JUDGMENT AGGREGATION

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1 A new brand of aggregation theory

It is a commonplace idea that collegial institutions generally make better decisions than those in which a single individual is in charge. This optimistic view, which can be traced back to French Enlighment social theorists like Rousseau and Condorcet, permeates today's western judiciary organization, which normally entrusts collegial courts with the competence to rule on the more complex cases; think of constitutional courts like the U.S. Supreme Court with its nine judges. However, the following, by now classic example from legal theory challenges the orthodoxy. A plaintiff has brought a civil suit against a defendant, alleging a breach of contract between them. The court is composed of three judges A, B and C, who will determine whether or not the defendant must pay damages to the plaintiff (d or $\neg d$). The case brings up two issues, i.e., whether the contract was valid or not (v or $\neg v$), and whether the defendant was or was not in breach of it (b and $\neg b$). Contract law stipulates that the defendant must pay damages if, and only if, the contract was valid and he was in breach of it. Suppose that the judges have the following views of the two issues and accordingly of the case:

A	v	$\neg b$	$\neg d$
B	$\neg v$	b	$\neg d$
C	v	b	d

In order to rule on the case, the court can either directly collect the judges' recommendations on it, or collect the judges' views of the issues and then solve the case by applying contract law to these data. If the court uses majority voting, the former, *case-based* method delivers $\neg d$, whereas the latter, *issue-based* method returns first v and b, and then d. This elegant example is due to legal theorists Kornhauser and Sager (1993). They describe as a *doctrinal paradox* any similar occurrence in which the two methods give conflicting answers. What makes the discrepancy paradoxical is that each method is commendable on some ground, i.e., the former respects the judges' final views, while the latter provides the court with a rationale, so one would wish them always to be compatible. The legal literature has not come up with a clear-cut solution, although the issue-based method generally attracts more sympathy (see Nash, 2003). This persisting difficulty casts doubt on the belief that collegial courts would be wiser than individual ones. Clearly, with a single judge, the two methods coincide unproblematically.

An entire body of work, now referred to as *judgment aggregation theory,* has grown out of Kornhauser and Sager's doctrinal paradox. As an intermediary step, their problem was rephrased by political philosopher Pettit (2001), who wanted to make it both more widely applicable and more analytically tractable. What he calls the *discursive dilemma* is, first of all, the generalized version of the doctrinal paradox in which a group, whatever it is, can base its decision on either the *conclusion-based* or the *premiss-based* method, whatever the substance of conclusions and premisses. What holds of the court equally holds of a political assembly, an expert committee, and many deliberating groups; as one of the promoters of the concept of deliberative democracy, Pettit would speculatively add political society as a whole. Second, and more importanly for our purposes, the discursive dilemma shiftes the stress away from the conflict of methods to *the logical contradiction within the total set of propositions that the group accepts*. In the previous example, with $d \longleftrightarrow v \wedge b$ representing contract law, the contradictory set is

$$\{v, b, d \longleftrightarrow v \wedge b, \neg d\}\,.$$

Trivial as this shift seems, it has far-reaching consequences, because all propositions are now being treated alike; indeed, the very distinction between premisses and conclusions vanishes. This may be a questionable simplification to make in the legal context, but if one is concerned with developing a general theory, the move has clear analytical advantages. It may be tricky to classify the propositions into the two groups and to define what the two methods consist of. It is definitely simpler, and and it is arguably sufficient, to pay attention to sets of accepted propositions - more briefly *judgment sets* - and inquire when and why the collective ones turn out to be inconsistent, given that the individual ones are consistent by assumption. This is already the problem of judgment aggregation.

