who votes for this allocation). Hence, *at least two* agents vote for $(0, 0, 100)$ in equilibrium. If *exactly two* agents vote for $(0, 0, 100)$, the outcome under the mean rule is either $(20, 40, 40)$ if only one agent votes for $(100, 0, 0)$, or $(40, 20, 40)$ if only one agent votes for $(0, 100, 0)$. In the first case, three agents prefer higher expenditure on the third project and lower expenditure on the second project. In the second case, three agents prefer higher expenditure on the third project and lower expenditure on the first project. Thus in both cases, at least one agent would deviate from her vote as only two agents vote for $(0, 0, 100)$. The only remaining Nash equilibrium candidate is thus the case in which *three* agents vote for $(0, 0, 100)$, and one agent for $(0, 100, 0)$ and $(100, 0, 0)$, respectively. The resulting outcome under the mean rule is $(20, 20, 60)$, and one easily verifies that all agents indeed choose mutual (unique) best responses. The arguments for the other peak distributions is similar.

Fig. 7 illustrates the peak distributions and Nash equilibria under the mean rule.

Focal Nash equilibria under the normalized median rule

Proposition 2 implies that truth-telling of all agents is a Nash equilibrium for distributions III and IV under the normalized median rule. On the other hand, as is easily verified, sincere voting of all agents is *not* a Nash equilibrium for distributions I and II. Indeed, in both cases participant 1 can strictly improve the truthful outcome by appropriate misrepresentation and get her most preferred allocation if all other participants vote sincerely. That voting sincerely for participants 2-5 is indeed individually optimal given the optimal misrepresentation of participant 1 can be proved by arguments similar



(a) I

(b) II

(c) III

(d) IV

Figure 7: Nash equilibria, mean rule

to that used in the proof of Proposition 2.

Regression Results Mean Rule

In order to get a better insight on the factors that influence truth-telling, we run a regression of the '*Peak-Vote-Distance*,' i.e. the absolute deviation of the vote from the true peak, on various independent variables. The results of the regression with fixed effects for the subjects can be found in Table 6. We include a total of 1,440 observations, where each of the 60 subjects makes 24 decisions over time. We consider the voting behavior over periods and expect that the distance to the peak increases over time. Three further distance measures are given: 'Peak-Nash-Distance', 'Nash-Vote-Distance' and the distance between the peak and the result of the previous period ('P-PR-Distance'), which we expect to have a positive effect on the deviation from truth-telling. We construct a dummy variable depending on the position of the Nash strategy: at the edge of the simplex, truth-telling or neither one. We include also a dummy variable for the peak distribution and information level.

We find a positive and significant correlation with the variable ‘period’, indicating a higher degree of deviation from truth-telling over time. As anticipated, truth-telling decreases slightly with an increasing distance between the true peak and the theoretical Nash strategy, since a greater ‘Peak-Nash-Distance’ indicates that participants have to deviate more from their true peak to play their Nash strategy.

Contrary to our expectations, the deviation from the peak is positively affected by the distance between the theoretical Nash strategy and the actual vote, i.e. ‘Nash-Vote-Distance’. This implies that the higher the deviation of the actual vote from the predicted Nash strategy, the more extreme participants tend to ‘lie.’ Given Nash play of the other four voters, this strategy results in a lower payoff as the outcome moves further away from the peak. After the experiment, we asked the participants about their approach to the voting. Some argued that they tried to deceive the others in one period by votes that lead to a lower payoff in order to receive a higher payoff in the next period. This behavior might explain the results that may seem non-strategic at first sight. Participants also tend to significantly less truth-telling with an increasing distance between own peak and the result of the previous round (‘P-PR-Distance’); this hints at a learning effect over periods of increasing strategic voting. Although in absolute numbers the difference of truth-telling across the peak distributions is low, we find a significant and high difference in the extent of truth-telling depending on the theoretical Nash strategy.

