Axiomatization of plurality refinements

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Abstract

Plurality rule uniquely satisfies anonymity, monotonicity, neutrality, and tops-onlyness. However, it is not always able to produce resolute outcomes. We study singleton-valued refinements of plurality rule that satisfy all but one of these four axioms. Monotonicity is preserved by all refinements of plurality, whereas no refinement satisfies the remaining three except for a very limited case. We explore what dropping one of the three remaining axioms brings about towards singleton-valued refinements.

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1 Introduction

We consider a collective choice problem where the number of individuals and the number of alternatives are both fixed. Each individual has a strict ranking of alternatives interpreted as his preference. The plurality rule (or simply, plurality) picks the alternatives which are ranked at the top by the highest number of individuals. Although there are several explorations of plurality in variable size societies[1], an analysis in a fixed size society is rather recent: Kelly and Qi (2016) characterize plurality on a fixed population in terms of four conditions, namely anonymity, monotonicity, neutrality, and tops-onlyness.

Anonymity and neutrality refer to the equal treatment of individuals and alternatives, respectively. Monotonicity implies positive responsiveness to an alternative's increase of support in the society. Tops-onlyness economizes information by requiring the social choice to only depend on the alternatives that are top ranked by the individuals and not on the rest of their rankings. It is worth noting that the conjunction of monotonicity, neutrality and tops-onlyness implies Pareto efficiency.

Plurality is irresolute in the sense of producing tied outcomes at certain preference profiles. When a resolute choice has to be made, it is of interest to consider refinements of plurality that announce a single winner at every preference profile. Refinements of plurality will fail to simultaneously satisfy the four conditions that characterize plurality, due to the said characterization but also due to the classical results of Moulin (1980, 1991) that show a logical incompatibility between anonymity and neutrality for resolute social choice rules.

We explore the extent to which we can preserve these four conditions while refining plurality. We first consider monotonicity and show that it is preserved by every refinement of plurality. Thus, the impossibility of simultaneously satisfying the axioms that characterize plurality comes from conjoining the remaining three. We show that plurality admits no anony-

[1] Hartfield (1978) delivers the earliest such characterization, followed by Roberts (1991), Ching (1996), Yeh (2008), and Ju (2005).

mous, neutral and tops-only refinement except for the very restrictive case of two alternatives and an odd number of individuals. This finding reflects the well-known tension between anonymity and neutrality shown by Moulin (1980). As a result, we first consider refinements of plurality that satisfy only one of anonymity and neutrality.

For anonymity and tops-onlyness, we consider choice functions that pick a single alternative from any given set of alternatives. Every choice function is capable to induce a refinement of plurality by picking a single alternative from the set of tied plurality winners. Moreover, this refinement would be anonymous and tops-only. Nevertheless, plurality admits anonymous and tops-only refinements that cannot be induced by a choice function. The generality of choice functions being insufficient to induce all anonymous and tops-only refinements of plurality, we identify the extent to which the concept must be generalized.

For neutrality and tops-onlyness, we consider priority functions that pick a single individual from any given set of individuals. At each preference profile, winning individuals are those whose top-ranked alternatives are among the plurality winners. Every priority function is capable to induce a refinement of plurality by announcing a single individual from the set of winning individuals and picking the top-ranked alternative of this individual. Moreover, this refinement is neutral and tops-only. Nevertheless, plurality admits neutral and tops-only refinements that cannot be induced by a priority function. The generality of priority functions being insufficient to induce all neutral and tops-only refinements of plurality, we identify the extent to which the concept must be generalized.

Finally, we explore the possibility of simultaneously satisfying anonymity and neutrality, dispensing with tops-onlyness. As shown by Campbell and Kelly (2015), even without the plurality constraint, these two axioms are severely restrictive.[2] However, one might wonder about the exact prospects in the context of plurality refinements. To that end, we consider the iter-

[2]Campbell and Kelly (2015) find, *inter alia*, that when the number of alternatives exceeds the smallest prime dividing the number of individuals, a resolute social choice rule is anonymous and neutral only if it chooses alternatives that are in the bottom half of preferences of all individuals.

ative plurality rule, which applies plurality to the profile restricted to plurality winners iteratively until it stabilizes. Iterative plurality is anonymous and neutral but not tops-only. Thus, in case it refines plurality, iterative plurality exemplifies an anonymous and neutral plurality refinement. More interestingly, we show that when iterative plurality does not refine plurality, plurality does not admit any anonymous and neutral refinement. As such, iterative plurality can be identified as the lower bound in refining plurality, as well as a simple method to check whether plurality admits an anonymous and neutral refinement.

