Impartial Ordinalism

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Introduction I

- Two core problems of Ordinal Preference Aggregation:
  1. Context-dependence inescapable per Arrow (1951); threatens social choice with unreliability
  2. Appropriate basis for choice evaluation

- Alternative basis for evalation
  1. **(Borda school)** Positional rank as a basis for imputed preference intensity
     - severe reliability issues
  2. **(Condorcet school)** Patterns of pairwise majorities (C2)
     - adresses reliability issues (maxmin and variants)
     - at price of informational impoverishment ?
  3. **(Impartial Ordinalism):** Ordinal dominance
     - aims at middle way

Introduction II

- Varieties of Impartial Ordinalism reflecting different "reliability stances".

Reliability Stance	Context Dependence	Choice rule
Satisficing	minimal	Essential Set
Hedging	minimal	Maximal Lottery
Optimizing	informative yet reliable	Pluri-Borda (et al.)
Sang-Froid	unrestricted	Borda

Ordinal Preference Aggregation Problems I

We will employ a variable-agenda, fixed-population framework.

- Let A^* denote a (finite) 'universe' of possible alternatives.

- A **choice set** or "agenda" A is a finite subset of A^*; their family is denoted by $\mathcal{A} = 2^{A^*}_{\setminus\varnothing}$.

- Each individual $i \in I$ has a preference relation described by a linear or weak order P_i on A. For any given $A \in \mathcal{A}$, let $\mathcal{L}(A)$ denote the set of linear orders on A.

Ordinal Preference Aggregation Problems II

- A **profile** μ is a rational-valued probability distribution on $\mathcal{L}(A)$, with $\mu_{P.}$ denoting the relative frequency of individuals with strict preference ordering P.
  - $\mu_P = \frac{|\{i:P_i=P\}|}{|I|}$.

- An **aggregation problem** is a pair (A, μ).

- A **social choice correspondence** C maps aggregation problems $(A, \mu) \in \mathfrak{D}$ to non-empty subsets of A.
  - Often sufficient to focus on *single-profile domains* $\mathfrak{D} = \{2^{A^*}_{\setminus\varnothing}\} \times \{\mu\}$.

Ordinal Preference Aggregation Problems III

Cardinal Ignorance

- Cardinal Ignorance: choice must *exlusively* rely on information given by preference rankings

1. No informational basis for interpersonal comparisons
2. No information about strength of preference available
   - by assumption: no individually elicited info,
     but also no appeal to background information obtained elsewhere.
     e.g. empirical knowledge of putative 'typical' strong preference for top.

Ordinal Preference Aggregation Problems IV

- "Cardinal Ignorance"
  - poses question of *informational adequacy*, not merely "*meaningfulness*"
    - top is ordinally meaningfully defined, but that does not entail/justify preference tops playing a distinct role under C.I.
- C.I. not an axiom, but serves as a background rationale for choice axioms
- Q: is C.I. common ground in conceptualizing the problem of ordinal aggregation after Arrow?

Decision-Problems under Ordinal Uncertainty

- Single decision-maker, uncertainty about future preferences
  - DM has only ordinal non-comparable information about future preferences *ex hypothesis*

- I is state space;
  μ probability distribution over P_i

- Possible applications include
  1. Moral uncertainty
     - moral theories / value judgements may be 'intrinsically ordinal'; psychological 'strength of preference' irrelevant
  2. Social choice behind veil of ignorance.

Ordinal Preference Aggregation Problems VI

- Relevance of single-person interpretation:
  1. Ordinal aggregation matter of *decision-theoretic rationality*, not ethics or political philosophy per se.
  2. But application in multi-person context to e.g. voting itself has substantive normative content
     - "impartiality" as choice behind veil of ignorance
     - impartiality as political equality

- Ordinal preference aggregation: non-standard model of "state-dependent preferences" for decision theory
  - non-standard: states *fully described* by ordinal rankings
  - standard Savage framework: implicit ex-ante cardinalization and comparison by decision maker
    - State-Dependent Expected Utility

Ordinal Preference Aggregation Problems VII

- Application of Expected Utility?

Axiom (Sure-Thing Principle)

If $C(A,\mu) \cap C(A,\mu') \neq \emptyset$, *then, for any* $\alpha \in [0,1]$,
$C(A, \alpha\mu + (1-\alpha)) = C(A,\mu) \cap C(A,\mu')$.