In a further step, List and Pettit (2002) introduce an aggregation mapping F, which takes profiles of individual judgment sets $(A_1, ..., A_n)$ to collective judgment sets A, and subject F to axiomatic conditions which they demonstrate are logically incompatible. Both the proposed formalism and impossibility conclusion are in the vein of social choice theory, but they are directed at the discursive dilemma, which the latter theory could not explain in its own terms. At this stage, the new theory exists in full, having defined its object of study - the F mapping, or *collective judgment function* - as well as its method of analysis - it consists in axiomatizing F and investigating subsets of axioms to decide which result in an impossibility and which, to the contrary, support well-behaved rules (such as majority voting).

List and Pettit's impossibility theorem was shortly succeeded, and actually superseded, by others of growing sophistication, due to Pauly and van Hees (2006), Dietrich (2006), Dietrich and List (2007a), Mongin (2008), Nehring and Puppe (2008, 2010), Dokow and Holzman (2009, 2010a and b), Dietrich and Mongin (2010). This lengthy, but still incomplete list, should be complemented by two papers that contributed differently to the progress of the field. Elaborating on earlier work in social choice theory by Wilson (1975), and in a formalism that still belongs to that theory, Nehring and Puppe (2002) inquired about the *agendas* of propositions for which axiomatic conditions clash. Agendas are the rough analogue of preference domains in social choice theory. This concept has raised to prominence in mature judgment aggregation theory, and Nehring and Puppe's problem was solved in full generality by Dokow and Holzman (2010a). On a different score, Dietrich (2007) showed that the whole formalism of the theory could be deployed without making reference to any specific logical calculus. Only a few elementary properties of the formal language and of the logic need assuming for the theorems to carry through. The so-called *general logic* states the requisits (see Dietrich and Mongin, 2010, for an up-to-date version). The first papers relied on propositional calculi, which is now recognized to be unnecessarily restrictive.

Section 2 provides a logical, and more specifically syntactical, framework for the F function, using the general logic as a background. It states the axiomatic conditions on F that have attracted most attention, i.e., systematicity, independence, monotonicity and unanimity-preservation. The issue of agendas arises in section 3, which presents an impossibility theorem in three variant forms, due to Nehring and Puppe, Dokow and Holzman, and Dietrich and Mongin respectively. This is but a sample of the work done in the theory at large; more material can be found in List and Puppe's (2009) and Mongin and Dietrich's (2010) accounts. Section 4 sketches a comparison with social choice theory, centering on Dietrich and List (2007a)'s proof of Arrow's (1951) impossibility theorem in terms of the new apparatus.

Two topics are omitted here. The first is *probability aggregation*, which gave rise to a specialized literature already long ago (see Genest and Zidekh's 1986 survey for the main results). Both commonsense and the traditional philosophy of judgment classify judgments into certain and uncertain ones, so this topic should really be part of judgment aggregation theory. However, we rely here on the habit taken by its contributors to view judgment only through the lenses of logic or related analytical tools. The second omission is *belief merging*, a topic which has recently emerged among theoretical computer scientists. They investigate belief sets, which are akin to the judgment sets defined here, and aggregate them in terms of algorithmic rules that suggest ways of escape from the above impossibility theorems (for an argument to this effect, see Pigozzi, 2006).

2 A logical framework for judgment aggregation theory

By definition, a *language* $\mathcal{L}$ for judgment aggregation theory is any set of formulas $\varphi, \psi, \chi, \ldots$ that is constructed from a set of logical symbols $\mathcal{S}$ containing $\neg$, the Boolean negation symbol, and that is closed for this symbol (i.e., if $\varphi \in \mathcal{L}$, then $\neg\varphi \in \mathcal{L}$). In case $\mathcal{S}$ contains other elements, such as symbols for the remaining Boolean connectives or modal operators, they satisfy the appropriate closure properties. A *logic* for judgment aggregation theory is any set of axioms and rules that regulates the inference relation $\vdash$ on $\mathcal{L}$ and associated technical notions - such as logical truth and contradiction, consistent and inconsistent sets - in accordance with the general logic. Informally, the main requisits are that $\vdash$ be monotonic and compact, and that any consistent set of formulas can be extended to a complete consistent set. ($S \subset \mathcal{L}$ is *complete* if, for all $\varphi \in \mathcal{L}$, either $\varphi \in S$ or $\neg\varphi \in S$.) Monotonicity means that inductive logics are excluded from consideration, and compactness (which is needed only in specific proofs) that some deductive logics are. The last requisit is the standard Lindenbaum extendability property.