We find that deviation from the true peak (the extent of lying) is significantly lower if the theoretical Nash strategy is to vote zero for only one project or truth-telling (as captured by the dummy variable ‘edgetruth_d’), compared to voting zero for two projects. We conclude that the degree of lying is lower if the theoretical Nash strategy is not to choose a vertex. Since the Nash strategy is not straightforward in these cases, it seems reasonable to vote for an allocation close to the peak. The dummy variable ‘peak_d’ takes the value 1 if the votes belong to peak distribution III and 0 else. For two peaks in this distribution, the Nash strategy is to vote zero for the project with the highest peak value and therefore not easy to detect. We find a negative and significant coefficient for the peak dummy, indicating that for this distribution the deviation from the peak is lower. Subjects may face more difficulties in finding the Nash strategy and may therefore find it natural to vote for an allocation closer to the peak as compared to the other peak distributions. We also find a significant effect of the degree of information on the distance from the peak. The positive coefficient of the dummy variable ‘info_d’ implies that full information increases the extent to which voters deviate from their peak.

	Dependent variable:
	Peak-Vote-Distance
period	3.935***
	(0.722)
Peak-Nash-Distance	0.168**
	(0.069)
Nash-Vote-Distance	0.253***
	(0.050)
P-PR-Distance	0.030**
	(0.013)
edgetruth_d	−14.012***
	(4.012)
peak_d	−23.498***
	(2.516)
info_d	16.230***
	(1.634)
Observations	1,440
R^2	0.261
Adjusted R^2	0.226
F Statistic	69.439*** (df = 7; 1373)
Note:	$^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$
	Standard errors are clustered by group

Table 6: Regression results mean rule: peak-vote-distance

We further run a fixed effects regression of the '*Nash-Vote-Distance*' to get a better insight on the deviation from votes to the Nash strategy. Table 7 shows the regression results, again including a total of 1,440 observations for 60 subjects over 24 rounds. The explanatory variables are the same as in the previous analysis for truth-telling.

The coefficient of 'period' has the anticipated negative sign and is significant. The negative sign supports the assumption of a learning effect in playing the Nash strategy over time: the more advanced the voting game, the closer the votes are to the Nash strategy. We can thus confirm Hypothesis H1.3: Nash play increases over time under the mean rule. Another indicator for the adapting of voting behavior is the negative and significant coefficient of the distance between the peak and the result of the previous period ('P-PR-Distance'), which reflects the higher gain in utility by Nash play if the peak is distant from the social outcome of the last round.

	Dependent variable:
	Nash-Vote-Distance
period	-3.350^{***}
	(1.069)
Peak-Nash-Distance	0.359^{***}
	(0.083)
Peak-Vote-Distance	0.583^{***}
	(0.107)
P-PR-Distance	-0.063^{***}
	(0.020)
edgetruth_d	20.431^{***}
	(6.691)
peak_d	21.872^{***}
	(6.345)
info_d	-24.673^{***}
	(2.410)
Observations	1,440
R^2	0.294
Adjusted R^2	0.260
F Statistic	81.736^{***} (df = 7; 1373)
Note:	$^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$
	Standard errors are clustered by group

Table 7: Regression results mean rule: Nash-vote-distance

The positive and significant correlation of the deviation of the vote to the theoretical Nash strategy and the 'Peak-Nash-Distance' highlights the growing difficulties of finding the corresponding Nash equilibrium the more remote the Nash strategy is from the peak. The distance between Nash play and vote increases with a higher 'Peak-Vote-Distance', indicating that manipulation occurs for subjects with a Nash strategy that is more 'difficult' to predict but not towards the Nash equilibrium. We also find a higher deviation from the Nash strategy if the focal strategy is to vote zero only for one project (edge) or to vote for the actual peak (truth). The positive coefficient indicates that strategies in the vertex are easier to identify, and we could see in the last section that subjects with non-vertex Nash strategies tend to truth-telling. Subjects also vote more often for allocations that deviate more from the Nash strategy at peak distribution III as compared to the other peak distributions, see the coefficient of the dummy-variable 'peak_d' in Table 7.

We also find an effect of the degree of information on the distance to the Nash strategy since the coefficient of the dummy variable 'info_d' is negative and significant. This result implies that with full information, the votes go further into the direction of the Nash strategy as compared to the no information treatment and thus further supports Hypothesis H1.2.

Regression Results Normalized Median Rule

To get further insight on the variables that impact the deviation from truth-telling, we regress on the distance between the peak and the vote for the normalized median rule. The results of the fixed effects regression can be found in Table 8, again with 1,440 observations derived from 60 subjects over 24 rounds.