- a.k.a. Population Consistency, Reinforcement

- By classical result of Young (1974), this characterizes *EU with positional cardinal utilities.*
  - Young's result as representation theorem a la vNM and Savage

Ordinal Preference Aggregation Problems VIII

- Borda rule as expression of Cardinal Ignorance
  - direct argument via equal utility-differences
  - axiomatic argument
    - Cancellation axiom of Young
    - Modified IIA axiom of Maskin (2020)

- But is this sufficient to handle Cardinal Ignorance ?
  - analogy of problem of 'ignorance utilities' to 'ignorance probabilities'; arguably, in both settings, ignorance precludes EU

The Borda Rule's Unreliability I

- The Borda rule is *unreliable*, i.e.
  - differences in choice not adequately justified by differences in information.

	alphas	betas
	99%	1%
Top	a	b
	b	c_1
.	c_1	.
	.	c_m
Bottom	c_m	a

- If $m \geq 100$, the Borda rule selects $\{b\}$
  - Is this plausible at all, *on ordinal information alone*??

The Borda Rule's Unreliability II

- Counterexample is
  - **general:** applies to all (non-augmented) problems
  - **severe**: can induce all Pareto optimal alternatives at initial profile by adding Pareto dominated alternatives
  - **significant:** generalizes to positional scoring rules
  - **robust:** presumably extends to inclusion of nearly dominated / near clones
  - **elusive:** just how far does it reach ?

- Upshot: under Cardinal Ignorance, 'positional rank' is not reliably useable,
  hence no adequate basis to **impute** preference intensities

Context-Independence Axioms I

1. Independence of Pareto Dominated Alternatives
2. Independence of Clones
    - Ind of Exchangeable Clones
    - Ind of Pareto-Dominated Clones
3. Independence of Ordinally Dominated Alternatives
4. Independence of Unchosen Alternatives

- Arguably, Independence of Pareto Dominated Alternatives and Independence of Exchangeable Clones as normative minimum
- Independence of Pareto-Dominated Clones (2c) as 'absolute Arrowian minimum'
- Independence of Unchosen Alternatives as 'Arrowian maximum'

Context-Independence Axioms II

- Comparisons to Pareto-dominated alternatives:
  dubious ‘signal’ but potentially much ‘noise’;
  hence ignore in determining best choice:

Axiom

C satisfies **Independence of Pareto-Dominated Alternatives** *if, for any $(A, \mu) \in \mathfrak{D}$ and any $b \in A$ such that b is Pareto-dominated in A,*

$$C(A \backslash b, \mu) = C(A, \mu) \backslash \{b\}.$$

Independence of Clones

- A set $B \subseteq A$ is a **cluster of clones** at μ if, for all $b, b' \in B$ and $a \in A \backslash B$: aRb iff aRb'.

Axiom

C satisfies **Independence of Clones** *if, for any $(A, \mu) \in \mathfrak{D}$ and any $\varnothing \neq B' \subseteq B \subseteq A$ such that B is a cluster of clones at μ,*

$$C\left(B' \cup A \backslash B, \mu\right) = C\left(A, \mu\right) \cap \left(B' \cup A \backslash B\right).$$

Context-Independence Axioms IV

- Arguably too restrictive, especially for GBR, and especially from expansive reliability stance (N2018)
  - hence restriction to exchangeable clusters of clones.

- B is **exchangeable** at μ if μ is invariant on arbitrary permutations of B.

Axiom

C satisfies **Independence of Exchangeable Clones** *if for any $(A, \mu) \in \mathfrak{D}$ and any $\varnothing \neq B' \subseteq B \subseteq A$ such that B is a cluster of exchangeable clones at μ,*

$$C\left(B' \cup A \backslash B, \mu\right) = C\left(A, \mu\right) \cap \left(B' \cup A \backslash B\right).$$

The Basis for Choice Value I

1. Imputed Preferences Intensities (Borda school)
   - Key axiom: Reinforcement
   - severe reliability issues
2. Pairwise Majorities (C2, Condorcet school)
   - Key axiom: Top cycle selection ('Smith dominance')
   - reliability issues can be addressed
   - is informational restriction necessary price to pay?
3. **Impartial Ordinalism**
   - Key axiom: Ordinal Admissibility
   - flexible treatement of reliability;
     possible tradeoff between context-independence and informativeness.