Section 2 introduces basic notions and notations. Section 3 delivers results and Section 4 makes some concluding remarks.

2 Basic notions and notation

Writing $\mathbb{N}$ for the set of natural numbers and picking $m, n \in \mathbb{N} \setminus \{1\}$, we conceive a *social choice problem* as a set A of alternatives with $\#A = m$ and a set N of individuals with $\#N = n$. We refer to (m, n) as the *size* of the social choice problem (A, N).

Writing $\mathcal{L}(X)$ for the set of linear orders, *i.e.,* complete, asymmetric, and transitive binary relations on a given finite set X, let $P_i \in \mathcal{L}(A)$ denote the *preference* of $i \in N$.[3] An $n-$tuple of such individual preferences indicates a *(preference) profile* $P_N \in \mathcal{L}(A)^N$. A *social choice rule* (SCR) is a mapping $f : \mathcal{L}(A)^N \to \mathcal{A}$, where $\mathcal{A} = 2^A \setminus \{\varnothing\}$ is the set of non-empty subsets of A. A *social choice function* (SCF) is an SCR f with $|f(P_N)| = 1$ for all $P_N \in \mathcal{L}(A)^N$. For an SCF f, we write $f(P_N) = x$ in place of $f(P_N) = \{x\}$. Given an SCR f, we say that an SCF g is a refinement of f iff $g(P_N) \in f(P_N)$ $\forall P_N \in \mathcal{L}(A)^N$.

We let $P_N|_B$ denote the restriction of $P_N \in \mathcal{L}(A)^N$ to those alternatives in $B \in \mathcal{A}$ so that $P_N|_B \in \mathcal{L}(B)^N$ and $xP_iy \iff xP_i|_By$ for all $x, y \in B$ and $i \in N$.

[3]So, given any distinct $x, y \in A$ and $P_i \in \mathcal{L}(A)$, precisely one of xP_iy and yP_ix holds. Moreover, xP_iy and yP_iz implies xP_iz for all $x, y, z \in A$ and $P_i \in \mathcal{L}(A)$. Finally, xP_ix does not hold for any $x \in A$.

For any $i \in N$ and $P_N \in \mathcal{L}(A)^N$, let τ_{P_i} denote the alternative on top of preference P_i. Thus, $\tau_{P_i} P_i y$ for all $y \in A \diagdown \{\tau_{P_i}\}$ and $i \in N$. We write $\tau(P_N) = (\tau_{P_i})_{i \in N}$ for the *tops profile* of P_N. For any alternative $x \in A$ and profile $P_N \in \mathcal{L}(A)^N$, let $n_{P_N}(x)$ denote the number of individuals who put x on top of their preferences in P_N. Thus, $n_{P_N}(x) = \#\{i \in N : \tau_{P_i} = x\}$.

For any non-empty finite set X, a *permutation* on X is a bijection $\sigma : X \leftrightarrow X$. Let Σ_X be the set of all permutations on X. We write, with a slight abuse of notation, $\sigma(P_N) = (P_{\sigma(i)})_{i \in N}$ for the profile obtained from $P_N \in \mathcal{L}(A)^N$ by a permutation $\sigma \in \Sigma_N$. An SCR is *anonymous* iff $f(P_N) = f(\sigma(P_N))$ $\forall P_N \in \mathcal{L}(A)^N$ $\forall \sigma \in \Sigma_N$. Again, by an abuse of notation, we write $\sigma(P_i)$ for the preference obtained from $P_i \in \mathcal{L}(A)$ by a permutation $\sigma \in \Sigma_A$ on A, *i.e.*, $x\, P_i\, y \iff \sigma(x)\, \sigma(P_i)\, \sigma(y)\ \forall x, y \in A$. Moreover, we set $\sigma(P_N) = (\sigma(P_i))_{i \in N}$ $\forall P_N \in \mathcal{L}(A)^N$. An SCR is *neutral* iff $f(\sigma(P_N)) = \sigma(f(P_N))$ $\forall P_N \in \mathcal{L}(A)^N$, $\forall x \in A$, and $\forall \sigma \in \Sigma_A$. An SCR f is *(Pareto) efficient* iff given any $P_N \in \mathcal{L}(A)^N$ and any $x \in f(P_N)$, $\nexists y \in A \backslash \{x\}$ with $y\ P_i\ x\ \forall i \in N$.