Among the many calculi that enter these framework, propositional examples stand out. They need not be classical, i.e., $\mathcal{S}$ may contain modal operators, like those of deontic, epistemic and conditional logics, each of them leading to a potentially relevant application. Each of these extensions should be double-checked, because some fail compactness (e.g., Heifetz and Mongin's 2001 probabilistic logic). Although this may not be so obvious, first-order calculi are also permitted. When it comes to them, $\mathcal{L}$ is the set of *closed* formulas - those without free variables - and the only question is whether $\vdash$ on $\mathcal{L}$ complies with the general logic.

In $\mathcal{L}$, a subset X is fixed to represent the propositions that are in question for the group; this is the *agenda*, one of the novel concepts of the theory and currently its main focus of attention. In all generality, X need only to be non-empty, with at least one contingent formula, and to be closed for negation. The discursive dilemma reconstruction of the court example leads to the agenda

$$\overline{X} = \{v, b, d, d \leftrightarrow v \wedge b, \neg v, \neg b, \neg d, \neg(d \leftrightarrow v \wedge b)\}.$$

The theory represents judgments by subsets $B \subset X$, which are initially unrestricted. These *judgment sets* - another notion specific to the theory - will be denoted by $A_i, A'_i, ...$ when they belong to the individuals $i = 1, ...n$, and by $A, A', \ldots$ when they belong to the group as such. A formula φ from one of these sets represents a proposition, in the ordinary sense of a semantic object endowed with a truth value. If φ is used also to represent a judgment, in the sense of a cognitive operation, this is in virtue of the natural interpretive rule:

> *(R) i judges that φ iff $\varphi \in A_i$, and the group judges that φ iff $\varphi \in A$.*

Standard logical properties may be applied to judgment sets. For simplicity, we only consider two cases represented by two sets of judgments sets:

- the unrestricted set 2^X;
- the set D of *consistent* and *complete* judgment sets (consistency is defined by the general logic and completeness is as above, but relative to X).

Thus far, the theory has been able to relax the completeness, but not the consistency assumption (see, e.g., Dietrich and List, 2008).

The last specific concept is the *collective judgment function* F, which associates a collective judgment set to each profile of judgment sets for the n individuals:

$$A = F(A_1, \ldots, A_n).$$

The domain and range of F can be defined variously, but we restrict attention to $F : D^n \to 2^X$, our baseline case, and $F : D^n \to D$, our target case, since it means that the collective sets obey the same stringent logical constraints as the individual ones. The present framework captures the simple voting rule of the court example, as well as less familiar examples. Formally, define *formula-wise majority voting* as the collective judgment function $F_{maj} : D^n \to 2^X$ such that, for every profile $(A_1, \ldots, A_n) \in D^n$,

$$F_{maj}(A_1, ..., A_n) = \{\varphi \in X : |\{i : \varphi \in A_i\}| \geq q\},$$
$$\text{with } q = \tfrac{n+1}{2} \text{ if } n \text{ is odd and } q = \tfrac{n}{2} + 1 \text{ if } n \text{ is even.}$$

Here, the range is not D because there can be unbroken ties, and so incomplete collective judgment sets, when n is even. More strikingly, for many agendas, the range is not D either when n is odd, because there are inconsistent collective judgment sets, as the court example neatly shows. By varying the value of q between 1 and n in the definition, one gets specific quota rules F^q_{maj}. One would expect inconsistency to occur with low q, and incompleteness with large q. Nehring and Puppe (2002, 2008) and Dietrich and List (2007b) investigate the F^q_{maj} in detail. Generally, the framework of agendas, judgment sets and collective judgment functions proves extremely convenient for a technical study of voting rules.