Ordinal Dominance I

- With $|A| = 2$, the ordering by majority comparison is arguably compelling.
  - e.g. May's (1952) theorem.
  - normative force relies on *Argument from Ignorance*
    - ignorance of/abstention from interpersonal comparions, rights, ...
- How to extend Argument from Ignorance to $|A| > 2$?

Ordinal Dominance II

- Will make essential use of the matrix of pairwise majority margins $M(a, b) : A \times A \to [-1, 1]$ given by

$$M(a, b) := \mu(\{P : aPb\}) - \mu(\{P : bPa\}).$$

- a **ordinally dominates** b in (A, μ) iff, for all $e \in A$,

$$M(a, e) > M(b, e).$$

  Impartial Ordinalism takes ordinal dominance to be *decisive* reason to reject b as inferior.

  - does not rely on any attribution of preference intensities – explicit or implicit
  - does not assume Fishburn's $C2$ (only M matters to determine C).

Ordinal Dominance III

The alternative a is **ordinally undominated** in (A, μ) ("$a \in OU(A, \mu)$") if there does not exist any other alternative $b \in A$ ordinally dominating it.
C is **ordinally undominated** if $C(A, \mu) \subseteq OU(A, \mu)$ for all $(A, \mu) \in \mathfrak{D}$.

- Ordinal Dominance seems fundamental, but rarely considered in the literature.
  - Versions of ordinal dominance introduced by Dutta-Laslier (1999).

Ordinal Dominance IV

- Positive examples:
  1. Condorcet Winner;
     many Condorcet extensions
  2. Borda Winner
     $Borda(A, \mu) = \arg\max_{a \in A} \sum_{e \in A} M(a, e) \frac{1}{|A|}$.

- Counterexamples:
  1. Plurality-based rules
     - Plurality rule, Plurality with runoff, IRV ("ranked choice").
     - apparent general consensensus that these can't be first-best ordo-welfarist
  2. Positional scoring rules other than Borda.
     - popular, but *justification for such rules requires appeal to "background information"*

Ordinal Admissibility I

One can naturally extend the ordinal dominance to randomized ("lotteries") $p \in \Delta(A)$ by considering *expected pairwise majorities*

$$M(p, q) := \sum_{a,b \in A} p_a M(a, b) q_b.$$

- The lottery p **ordinally dominates** q iff, for all $e \in A$,

$$M(p, e) > M(q, e).$$

  - could explicitly introduce lotteries via stochastic social choice setting and add 'purification' axiom.

- An alternative $a \in A$ is **ordinally admissible** if there does not exist a lottery $p \in \Delta(A)$ such that p ordinally dominates the degenerate lottery δ_a.

Ordinal Admissibility II

Example (Condorcet Loser)

- *A Condorcet loser is never ordinally admissible.*
- Here, Condorcet loser d is Maxmin winner.
- Lottery $p = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ ordinally dominates d;
  table gives expected maj. margins $M(p, e)$, with $0 < \varepsilon < \frac{1}{3}$.

	a	b	c	d
a	0	$-\frac{1}{3}$	$\frac{1}{3}$	ε
b	$\frac{1}{3}$	0	$-\frac{1}{3}$	ε
c	$-\frac{1}{3}$	$\frac{1}{3}$	0	ε
d	$-\varepsilon$	$-\varepsilon$	$-\varepsilon$	0
p	0	0	0	ε

Ordinal Admissibility III

Example ('Almost' Condorcet Loser)

- M given up to (sufficiently small) scaling factor σ
- d is unique Schulze, leximin winner;
  d is (non-unique) Ranked Pairs, Split Cycle winner
  - can be made unique by perturbation of M
- Lottery $p = (\frac{1}{3}, \frac{1}{3} + 4\varepsilon, \frac{1}{3} - 4\varepsilon)$ ordinally dominates d; $\varepsilon \leq 1/12$.

	a	b	c	d
a	0	-1	1	-7ε
b	1	0	-1	5ε
c	-1	1	0	5ε
d	7ε	-5ε	-5ε	0
p	8ε	-4ϵ	-4ϵ	ε

Ordinal Admissibility IV

Proposition

The alternative $a \in A$ is ordinally admissible in A if and only if there exists a weight vector $w \in \Delta(A)$ such that, for all $b \in A$,

$$\sum_{e \in A} M(a,e) w_e \geq \sum_{e \in A} M(b,e) w_e. \tag{1}$$

- Proof by standard separation argument. □

Ordinal Admissibility V

- Consider *sets* of alternatives G as potential output of a choice correspondence at (A, μ)
  - G as recommendation.
  - G is '*adequately decisive*' if any probabilitic selection from G is ordinally admissible.