An SCR f is *monotonic* if for any $x \in A$ and $P_N, P'_N \in \mathcal{L}(A)^N$ such that there exists $i \in N$ with $\tau_{P_i} \neq x$, $\tau_{P'_i} = x$, and $y P_i z \iff y P'_i z$ for all $y, z \in A \diagdown \{x\}$ while $P_j = P'_j$ for all $j \neq i$, we have $x \in f(P_N)$ implies $f(P'_N) = x$. Finally, an SCR f is *tops-only* if for any $P_N, P'_N \in \mathcal{L}(A)^N$ with $\tau(P_N) = \tau(P'_N)$ we have $f(P_N) = f(P'_N)$.

3 Results

The plurality rule is the SCR $\Pi : \mathcal{L}(A)^N \rightarrow \mathcal{A}$ that assigns to every profile the alternatives that are put on top by the highest number of individuals. Thus, formally,

$$\Pi(P_N) = \{x \in A : n_{P_N}(x) \geq n_{P_N}(y)\ \forall y \in A\}.$$

Theorem 1 (Kelly and Qi, 2016) *An SCR $f : \mathcal{L}(A)^N \rightarrow \mathcal{A}$ is tops-only, anonymous, monotonic, and neutral if and only if $f = \Pi$.*

Let Φ denote the set of SCFs that are refinements of Π. Thus, $f \in \Phi$ implies $f(P_N) \in \Pi(P_N)$ for all $P_N \in \mathcal{L}(A)^N$. We ask the extent to which refinements of plurality preserve the conditions that characterizes plurality. We first consider monotonicity and show that every refinement of plurality is monotonic. Let Φ_M denote the set of monotonic refinements of Π.

Theorem 2 $\Phi_M = \Phi$.

Proof: That $\Phi_M \subseteq \Phi$ is straightforward. For the converse, take any refinement $f \in \Phi$ and $P_N \in \mathcal{L}(A)^N$ such that $f(P_N) = x$ for some $x \in A$ with $\tau_{P_i} \neq x$ for some $i \in N$. We have $x \in \Pi(P_N)$ by definition. Let P'_N be such that $\tau_{P'_i} = x$, $yP_iz \iff yP'_iz$ for all $y, z \in A \diagdown \{x\}$ while $P_j = P'_j$ for all $j \neq i$. By monotonicity $\{x\} = \Pi(P'_N)$ implying $f(P'_N) = x$. $\square$

Thus, plurality can be refined without worrying about monotonicity. So the impossibility of simultaneously satisfying the axioms that characterize plurality comes from conjoining the remaining three. In fact, there is a well-known general tension between anonymity and neutrality shown by Moulin (1980). We quote the relevant result below (see Ozkes and Sanver, 2021, for alternative formulations). Define $\mathcal{D}^*_m(n) = \{k \text{ is a prime} : k \leq m \text{ and } k \mid n\} \cup \{1\}$.

Theorem 3 (Moulin, 1980) *There exists an anonymous, efficient, and neutral SCF iff* $\mathcal{D}^*_m(n) = \{1\}$.

This tension is more severe for plurality when tops-onlyness is also added, ruling out all possible sizes of social choice problems except the very restrictive case of two alternatives and an odd number of individuals as we show in Theorem 4.

Theorem 4 Π *admits an anonymous, neutral, and tops-only refinement if and only if* n *is odd and* $m = 2$.

