On Pendular Voting

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Introduction

Mission – New Forms of Democracy at ETH Zurich:
We assess the potential and the limitations of a number of unprecedented ideas designed to renew and strengthen democratic decision-making, with a view to re-establish democratic sustainability and, wherever feasible, using digitization for that purpose.

History-bound Reelections

Problem: Most races for the House of Representatives in US are not competitive, incumbents find it easier to be reelected than first-time candidates to be elected.

Solution: Score-replication Rule Idea (two-candidates race)

1. If a candidate is not the incumbent, a simple majority of votes will suffice for election.
2. If a candidate is the incumbent, s/he must obtain the highest percentage of votes s/he has ever obtained for this same office.
3. If the incumbent fails to achieve this highest past percentage, s/he will not be elected (and challenger is elected or a run-off with two new candidates takes place).

History-bound Reelections: Example

Two-candidates election race.

Practical Score-replication Rule:
Every incumbent has to achieve $\max[1/2, s - \Delta]$ *to be reelected, with* s *being the highest previously-obtained vote-share.*

First results (AEJ Micro 2021): If $\Delta > 0$ is chosen appropriately,

- able office-holders are reelected, less able ones deselected,
- office-holders indulge less in their own preferences, welfare is improved.
- races become competitive again.

Variant: The Score-replication Rule can be *offered* by the incumbent during the electoral campaign.

Assessment Voting: Voting between Two Alternatives

- Normative goals:
  - Implement the will of the majority (if same preference intensity) **(G1)**
  - "Minimize" costs **(G2)**
  - One person, one vote **(G3)**
- Four options:

	G1	**G2**	**G3**
Voluntary Voting		✓	✓
Compulsory Voting	✓		✓
Random Sample Voting	✓	✓	
Assessment Voting	✓	✓	✓

Assessment Voting for Direct Democracy

Two-round voting procedure:

- First round: Randomly-chosen assessment group (e.g. 50'000 or 100'000 citizens) votes on proposal A versus S (are incentivized to vote).
- Members of this group have then exercised their right to vote irrevocably.
- Once the first-round results are published, initiators have the possibility to withdraw their proposal.
- Second round: If the initiative is not withdrawn, the remaining voters vote.
- The results of both rounds are added and a majority rule is applied.

$\Rightarrow$ Advantages: The majority's preferred alternative is chosen, saves voting costs (JET 2021).

Referenda

Referenda have become more popular in representative democracies (often so-called "elite referenda"), and they are standard in direct democracies.

Problems:

- How to find/induce good proposals?
  (only a few proposals can be voted on in mass voting. E.g. Brexit: many variants were available.)
- What to do when preferences are unknown?

The Swiss System with Three Alternatives

Initiative A, counterproposal B (parliament), status quo S. Alternatives typically can be ordered: Typically S,B,A.

Voting rule: (voting: A versus S, B versus S, A versus B)

- If A is approved and B is not, A is implemented.
- If B is approved and A is not, B is implemented.
- If A and B are approved, the winner is determined by the tie-break question ("Stichfrage" A versus B).
- Otherwise the status quo prevails.

Fact: Suppose single-peaked preferences.
Incentives for strategic voting in some constellations: Supporters of A vote against B (even if they like it) to avoid the tie-break question.

Questions

- **How should counterproposals (and initial proposals) be made?**
- **How to deal with uncertainty about preferences of citizens?**
- **Which tie-breaking rule should be applied with more than two proposals?**

Pendular Voting: Main Idea and Model

Procedure:

- **Stage 1**
  - Initiative (proposal A) by admissible political agent (government, parliament, initivative group) is made to change the status quo S.
  - Random sample of the population votes (A versus S).
  - Result are published (preferences may be revealed).
- **Stage 2**
  - Second proposal B according to result of first stage, either moderate or ambitious counterproposal (e.g. by counterproposal commission).
  - Entire electorate (including random sample) votes (A, B, S): Order SAB or SBA.

Tie-breaking rule: *If there is a cycle: Middle alternative is chosen (A or B).*

Pendular Voting: Model

- one-dimensional linear policy $\theta \in [0, 1]$
- status quo: $S = 0$
- continuum of citizens of unit measure; citizen identified by type $z \in [0, 1]$
- utilities of citizen z: $u(z, \theta)$, single-peaked with peak z (and symmetric in some applications)

Pendular Voting: Model

- distribution of citizen type z:
  - N states of nature, $k = 1, ..., N$
  - family of probability distributions f_k (density), F_k (probability distribution), $\hat{z}_k$ median in state k (Condorcet winner in state k)
  - common prior over N (every voter will update, based on type z)
  - Remark: Nobody knows the Condorcet Winner (peak of median voter)
- objective in Stage 0: minimization of expected distance between outcome and peak of median voter
- notation
  λ (small) measure of electorate voting in Stage 1
  $\bar{\theta}$ initial proposal, δ vote share in favor of $\bar{\theta}$ in Stage 1, new proposal $\alpha(\bar{\theta}, \delta)$

$$\alpha(\bar{\theta}, \delta) = \begin{cases} \bar{\theta} + \mu_1 & \text{if } \delta \geq 1/2, \\ \bar{\theta} - \mu_2 & \text{if } \delta < 1/2. \end{cases}$$

Pendular Voting: Model

- Voting in Stage 2 (simultaneously) ($x_0 = 0, x_1$ middle alternative ($\bar{\theta}$ or $\alpha(\bar{\theta}, \delta)), x_2$)
  $x_0 \leftrightarrow x_1$
  $x_0 \leftrightarrow x_2$
  $x_1 \leftrightarrow x_2$
- Voting rule and breaking cycles:
  - If any alternative wins two of the three pairwise votes, then it becomes the outcome of the voting procedure.
  - If each of the three votes is won by a different alternative, i.e. if there is a cyclic outcome, then the intermediate alternative x_1 becomes the outcome of the voting procedure.