Having defined and exemplified F functions, we introduce some axiomatic properties they may satisfy.

Systematicity. For all formulas $\varphi, \psi \in X$ and all profiles $(A_1, \ldots, A_n)$, $(A'_1, \ldots, A'_n)$, if $\varphi \in A_i \Leftrightarrow \psi \in A'_i$ for every $i = 1, \ldots, n$, then

$$\varphi \in F(A_1, \ldots, A_n) \Leftrightarrow \psi \in F(A'_1, \ldots, A'_n).$$

Independence. For every formula $\varphi \in X$ and all profiles $(A_1, \ldots, A_n)$, $(A'_1, \ldots, A'_n)$, if $\varphi \in A_i \Leftrightarrow \varphi \in A'_i$ for every $i = 1, \ldots, n$, then

$$\varphi \in F(A_1, \ldots, A_n) \Leftrightarrow \varphi \in F(A'_1, \ldots, A'_n).$$

Monotonicity. For every formula $\varphi \in X$ and all profiles $(A_1, \ldots, A_n)$, $(A'_1, \ldots, A'_n)$, if $\varphi \in A_i \Rightarrow \varphi \in A'_i$ for every $i = 1, \ldots, n$, with $\varphi \notin A_j$ and $\varphi \in A'_j$ for at least one j, then

$$\varphi \in F(A_1, \ldots, A_n) \Rightarrow \varphi \in F(A'_1, \ldots, A'_n).$$

Unanimity preservation. For every formula $\varphi \in X$ and every profile $(A_1, \ldots, A_n)$, if $\varphi \in A_i$ for every $i = 1, \ldots, n$, then $\varphi \in F(A_1, \ldots, A_n)$.

By definition, F is a *dictatorship* if there is a j such that, for every profile $(A_1, \ldots, A_n)$,

$$F(A_1, \ldots, A_n) = A_j.$$

Given the unrestricted domain, there can only be one such j, to be called the *dictator.* The last property is

Non-dictatorship. F is not a dictatorship

It is routine to check that F_{maj} satisfies all the list. Systematicity means that the group, when faced with a profile of individual judgment sets, gives the same answer concerning a formula as it would give concerning a *possibly different* formula, when faced with a *possibly different* profile, supposing that the individual judgments concerning the first formula in the first profile are the same as those concerning the second formula in the second profile. Independence amounts to restricting this requirement to $\varphi = \psi$. Thus, it eliminates one aspect of Systematicity - i.e., the identity of the formula does not matter - while preserving another aspect - i.e., the collective judgment of φ depends only on individual judgments of φ. That is, by Independence, the collective set A is defined *formula-wise* from the individual sets $A_1, \ldots, A_n$.

Systematicity was the condition underlying List and Pettit's (2002) impossibility theorem, but henceforth, the focus of attention shifted to Independence. The former has little to say for itself except that many voting rules satisfy it, but the latter can be rephrased, and thus defended, as a *non-manipulability* condition. (If a referee is in charge of defining the agenda, it would be impossible for him to upset the collective judgment on a formula by adding or withdrawing other formulas.) Some writers consider Monotonicity as a natural addition to Independence. It requires that, when a collective result favours a subgroup's judgment, this still holds if more individuals join the subgroup. It can be defended in terms of democratic responsiveness, though perhaps not so obviously as the last conditions, i.e., Unanimity-preservation and Non-dictatorship.

The problem that has gradually raised to the fore is to characterize – in the mathematical sense of necessary and sufficient conditions – the agendas X such that no $F : D^n \to D$ satisfies Non-dictatorship, Independence, and Unanimity-preservation. There is a variation of this problem with Monotonicity as a further axiomatic condition. The next section provides the answers.