Proof: To see the "if" part, it suffices to observe that when n is odd and $m = 2$, Π is a social choice function. We now show the "only if" part. The case where n is even or $n = 3$ are covered by the general impossibility established by Theorem 3, with no specific reference to the plurality rule. Now consider the case where $n \geq 5$ is odd and $m > 2$. Let $xP_iyP_iz_1P_iz_2 \cdots z_{n-3}P_iz_{n-2}$ for all

$i \in N_1$, $yP_ixP_iz_1P_iz_2 \cdots z_{n-3}P_iz_{n-2}$ for all $i \in N_2$, and $z_1P_jz_2 \cdot x \cdot y \cdot z_{n-3}P_jz_{n-2}$, where $N_1 \cap N_2 = \emptyset$, $|N_1| = |N_2| = |N-1|/2$ and $\{j\} = N \backslash N_1 \cup N_2$. Let $\sigma : A \leftrightarrow A$ be such that $\sigma(x) = y$ and $\sigma(z_t) = z_t$ for all $t \in \{1, 2, ..., n-2\}$. Let $P'_N \in \mathcal{L}(A)^N$ be such that $\sigma(P_i) = P'_i$ for all $i \in N_1 \cup N_2$ and $P_j = P'_j$. Without loss of generality, assume that a refinement f of Π is such that $f(P_N) = x$. By neutrality, we have that $f(\sigma(P_N)) = y$. By tops-onlyness, $f(\sigma(P_N)) = y$ implies $f(P'_N) = y$, as well. However, for any $\rho : N \leftrightarrow N$ such that $\rho(j) = j$ and $\{\rho(i)\}_{i \in N_1} = N_2$ we have $f(P'_N) = f(P_N) = x$ by anonymity, a contradiction. □

As a result, we consider refinements of plurality that fail only one of anonymity, neutrality, and tops-onlyness, and start by refinements that are anonymous and tops-only.

We write $C : \mathcal{A} \to A$ for a choice function satisfying $C(X) \in X$ for all $X \in \mathcal{A}$. Every choice function C induces a refinement f_C of plurality by setting $f_C(P_N) = C(\Pi(P_N))$ for all $P_N \in \mathcal{L}(A)^N$.

Remark 1 *f_C is anonymous and tops-only for any choice function C.*

Remark 2 *As a particular case of refining plurality by a choice function, we have the well-known method of fixing a tie-breaking linear order T over alternatives and picking at each P_N the alternative in $\Pi(P_N)$ that is highest ranked in T. In fact, this is equivalent to refining plurality by a singleton-valued choice function $C : \mathcal{A} \to A$ that satisfies the weak axiom of revealed preference.*[4]

Can every anonymous and tops-only refinement of plurality be induced by a choice function? The answer is negative. To see this, take two tie-breaking linear orders T and $-T$ such that $-T$ is the reverse of T. Use T to refine plurality at preference profiles where every alternative is top-ranked by at least one individual, and use $-T$ otherwise.

[4]A choice function C satisfies the weak axiom of revealed preference (WARP) if for any $a, b \in A$ and $B \in \mathcal{A}$, having $a, b \in B$ and $a \in C(B)$ implies that if $b \in B'$ and $b \in C(B')$ for some $B' \in \mathcal{A}$, we must have $a \notin B'$. WARP is a consistency requirement on choice functions originated from the revealed preference theory.

The generality of choice functions being insufficient to induce all anonymous and tops-only refinements of plurality, we ask the extent to which the concept of a choice function must be generalized. For any $P_N \in \mathcal{L}(A)^N$, let $\delta_{P_N} : A \to \mathbb{N}$ identify the number of individuals that put each alternative on top, *i.e.*, $\delta_{P_N}(x) = |\{i \in N : \tau_{P_i} = x\}|$. Write $G_\delta(\Pi) = \{(\delta_{P_N}, \Pi(P_N))\}_{P_N \in \mathcal{L}(A)^N}$ and define a generalized choice function as a mapping $C : G_\delta(\Pi) \to A$ with $C(\delta_{P_N}, \Pi(P_N)) \in \Pi(P_N)$. Let Γ be the set of generalized choice functions and $\Phi_{T,AN}$ be the set of tops-only and anonymous refinements of Π.

Theorem 5 $\Phi_{T,AN} = \{f_C\}_{C \in \Gamma}$.