- The set $G \subseteq A$ is **jointly ordinally admissible (j.o.a.)** for (A, μ) if, for no lottery $p \in \Delta(A)$ with support contained in G, there exist another lottery $q \in \Delta(A)$ such that q ordinally dominates p.
  A choice correspondence C is **jointly ordinally admissible (JOA)** if $C(A, \mu)$ is jointly ordinally admissible for all $(A, \mu) \in \mathfrak{D}$.

Generalized Borda Rules

An **index** (weighting function) ρ will be any function that assigns to any aggregation problem (A, μ) a non-negative vector of weights $\rho(A, \mu) \in \Delta(A)$. Any i.w. function ρ induces a choice correspondence B_ρ given by

$$B_\rho(A, \mu) := \arg\max_{a \in A} \sum M(a, e) \rho_e (A, \mu) .$$

A correspondence $C \subseteq B_\rho$ will be called a **generalized Borda rule** (GBR) based on index ρ; if $C = B_\rho$ it is the *exact* GBR based on ρ.

Theorem

A choice correspondence C is jointly ordinally admissible if and only if C is a generalized Borda rule.

Normativity of Impartial Ordinalism

Normative Claims:

1. **Constructive**
   - JOA plausible and useful via GBR representation;
     in particular, enables well-structured repertoire of (new) SCRs satisfying various reliability conditions flexibly and transparently
2. **Veridical**:
   - JOA as necessary implication of Cardinal Ignorance.

Ordinal Admissibility VIII

Criticizing Ordinal Dominance and Ordinal Admissibility?

- Single-profile, single-agenda axioms;
  unconditional implications
  WYSWYG
- hence: should be *easy* to criticize *directly* implications for particular aggregation problems (if worth criticizing)
  - e.g. Plurality, Instant Runoff – do you have a sound argument *in some particular problem* for violating OD/OA *under Cardinal Ignorance*?
- conversely, should be *hard* to criticize these *indirectly* in the absence of unsuccessful direct criticism;
  - e.g. conflict with other apparent desiderata

Ordinal Admissibility IX

Doubting Ordinal Dominance ?

- Plausible that the M-vector comparison is good prima-facie reason to choose *a* over *b*,
  but is it a *decisive* reason?
  - What abou other ordinal facts about *a* vs *b*?
    E.g. rank comparisons, choice-pluralities in non-binary comparisons?
- Have no proof that such counterargument impossible –
  what would such proof look like?
- Counter-counterargumenst:
  - Decisiveness as defeasible hypothesis:
    there don't seem to be convincing alternatives/augmentations on the horizon
  - candidate weakenings would fail to be *reliable*,
    so no sound candidate for *argument from insufficient reason.*

Ordinal Admissibility X

Doubting Ordinal Admissibility

- Introducing probabilities?
  - In single-person decision problem, $M(a, b)$ are (differences of) probabilities;
    hence $M(p, a)$ and $M(p, q)$ are probabilities also.
  - hence comparison of lotteries creates no additional issues;
  - implied cardinality in use of M-margins follows from rules of probability.
    - dto. for head count in multi-person setting

Ordinal Admissibility XI

- Neglected potential *risk* in lotteries?
  - If one *had* cardinal background information, there would be valid concern about risk
  - But with Cardinal Ignorance, no basis to define “aversion to risk”
    - in particular, if Ordinal Dominance is accepted, the relevance of positional rank has already been denied; hence no basis to introduce aversion to positional risk.
    - consistency with first-order stochastic dominance extension a la Brandt et al.

Satisficing I

- Varieties of Impartial Ordinalism reflecting different "reliability stances".

Reliability Stance	Context Dependence	Choice rule
Satisficing	minimal	Essential Set
Hedging	minimal	Maximal Lottery
Optimizing	informative yet reliable	Pluri-Borda (et al.)
Sang-Froid	unrestricted	Borda

Satisficing II

- "Satisficing": aim at 'good enough' decisions:
  1. Minimize reliability hazards by minimizing context-dependence
  2. Accept opportunity cost of leaving potentially useful information aside.
  3. Good enough: adequate decisiveness ensured by JOA.

- Under Impartial Ordinalism, committed to choosing undominated alternative;
  so, information about dominated ones arguably *dispensable*.