3 An impossibility theorem in three forms

The promised answers depend on further technical notions. First, a set of formulas $\mathcal{S} \subset \mathcal{L}$ is called *minimally inconsistent* if it is inconsistent and all its proper subsets are consistent. With a classical propositional calculus, this is the case for

$$\{v, b, d \leftrightarrow v \wedge d, \neg d\},$$

but not for

$$\{\neg v, \neg b, d \leftrightarrow v \wedge b, d\}.$$

Second, for $\varphi, \psi \in X$, it is said that φ *conditionally entails* ψ – denoted by $\varphi \vdash^* \psi$ – if $\varphi \neq \neg\psi$ and there is some minimally inconsistent $Y \subset X$ with $\varphi, \neg\psi \in Y$. This is trivially equivalent to requiring that $\{\varphi\} \cup Y' \vdash \psi$ holds for some minimal auxiliary set of premisses Y' that is contradictory neither with φ, nor with $\neg\psi$.

Now, an agenda X is said to be *path-connected* (another common expression is *totally blocked*) if, for every pair of formulas $\varphi, \psi \in X$, there are formulas $\varphi_1, \ldots, \varphi_k \in X$ such that

$$\varphi = \varphi_1 \vdash^* \varphi_2 \vdash^* \ldots \vdash^* \varphi_k = \psi.$$

Loosely speaking, agendas with this property have many, possibly roundabout logical connections. Finite agendas can be represented by directed graphs: the formulas φ, ψ are the nodes and there is an arrow pointing from φ to ψ for each conditional entailment $\varphi \vdash^* \psi$. The court agenda $\overline{X}$ is path-connected, as the picture below of conditional entailments illustrates (it does not represent all existing conditional entailments, but sufficiently many for the reader to check the claim).

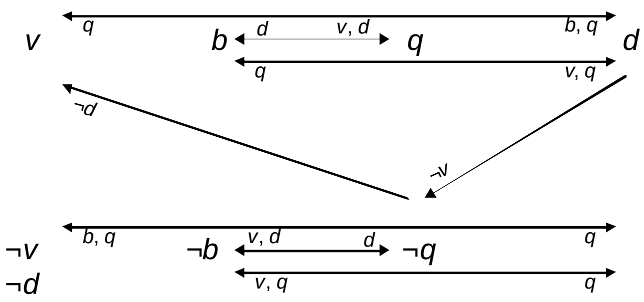

Figure 1: The court agenda in the discursive dilemma version

(Here and in the next figures, an arrow pointing from one formula to another means that the former conditionally entails the latter, and the unbolded formulas near the head of the arrow are a choice of auxiliary premisses; $d \leftrightarrow v \wedge b$ is abridged as q.)

Now, we are in a position to state a version of our sample theorem (see Dokow and Holzman, 2010a, and Nehring and Puppe, 2010a; it originates in Nehring and Puppe, 2002). From now we assume that $n \geq 2$.

Theorem (first form). If X is path-connected, then no $F : D^n \to D$ satisfies Non-dictatorship, Unanimity preservation, Monotonicity and Independence. The agenda condition is also necessary for this conclusion.

As an illustration of the sufficiency part, let us take $\overline{X}$ and F_{maj}, assuming that n is odd, so that F_{maj} has range D if and only if $F_{maj}(A_1, ..., A_n)$ is consistent for all profiles $(A_1, ..., A_n)$. The court example is a profile to the contrary, hence it shows that D is not the range of F_{maj}. The theorem entails the same negative conclusion, since F_{maj} satisfies the four axioms and $\overline{X}$ has just been shown to be path-connected. This abstract deduction supersedes the empirical finding by making it a particular case of a generality. The discursive dilemma will occur any time the agenda is path-connected; then the axioms become incompatible with collective consistency. Conversely, when the agenda is not path-connected, there is no discursive dilemma. This important additional result corresponds to the necessity part of the theorem, which we do not illustrate here.