Proof: That $\Phi_{T,AN} \supseteq \{f_C\}_{C \in \Gamma}$ is left to the reader. To show $\Phi_{T,AN} \subseteq \{f_C\}_{C \in \Gamma}$, Take any $f \in \Phi_{T,AN}$. For any $P_N, P'_N \in \mathcal{L}(A)^N$ with $\Pi(P_N) = \Pi(P'_N)$, $\delta_{P_N} = \delta_{P'_N}$ implies $f(P_N) = f(P'_N)$. Thus, the choice function $C_f : G_\delta(\Pi) \to A$ with $C_f(\delta_{P_N}, \Pi(P_N)) = f(P_N)$ is well-defined. Naturally $C_f \in \Gamma$, implying $f \in \{f_C\}_{C \in \Gamma}$. $\square$

We say that a refinement of plurality $f : \mathcal{L}(A)^N \to A$ is *coherent* iff $f(P_N) = f(P'_N)$ for any two profiles P_N and P'_N with $\Pi(P_N) = \Pi(P'_N)$. Coherence characterizes refinements of plurality obtained by a generalized choice function $C \in \Gamma$ independent of δ_{P_N}, or equivalently, by a choice function $C : \mathcal{A} \to A$.

Proposition 1 *A refinement of plurality $f : \mathcal{L}(A)^N \to A$ is coherent iff there exists a choice function $C : \mathcal{A} \to A$ such that $f(P_N) = C(\Pi(P_N))$ for all $P_N \in \mathcal{L}(A)^N$.*

Proof: We leave the "if" part to the reader and show the "only if" part. Let f be a coherent refinement of plurality. Define the choice function $C_f : \mathcal{A} \to A$ such that $C_f(\Pi(P_N)) = f(P_N)$ for all $P_N \in \mathcal{L}(A)^N$. Coherence ensures that C_f is well-defined. Note that $\mathcal{A} \diagdown \{\Pi(P_N)\}_{P_N \in \mathcal{L}(A)^N}$ can be non-empty, in which case we set $C_f(X) \in X$ arbitrarily for any $X \in \mathcal{A} \diagdown \{\Pi(P_N)\}_{P_N \in \mathcal{L}(A)^N}$. $\square$

Remark 1 and Proposition 1 show that although coherence seems mild, it implies anonymity and tops-onlyness.

We focus next on refinements of plurality that are neutral and tops-only. A *priority function* is a choice function $\theta : 2^N \to N$ over N satisfying $\theta(K) \in K$ for all $K \in 2^N$. Let $W(P_N) = \{i \in N : \tau_{P_i} \in \Pi(P_N)\}$ be the set of *winning individuals* at profile $P_N \in \mathcal{L}(A)^N$. Every priority function θ induces a refinement f_θ of plurality by setting $f_\theta(P_N) = \tau_{P_{\theta(W(P_N))}}$ for all $P_N \in \mathcal{L}(A)^N$.

Remark 3 *f_θ is neutral and tops-only for any priority function θ.*

Remark 4 *As a particular case of refining plurality by a priority function, one can fix a linear order T over individuals and pick at each P_N the alternative that is top-ranked by the individual in $W(P_N)$ who is highest ranked by T.*

Can every neutral and tops-only refinement of plurality be induced by a priority function? The answer is negative. To see this, consider a refinement that picks the alternative that is top-ranked by the winning individual who is highest ranked by $T \in \mathcal{L}(N)$ whenever every alternative appears on top at least once, while uses the reverse order $-T$ otherwise.

The generality of priority functions being insufficient to induce all neutral and tops-only refinements of plurality, we ask the extent to which the concept of a priority function must be generalized.

Define $GW_\tau(\Pi) = \{(\tau(P_N),\ W(P_N))\}_{P_N \in \mathcal{L}(A)^N}$. A generalized priority function is a mapping $\theta : GW_\tau(\Pi) \to N$ such that $\theta(\tau(P_N),\ W(P_N)) \in W(P_N)$ for all $P_N \in \mathcal{L}(A)^N$. Write Θ for the set of generalized priority functions for N and $\Phi_{T,NE}$ for the set of tops-only and neutral refinements of Π.

Theorem 6 $\Phi_{T,NE} = \{f_\theta\}_{\theta\in\Theta}$.