Pendular Voting: Model

- Assumption: Citizens with the same preference ordering vote in the same way ("coordinate voting", "group-strategy-proofness", conservative assumption: results robust to manipulations by large groups).
- Equilibrium Concept: perfect Bayesian Equilibrium

Equilibria – Results

Four possible groups:

$$x_2 \succ x_1 \succ x_0,$$
$$x_1 \succ x_2 \succ x_0,$$
$$x_1 \succ x_0 \succ x_2,$$
$$x_0 \succ x_1 \succ x_2.$$

Proposition 1. *In the second stage of the Pendular Voting procedure, there is an equilibrium in which all citizens vote sincerely.*

Remarks:

- No incentives for one of the group to vote strategically.
  Attempts of group $x_2 \succ x_1 \succ x_0$ to generate a cycle (vote against x_1 when $x_1 \leftrightarrow x_0$) is not profitable.
- There may be equilibria with strategic voting, but not always.

Fact:

Alternative ways to break cycles are not group strategy-proof
(e.g. winner determined by highest aggregate voting share).

Equilibria

Corollary

Corollary 1. In the equilibrium with sincere voting, the outcome of the second round of Pendular Voting is determined as follows:

- *If the group* $x_2 \succsim x_1 \succsim x_0$ *is a majority, then* x_2 *wins.*
- *If the group* $x_0 \succsim x_1 \succsim x_2$ *is a majority, then* x_0 *wins.*
- *In any other case,* x_1 *wins.*

In particular, the outcome is always the median voter's preferred policy out of the three proposals.

Equilibria

Suppose proposal θ, potential proposals α^+, α_- and 0.
Four groups:

- $Z_3 : \alpha^+ > \theta > \alpha_- > 0$
- $Z_2 : \theta > \alpha_-$ (or α^+) $> ...$
- $Z_1 : \alpha_- > ...$, subgroup $Z_1^1 : \alpha_- > 0 > \theta > \alpha^+$
- $Z_0 : 0 > \alpha_- > \theta > \alpha^+$

Equilibria

Theorem (1)

In the Pendular Voting procedure, the only group that could be tempted to manipulate is Z_0. It will do so if and only if it believes (with sufficient probability) to be in a state of nature where $Z_0 \cup Z_1^1$ forms a majority but where neither Z_0 nor Z_1 is a majority.

$$\text{If } \underline{\text{Not Manipulation}} \begin{cases} \text{No:} & Z_0 \cup Z_1, \\ \text{Yes:} & Z_2 \cup Z_3. \end{cases} \qquad \text{If } \underline{\text{Manipulation}} \begin{cases} \text{No:} & Z_1 \cup Z_2 \cup Z_3, \\ \text{Yes:} & Z_0. \end{cases}$$

Equilibria

Theorem (2)

Consider the Bayesian Nash Equilibrium of the Pendular Voting procedure given by Theorem 1 for the first stage, in combination with the equilibrium with sincere voting for the second stage; then:

- *The outcome is 0 if and only if* Z_0 *is a majority.*
- *The outcome is* α^+ *if and only if* Z_3 *is a majority.*
- *The outcome is* α_- *if and only if* $Z_0 \cup Z_1$ *is a majority (but* Z_0 *itself is not a majority).*
- *The outcome is* θ *if and only if none of the above holds.*

Implications

- Manipulation in first round can occur, but does not matter: Outcome is the same as if everybody voted sincerely in the first round.
- Explicit outcome characterization.
- So far, everything independent of beliefs.

Example. Suppose that the median voter type is uniformly distributed (continuum of states).

Case 1: Benevolent agenda-setter in first stage:

- Maximizes expected social welfare.
- Selects μ_1, μ_2 to minimize $\mathbb{E}[|\Omega_\theta^{PV}(\hat{z}) - \hat{z}|]$.

Then, the design parameters that maximize social welfare are

$$\mu_1^* = \mu_2^* = \frac{2}{7}.$$

Case 2: Suppose that the agenda-setter is selfish (and the median voter type is uniformly distributed and extreme, type $z = 1$ sets $\bar{\theta}$). Then, the design parameters that maximize social welfare are

$$\mu_1^* = \mu_2^* = \frac{1}{2}.$$

With this choice of parameters, the selfish agenda-setter will choose the proposal $\bar{\theta} = \frac{1}{2}$ and the expected social welfare of Pendular Voting is

$$\mathbb{E}[|\Omega^{PV}(\hat{z}) - \hat{z}|] = \frac{1}{8}.$$

Agenda Setting, Design of Pendular Voting, and Welfare

Overall design and comparisons

- Aggregating over all possible agenda-setters yields design parameters at the constitutional level.
- Comparison to one-round proposal-making (benevolent or selfish agenda-setter) $\Rightarrow$ Pendular Voting always dominates.
- Comparison to two-round proposal-making with two strategic agenda-setters $\Rightarrow$ Pendular Voting does dominate in standard cases, but not universally (e.g. first agenda-setter is known to be extreme).
- Alternative design of counterproposal for Pendular Voting:
  Second counterproposal by expert commission: more extreme or more moderate

Conclusion

- Assessment Voting for binary majority decisions for a large electorate
- Pendular Voting for mass voting with counterproposals
- Pendular Voting applicable to many countries in which referenda are undertaken (today, mostly so-called "elite referenda"), e.g. Brexit.