As it turns out, Monotonicity can be dropped from the list of axioms if the agenda is required to satisfy a further condition. Let us say that X is *even-number negatable* if there is a minimally inconsistent set of formulas $Y \subseteq X$ and there are distinct $\varphi, \psi \in Y$ such that $Y_{\neg\{\varphi,\psi\}}$ is consistent, where the set $Y_{\neg\{\varphi,\psi\}}$ is obtained from Y by replacing φ, ψ by $\neg\varphi, \neg\psi$ and keeping the other formulas unchanged. This seems to be an unpalatable condition, but it is not demanding, as $\overline{X}$ illustrates: take

$$Y = \{v, b, d, \neg(d \leftrightarrow v \wedge b)\} \text{ and } \varphi = v, \psi = b,$$

and there are alternative choices of Y. The next result was proved by Dokow and Holzman (2010a) as well as, for the sufficiency part, by Dietrich and List (2007a).

Theorem (second form). If X is path-connected and even-number negatable, then no $F : D^n \to D$ satisfies Non-dictatorship, Unanimity-preservation, and Independence. If $n \geq 3$, the agenda conditions are also necessary for this conclusion.

A further step of generalization is available. Unlike the work reviewed so far, it is motivated *not by the discursive dilemma, but by the doctrinal paradox*, and it is specially devised to clarify the premiss-based method, which is often proposed as a solution to this paradox (see Pettit, 2001, and Nash, 2003). Formally, we define the *set of premisses* to be a subset $P \subseteq X$, requiring only that it be non-empty and closed for negation, and we reconsider the framework to account for the difference between P and its complement $X \setminus P$. Adapting the axioms, we define

Independence on premisses: same statement as for Independence, but holding only for every $p \in P$.

Non-dictatorship for premisses: there is no $j \in \{1, \ldots, n\}$ such that $F(A_1, \ldots, A_n) \cap P = A_j \cap P$ for every $(A_1, \ldots, A_n) \in D^n$.

Now revising the agenda conditions, we say that X is *path-connected in* P if, for every pair $p, p' \in P$, there are $p_1, \ldots, p_k \in P$ such that

$$p = p_1 \vdash^* p_2 \vdash^* \ldots \vdash^* p_k = p',$$

and that X is *even-number negatable in* P if there are $Y \subseteq X$ and $\varphi, \psi \in Y$ as in the above definition for being even-number negatable, except that "$\varphi, \psi \in Y \cap P$" replaces "$\varphi, \psi \in Y$" (i.e., the negatable pair consists of premisses). These new conditions can be illustrated by court agendas in the doctrinal paradox style.

If we stick to the agenda $\overline{X}$, the subset

$$\overline{P} = \{v, b, d \leftrightarrow v \wedge b, \neg v, \neg b, \neg(d \leftrightarrow v \wedge b)\}$$

best captures the judges' sense of what counts as a premiss. However, the following construal may be more to the point. Suppose that judges do not vote on the law, but rather take it for granted and apply it - a realistic case from legal theory (see Kornhauser and Sager, 1993). We model this, first by discarding the formula $d \leftrightarrow v \wedge b$ from the agenda, which thus reduces to

$$\overline{\overline{X}} = \{v, b, d, \neg v, \neg b, \neg d\},$$

and second by including this formula into the inference relation, now defined by

$$S \vdash_{d \leftrightarrow v \wedge b} \psi \text{ if and only if } S \cup \{\varphi, d \leftrightarrow v \wedge b\} \vdash \psi.$$

In this alternative model, the set of premisses reads as

$$\overline{\overline{P}} = \{v, b, \neg v, \neg b\}.$$

Technically, the two construals are wide apart: $\overline{X}$ is both path-connected and even-number negatable in $\overline{P}$, whereas $\overline{\overline{X}}$ is even-number negatable, but not path-connected, in $\overline{\overline{P}}$, thus failing the more important agenda condition. The next two pictures - the first for $\overline{P}$ and the second for $\overline{\overline{P}}$ - illustrates the stark contrast.