Axiom

C satisfies **Independence of Dominated Alternatives (IDA)** *if*

$$C(A,\mu) = C(A',\mu) \text{ for all } A, A' \text{ such that } OU(A,\mu) \subseteq A' \subseteq A.$$

Satisficing III

- a is a Condorcet winner ("$a \in CW(A, \mu)$") if $M(a, b) \geq 0$ for all $b \in A$;
  - a is a **strict C. w.** ("$a \in CW^o(A, \mu)$") if $M(a, b) > 0$ for all $b \neq a$.
- (A, μ) is **strictly Condorcet ordered** if the majority tournament is strictly transitive
  - i.e. iff for all $A' \subseteq A$, $CW^o(A', \mu) \neq \varnothing$.

Proposition

If (A, μ) is strictly Condorcet ordered, C satisfies Ordinal Dominance and Independence of Ordinally Dominated Alternatives iff $C = CW$.

Proof. The strict Condorcet winner ordinally dominates the Condorcet loser. Proceed by induction. □

Satisficing IV

- Push still further: apply to C itself.

Axiom

C satisfies **Independence of Unchosen Alternatives (IUA)** *if*

$$C(A,\mu) = C(A',\mu) \text{ for all } A, A' : C(A,\mu) \subseteq A' \subseteq A.$$

- IUA standard axiom, esp. in tournament literature
  - e.g. Top Cycle, Pareto, Minimal Covering Set.
  - also called "Strong Superset Property"

- Within Imp. Ordinalism (JOA), IUA characterizes unique solution concept (generically), the **Essential Set** due to Dutta-Laslier (SCW 1999).

The Essential Set I

- p **maximal lottery** if $M(p, a) \geq 0$ for all $a \in A$.
  - a.k.a. randomized Condorcet winner
  - axiomatization as SSCR in Brandl et al. (2016).
  - $ML(A, \mu)$: their set; if unique, $\{\rho_{ML}\} := ML(A, \mu)$

- μ is **regular** if $|ML(B, \mu)| = 1$ for all non-empty $B \subseteq A$.
  - Generically, $|ML(A, \mu)| = 1$ by Laffond, Laslier, Le Breton (JET 1997).
- An alternative a is **essential** ($a \in ES(A, \mu)$) if it is contained in the support of some maximal lottery.
- If $|ML(A, \mu)| = 1$, equivalently

$$ES(A, \mu) = B_{\rho_{ML}}(A, \mu)$$

The Essential Set II

Theorem

Let C be a choice correspondence on the single-profile domain $2^{A^*}_{\setminus\varnothing} \times \{\mu\}$.

The Essential Set Correspondence C = ES satisfies Joint Ordinal Admissibility and Independence of Unchosen Alternatives.

Conversely, if μ *is regular and C satisfies Joint Ordinal Admissibility and Independence of Unchosen Alternatives, C = ES.*

- *Single-profile* result
  - Fixed finite number of individuals
  - Fixed feasible set and its subsets
  - Do not exploit restrictions obtained from special features of profiles (e.g. Pareto-dominated alternatives, clones)

- Corollary: any refinement of ES must violate IUA.

Cf. Axiomatization by Laslier (SCW 2000)

Theorem. *ES* is the smallest choice correspondence on the universal domain satisfying IUA, Borda-Regularity, Cloning Consistency, ...

- IUA shared with main result.
- "Cloning Consistency" defined in terms of M-equivalences
  - To exploit it, need to consider hypothetical/counterfactual agendas with and hypothetical/counterfactual preferences.
- Borda regularity says that
  "If $Borda(A, \mu) = A$, then $C(A, \mu) = A$"
  - ensures cardinal use of majority matrix.
  - Borda regularity entailed by conjunction of JOA and IUA; just special case of main result.

Generalization of Main Result

- C now SSCR, ie.
  - $C(A,\mu) \subseteq \Delta^A$ rather than $C(A,\mu) \subseteq A$

Axiom

(IUA) *For all* $p \in C(A,\mu)$, $p_a = 0$, *then* $C(A,\mu) = C(A \backslash a, \mu)$

Axiom

(OA) *For no* $p \in C(A,\mu)$, *there exists* $q \in \Delta^A$ *such that* q *is ordinally dominates* p.

- OA no less appealing for lotteries

Hedging: Extension to Stochastic Choice Rules II

Essential Reduction $C(A,\mu) = C(ES(A,\mu),\mu)$

Essential Support $\cup_{p \in C(A,\mu)} \operatorname{supp} p = ES(A,\mu)$.