Proof: That $\Phi_{T,NE} \supseteq \{f_\theta\}_{\theta\in\Theta}$ is left to the reader. To see $\Phi_{T,NE} \subseteq \{f_\theta\}_{\theta\in\Theta}$, take any $f \in \Phi_{T,NE}$. For any $P_N, P'_N \in \mathcal{L}(A)^N$ with $\tau(P_N) = \tau(P_N)$, we have $W(P_N) = W(P'_N)$ and $f(P_N) = f(P'_N)$. Thus, the generalized priority function $\theta^f_\tau : GW_\tau(\Pi) \to N$ such that $\theta^f_\tau(\tau(P_N), W(P_N)) = f(P_N)$ for all $P_N \in \mathcal{L}(A)^N$ is well-defined. Naturally $\theta^f_\tau \in \Theta$, implying $f \in \{f_\theta\}_{\theta\in\Theta}$. □

We close by exploring the possibility of simultaneously satisfying anonymity and neutrality, dispensing with tops-onlyness. To this end, we define the iterative plurality (rule) $\upsilon : \mathcal{L}(A)^N \to \mathcal{A}$, which selects the plurality winners after successive restriction of profiles to plurality winners. Formally, let $\Pi^{i+1}(P_N) = \Pi(P_N|_{\Pi^i(P_N)})$ for all $i \geq 1$ together with $\Pi^1(P_N) = \Pi(P_N)$, and define $\upsilon(P_N) = \Pi^k(P_N)$, where k is the minimal integer that satisfies $\Pi^k(P_N) = \Pi^{k+1}(P_N)$. Such an integer always exists given the finiteness of A.

The iterative plurality is anonymous and neutral while it fails tops-onlyness. It is a sub-correspondence of plurality that does not necessarily refine it. However, in case the iterative plurality refines plurality, it exemplifies a refinement of plurality that is anonymous and neutral but not tops-only. More interestingly, in case it does not refine plurality, plurality does not admit any anonymous and neutral refinement.

Theorem 7 *Plurality admits an anonymous and neutral refinement iff iterative plurality refines it.*

Proof: "If" part is straightforward. To see the "only if" part, note first that given Theorem 3, if plurality admits an anonymous and neutral refinement, as any refinement of plurality is efficient as well, we must have that $\mathcal{D}^*_m(n) = \{1\}$. Take any $P_N \in \mathcal{L}(A)^N$ and let k be the minimal integer that satisfies $\Pi^k(P_N) = \Pi^{k+1}(P_N)$. It is straightforward to note that $n_{P_N|_{\Pi^k(P_N)}}(x) = n_{P_N|_{\Pi^k(P_N)}}(y) = t$ for some $t \in \mathbb{N}$ for all $x, y \in \upsilon(P_N)$, and $n_{P_N|_{\Pi^k(P_N)}}(z) = 0$ for any $z \notin \upsilon(P_N)$. Thus, $|\upsilon(P_N)| \times t = n$. This implies that for any profile $P_N \in \mathcal{L}(A)^N$, $|\upsilon(P_N)|$ is a divisor of n. Thus, we have $\mathcal{D}^*_m(n) = \{1\}$ only if $|\upsilon(P_N)| = 1$ for all $P_N \in \mathcal{L}(A)^N$. □

4 Concluding remarks

Voting literature is rife with methods satisfying varying properties fit for many settings, particularly when preferences are expressed as rankings. In many practical situations, however, people are not asked for their rankings and plurality is probably the most widely used methods in that case. Legal opinion forming processes in the Supreme Court of the United States (Thurmon, 1992), first-past-the-post electoral systems (Blais, 2008), and state-level elections for most of the Electoral College in United States presidential elections (Miller, 2012) are among the prominent examples of its usage. To be sure, the use of plurality is not restricted to political or legal domains: Sport competitions, health care management (Calvao et al., 2016), and corporate governance (approximately 41% of Russell 3000 companies employed plurality rule in director elections as of 2019 (Tonello, 2020; Kastiel and Nili, 2021)) are among the uncommon examples in the literature.

Especially when decision making is made by committees where the number of voters is typically small, plurality risks to lead to a tie. In such as case, a refinement is necessary to reach a resolute outcome. With a refinement, certain axiomatic properties of plurality get inevitably lost. We draw a somewhat complete picture of the properties that could be preserved when plurality is refined.

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