	Dependent variable:
	Peak-Vote-Distance
period	1.933**
	(0.828)
info_d	-0.034
	(1.606)
PivotalVoter	-8.096^{***}
	(1.646)
PivotalTruth	-13.856^{***}
	(2.824)
Observations	1,440
R^2	0.056
Adjusted R^2	0.012
F Statistic	20.256*** (df = 4; 1376)
Note:	$^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$
	Standard errors are clustered by group

Table 8: Regression results normalized median rule: peak-vote-distance

We find that subjects deviate more from their true peak the higher the period. While we find that the shares of true votes are higher under full information, we are not able to find a significant effect of the information on the extent of 'lying.' We include two dummy variables to control for pivotal voters (participant 1 under peak distributions I and II and participant 3 under distributions III and IV) and pivotal voters whose focal Nash strategy is truth-telling (participant 3 under distributions III and IV). The results are as one would expect: pivotal voters deviate from truth-telling significantly less compared to non-pivotal voters, see the negative coefficient of the dummy variable 'PivotalVoter'. Moreover, the pivotal voters for whom truth-telling is a Nash strategy deviate significantly less from their true peak as compared to those pivotal voters who possess a manipulation possibility, as indicated by the negative coefficient of 'Pivotal-Truth'.

Experimental Instructions

In the following we reproduce the experimental instructions; the original instructions were in German.

1 Preliminary Remarks

Welcome to the experiment and thank you for your participation. At the beginning we would like to ask you to switch off your mobile phones and stop all communication. If you have any questions, please direct them as quietly as possible to the experiment leader and do not speak to the other participants. In this experiment you earn cash depending on your decisions and the decisions of the other participants. During the experiment your account balance will be displayed in the unit *ECU*. 100 *ECU* equals €1.00. At the end of the experiment you will be paid your last account balance. For your punctual appearance at the experiment you will receive an additional €5.00.

2 Mathematical Basics

First we would like to familiarize you with some mathematical basics that will be important for the experiment.

Treatment Mean Rule:

2.1 Average (arithmetic mean)

The average is a mean value defined as the quotient of the sum of all values and the number of values. The formula for calculation is

$$x = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{y_1 + \cdots + y_n}{n}$$

Example: Given are the five numbers $3, 19, 58, 25, 80$. The average is $x = 37$, because $\frac{1}{5} \cdot (3 + 19 + 58 + 25 + 80) = \frac{185}{5} = 37$.

Treatment Median Rule:

2.1 Median

The median of an odd number of values is the number that is in the middle position after sorting the values in ascending order.

Example: Given are the five numbers $3, 19, 58, 25, 80$. The median is $x =$25, namely the middle number $3, 19, \mathbf{25}, 58, 80$.

If several median values are determined, which in total should reach a certain value, an adjustment of the medians may be necessary. This can be achieved by normalization.

Normalization When adjusting the determined medians by normalization, the ratio of the median values to each other is maintained and the individual values are increased or decreased together until their sum reaches the targeted value.

Example: The amount of 100 is to be divided into three components A, B and C. The underlying values are as follows:

1. $A_1 = 80$, $B_1 = \ 0$, $C_1 = 20$
2. $A_2 = 20$, $B_2 = 70$, $C_2 = 10$
3. $A_3 = 10$, $B_3 = 50$, $C_3 = 40$

This results in the medians $M_A = 20$, $M_B = 50$ and $M_C = 20$ after separate consideration of A, B and C. Since $20 + 50 + 20 < 100$, an adjustment is necessary. The ratio between M_A, M_B and M_C is $2:5:2$, accordingly after normalization $M_A^* = 22\frac{2}{9}$, $M_B^* = 55\frac{5}{9}$ and $M_C^* = 22\frac{2}{9}$, so that $M_A^* + M_B^* + M_C^* = 22\frac{2}{9} + 55\frac{5}{9} + 22\frac{2}{9} = 100$.

2.2 Absolute Value Function

$|x|$ is the absolute value of x. This is also known as abs(x).

Example: $|-15 + 12| = |-3| = 3$.

