Which Voting Rule Is More Manipulable?
Results from Simulation Studies

Tobias Lindner MdB  Klaus Nehring  Clemens Puppe | Freudenstadt, 11 September 2011
A voting rule $F$ on a set $X$ of social states selects a (unique) $x \in X$ for every profile $(\succ^1, \succ^1, \ldots, \succ^n)$ of preferences over $X$.

$F$ is strategy-proof if $F(\succ^1, \ldots, \succ^i, \ldots, \succ^n) \succeq_i F(\succ^1, \ldots, \succ^i', \ldots, \succ^n)$

Theorem (Gibbard-Satterthwaite)

If $\#X \geq 3$, then the only strategy-proof voting rule on an unrestricted preference domain is the dictatorship of one individual.
Manipulability of Voting Rules

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The Median Voter Theorem

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Suppose that social states can be ordered from left to right such that all preferences are **single-peaked**, then there exist non-degenerate strategy-proof voting rules. E.g. choosing the median of the individual peaks defines an anonymous, neutral and strategy-proof voting rule.
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Unfortunately, for many economically relevant domains all strategy-proof voting rules are dictatorial, even under generalized single-peakedness.
Nehring/Puppe (2007/2010) demonstrate the existence of non-dictatorial and strategy-proof voting rules for classes of generalized single-peaked preferences. Unfortunately, for many economically relevant domains all strategy-proof voting rules are dictatorial, even under generalized single-peakedness.
The Median Voter Theorem revisited


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The Median Voter Theorem revisited

- What happens in such domains?
- If all non-dictatorial voting rules are manipulable, which of those are less manipulable than others? (Throughout, we will assume that $F$ respects unanimity: if all individuals happen to agree that $x$ is best, then $F(\ldots) = x$.)
- What is a good notion of “less manipulable”?
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Voting on the Simplex

Setting of the Simulation Studies

Preliminary Results

Freudenstadt, 11 September 2011
Agenda

1. Voting on the Simplex
   - Peaks-Only Preference Aggregation
   - Voting Rules

2. Setting of the Simulation Studies
   - Basic Settings and Parameters
   - Variables Measuring Manipulability

3. Preliminary Results
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3 Preliminary Results
The (K,L)-Simplex

**Definition ((K,L)-Simplex)**

A (K,L)-Simplex is the set

\[ X = \{ x \in \mathbb{R}^K : \sum_{j=1}^{K} x_j = L, x_j \geq 0 \} \]

**Economic Applications:**

- budget allocation problem where \( x_j \) is the money amount spent on (public) good \( j \) and \( L \) the total budget,
- aggregation of probability distributions (\( L = 1 \)), where \( x_j \) is the probability of \( j \)

If \( K \geq 3 \), all strategy-proof voting rules on \( X \) are dictatorial, even under generalized single-peaked preferences (See Nehring and Puppe 2010).
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Peaks-Only Preference Aggregation

- Let \( d(x, y) = \frac{1}{2} \sum_{j=1}^{K} |x_j - y_j| \) denote the **distance** between \( x \) and \( y \), e.g. in the public goods context the number of dollars that have to be shifted to get from allocation \( x \) to \( y \).
- We assume that an agent \( i \) has (generalized) single-peaked preferences on \( X \) (with respect to that distance) and submits a **proposal** \( w^i \in X \) (the peak).
- Denote by \( p(w) \) the number of agents who proposed \( w \).

**Definition (Voting Rule on the Simplex)**

A voting rule on the simplex is a mapping

\[
F(w^1, w^2, \ldots, w^n) = x \in X
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which assigns to each profile of peaks \((w^1, w^2, \ldots, w^n)\) an element \( x \) in the simplex.
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3. Preliminary Results
The Midpoint Rule

- A midpoint minimizes the sum of distances to every peak.

**Definition (Midpoint)**

An allocation $m(p) \in X$ is a **midpoint** if

$$m(p) = \text{argmin}_{x \in X} \sum_{w \in X} p_w d(x, w)$$

- Let $M(p)$ denote the set of midpoints. Evidently, $M(p)$ need not be a singleton but it is always non-empty.
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Natural single valued selection for Midpoint Rule: the “shadow voter selection”. Give every point in the simplex in addition a mass of $\epsilon$ (shadow voter): $\tilde{p}(w) := \epsilon + p(w)$

**Theorem**

For $\lim_{\epsilon \to 0}$ we have $\tilde{M}(p) \subseteq M(p)$ and $|\tilde{M}(p)| = 1$, i.e. a unique midpoint.
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Limited Manipulability of the Set of Midpoints

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**Theorem**

*Under the midpoint rule, an agent cannot move the midpoint (metrically) closer to his/her own peak by misrepresentation.*
This rule selects the mean of peaks:

**Definition (Mean Rule)**

\[ F^{\text{Mean}}(w^1, w^2, \ldots, w^n) := (\bar{w}_1, \bar{w}_2, \ldots, \bar{w}_K) \]

where

\[ \bar{w}_j = \frac{1}{n} \sum_{i=1}^{n} w^i_j \]
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**Definition (NMedian Rule)**

\[ F_{\text{NMedian}}(w^1, w^2, \ldots, w^n) := \left( \frac{w_1^{\text{med}}}{c} L, \frac{w_2^{\text{med}}}{c} L, \ldots, \frac{w_K^{\text{med}}}{c} L \right) \]

where \( w_j^{\text{med}} \) is the median of coordinate \( j \) and \( c = \sum_{j=1}^{K} w_j^{\text{med}} \).
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The “Sequential Median Rule” takes the coordinate wise median and adjusts it to the simplex in a fixed order:

**Example**

Let $K = 3$, $L = 21$ and $w_{1}^{med} = 7$, $w_{2}^{med} = 10$, $w_{3}^{med} = 6$. Note that $\sum_{j=1}^{3} w_{j}^{med} > L$. The $w_{j}^{med}$ are adjusted in ascending order $(1, 2, 3)$.

$$F^{\text{SeqMedian}}(7, 10, 6) = (7, 10, 4)$$
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Evidently, the SeqMedian Rule depends on the order of coordinates.
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   - Basic Settings and Parameters
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3. Preliminary Results
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- number of agents manipulating
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- number of manipulations
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Effect of Manipulation

- distance between outcome under true peaks and outcome under announced peaks (after convergence)
- average deviation (of an agent) between true and announced peaks
- sum of utility gains/losses
- comparison of sum of utilities to utilitarian maximum and expected utility under random dictatorship
  - normalize an agent’s utility such that $u^i(\vec{0}) := 0$ and $u^i(\vec{w}) := 1$
  - compute $s(x) := \sum_{i} u^i(x)$
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  - for later: “utility loss” = loss in this scale
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- sample size 1000
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- number of agents: 3 to 45
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<tr>
<th>MANIPULATIONS</th>
<th>MAXUTILGAIN</th>
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<tr>
<td>MEAN</td>
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<td>MIDPOINT</td>
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<td>NMEAN</td>
<td>2,4</td>
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<td>MIDPOINT</td>
<td>4,86%</td>
<td>4,51%</td>
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<td>2.96%</td>
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<td>0.40%</td>
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Freudenstadt, 11 September 2011

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NMedian: Distance

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Observations

- Midpoint seems to be invariant against variations of the CES-$\rho$ while NMedian does not.
- Similarities between rules which use the median (number of agents manipulating, manipulations).
- Mean is highly manipulable in all situations.
- Good welfare properties of the Midpoint and the NMedian.
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