Theorem

Let C be a **convex-valued** *SSCR on the single-profile domain $2^{A^*}_{\setminus\varnothing} \times \{\mu\}$ and μ regular. C satisfies IUA and OA if and only if it satisfies Essential Support and Essential Reduction.*

Corollary

If the SSCR C is **convex-valued and anti-convex-valued** *and μ regular, C satisfies IUA and OA iff $C = co(ES)$.*

An Impartially Ordinalist Rationale for Max Lotteries

- Brandl-Brandt-Seedig (EMA 2016) provide two closely related characterizations of ML.

- C has **universal domain** if $\mathcal{A} = \{A \subset A^*, A \text{ finite}\}$ for some infininite A, and $\mathcal{D} = \cup_{A \in \mathcal{A}} \{A\} \times \mathcal{L}(A)$.
  - C defined for arbitrarily large finite agendas and arbitrary profiles of linear orders.

Theorem (BBS 2016)

Let C be an SSCR satisfying convex-valuedness, generic single-valuedness, continuity (uhc) on the universal domain.

C satisfies Condorcet Consistency, Cloning-Invariance and Population Consistency if and only if $C = ML$.

Hedging: Extension to Stochastic Choice Rules IV

- Since OA&IUA implies Condorcet Consistency, this yields:

Theorem

Let C be an SSCR satisfying convex-valuedness, generic single-valuedness, continuity (uhc) on the universal domain.

C satisfies Ordinal Admissibility, IUA, Cloning-Invariance and Population Consistency if and only if $C = ML$.

- Shorter proof possible? All assumptions needed?

Hedging: Extension to Stochastic Choice Rules V

Issues?

1. Why lotteries better than their constituents?
    - violation of Anti-Convexvaluedness
    - GBR rationale: robust ('ambiguity-averse') choice under *complete agnosticism about 'correct' GBR index weights*
        - $ML(A, \mu) = \max_p \min_\rho M(p, \rho)$
        - So ML constitutes *agnostic baseline* within Impartial Ordinalism
2. Is such *agnosticism* the last word?

A Glimpse Beyond Satisficing: Lopsided Preferences I

- $A = \{1, ..., m\}$
- μ concentrated on single-peaked preferences
- μ is **(upward) lopsided** if, for any ranking $\succ$ in $\text{supp}\,\mu$ with top θ,

$$\text{if } x < \theta < y, \text{ then } y \succ x.$$

  - thus: top determines entire preference ordering.
  - single-peaked and single-crossing domain.

- Story: "**Necessary Evil**" – do as much as necessary, as little as possible;
  divergent perceptions on what is necessary.

A Glimpse Beyond Satisficing: Lopsided Preferences II

- If **know only tops** plus single-peakedness known (partial order), choice of median top is compelling.
  - Frugal aggregation

- If **additionally learn** that subtop preferences are **upward lopsided**, this should shift social choice upward.
  - Condorcet criterion ignores this information
  - But GBRs can incorporate this info reliably.

- E.g. **Pluri-Borda Rule**: GBR B_ρ with ρ given by $\rho(\mu) = \pi(\mu)$, where π is distribution of tops ("plurality index").
  - The Pluri-Borda rule satisfies Independence of Pareto Dominated Alternatives and Independence of Exchangeable Clones.

A Glimpse Beyond Satisficing: Lopsided Preferences III

- For simplicity, $A = [0, 1]$; preferences upward lopsided.

Proposition

With μ uniform , Pluri-Borda$(A, \mu) =$ Borda$(A, \mu) = 2/3$.

Proposition

With μ continuous, Pluri-Borda(A, μ) selects $2/3$-quantile; while Borda(A, μ) may select any quantile in $(0, 1)$

A Glimpse Beyond Satisficing: Lopsided Preferences IV

Remark on associated social orderings:

- The Condorcet ordering is given by the ranking of the median voter (by single crossing)
  - Hence $a = 1$ is ranked above all a below the median $\frac{1}{2}$; discontinuity at $\frac{1}{2}$.

- By contrast, the Borda=Pluri-Borda scores are quadratic (with μ uniform).
  - Hence if alternatives are ranked accordingly, $a = 1$ is indifferent to $\frac{1}{3}$, and the median is indifferent to $\frac{5}{6}$.

Thank you for your attention !