3 Structure of the Experiment

You will take part in an election to determine the respective level of funding for three projects. Four more people take part in the election. Each participant is assigned an individual most preferred allocation, which is labeled p_1, p_2, and p_3. p_1 describes the desired value for project 1, p_2, and p_3 for projects 2 and 3. A total of 100 monetary units is available and must always be used entirely. Thus your assigned values are always between 0 and 100 and add up to 100. Your most preferred allocation remains the same for several rounds, as do those of the other participants. As soon as your assigned allocation changes, the other participants' allocations change as well. Your payout depends on the difference between your most preferred allocation and the outcome of all five votes. You will find the exact payout function in section 3.3.

3.1 Submissions of Votes

Each of the five participants submits a vote which is included in the election. That means all five participants name three natural numbers between 0 and 100, which add up to 100 again. If the sum of your three values does not equal 100, or if your individual values are not natural numbers, you will receive an error message and will have to adjust your vote.

3.2 Election

On the basis of all five votes, the amount of funding for the three projects will be determined. To do this, the votes, i.e. five values for each of the three projects, are added together and divided by five (which is the average of all proposals). This results in the outcome x_1, x_2 and x_3.
(***Median Treatment:*** To do this, the votes, ..., are sorted by size and the third largest proposal is chosen (this corresponds to the median procedure). ...If the values x_1, x_2, and x_3 obtained by the median procedure do not add up to 100, the result is normalized as explained in Section 2.1 above.)

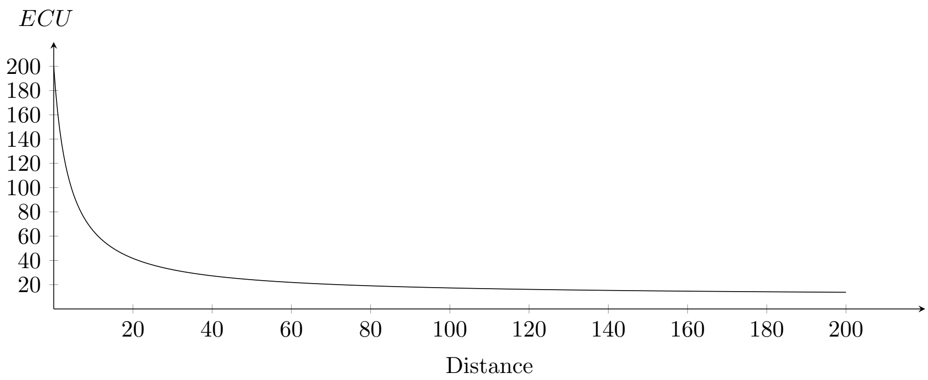
Sometimes, before the election, you may also find out the assigned most preferred allocations of the other participants.

3.3 Payment

The smaller the difference between the x_1, x_2, x_3 values obtained from the elections and the p_1, p_2, p_3 values assigned to you, the higher your payoff. Your individual payout f_i in the unit *ECU* is calculated as follows

$$f_i(p^i, x) = 10 + \frac{760}{4 + \sum_{j=1}^{3} |p_j^i - x_j|}$$

The following figure shows the payout function graphically. The distance is the sum of the absolute values of the differences of x_1, x_2, x_3 and p_1, p_2, p_3.



Example: If the election results in $x_1 = 15$, $x_2 = 50$ and $x_3 = 35$ and if your assigned most preferred allocation is $p_1 = 30$, $p_2 = 50$, $p_3 = 20$, then your distance from the outcome is $|15 - 30| + |50 - 50| + |35 - 20| = 30$. This would mean a payout of $f = 10 + \frac{760}{4+30} = 32.35$ *ECU*, so €0.3235.

4 Procedure of the experiment

The election will take place in several rounds. The procedure for each round is as follows:

1. You will get your values p_1, p_2 and p_3 and sometimes additionally the values of the other participants. These values remain the same for several periods.
2. You submit a vote.
3. You will be told the values x_1, x_2 and x_3 calculated from the votes of all participants and your payout.

5 Concluding remarks

Before the experiment starts, you will be asked some comprehension questions on the screen. At your place you will find paper and pen. We ask you to leave them in place when you leave the room. You can also use the calculator during the entire study.
The payout will take place at your seat. Please remain seated after the end of the experiment and wait until we open the door of your cabin. If you have any questions during the experiment, please open your door and wait until we come to you. Please close the cabin door now and start with the comprehension questions. Thank you very much!