(The first picture represents sufficiently many conditional entailments in $\overline{P}$ for the conclusion that $\overline{X}$ is path-connected in $\overline{P}$, and the second represents all conditional entailments in $\overline{\overline{P}}$, which are too few for $\overline{\overline{X}}$ to be path-connected in $\overline{\overline{P}}$.)

Having illustrated the new definitions, we state the result, due to Dietrich and Mongin (2010), which put them to use.

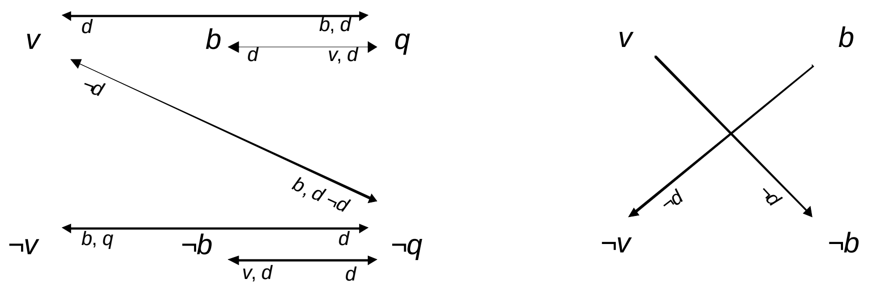

Figure 2: Two sets of premisses $\overline{P}$ (left) and $\overline{\overline{P}}$ (right) for the court agenda in the doctrinal paradox version

Theorem (third form). If X is path-connected and even-number negatable in P, there is no $F : D^n \to D$ that satisfies Non-dictatorship on premisses, Independence on premisses and Unanimity preservation. If $n \geq 3$, the agenda conditions are also necessary for this conclusion.

Note carefully that Unanimity-preservation retains its initial form, unlike the other two conditions. If it were also restricted to premisses, one would check that no impossibility follows. Thus, the statement is best interpreted as an impossibility theorem for the premiss-based method, granting the normatively defensible constraint that unanimity should be preserved on all formulas. This is like adding a whiff of conclusion-based method. Anyone who accepts this addition is committed to the unpleasant result that the premiss-based method is, like its rival, fraught with difficulties. As with the previous forms of the theorem, solutions are to be sought on the agenda's side by relaxing the even-number negatibility or - more relevantly - the path-connectedness condition. The $\overline{\overline{X}}, \overline{\overline{P}}$ reconstruction of the doctrinal paradox illustrates the way out; observe that F_{maj} is well-behaved in this case.

Legal interpretations aside, the third form of the theorem is more assertive than the second one. This is seen by considering $P = X$, a permitted limiting case. Having explored our sample theorem in full generality, we move to the comparative topic of this review.

4 A brief comparison with social choice theory

Judgment Aggregation Theory has clearly been inspired by social choice theory, and two legitimate questions are, how it formally relates, and what it eventually adds, to its predecessor. The F mapping resembles the *collective preference function* G, which takes profiles of individual preference relations to preference relations for the group. (The official terminology for G, i.e., the "social welfare function", is very misleading.) The normative properties posited on judgment sets are evocative of those, like transitivy, one encounters for preference relations, and the axiomatic conditions on F are most clearly related to those put on G: neutrality corresponds to systematicity, independence of irrelevant alternatives to independence, positive responsiveness to monotonicity, and the Pareto principle to unanimity preservation, not to mention the shared requisite of non-dictatorship.

Conceptually, a major difference lies in the *object* of the two aggregative processes. A *judgment*, as the acceptance or rejection of a proposition, is more general than a *preference* between two things. According to a plausible account, an agent, whether individual or collective, prefers x to y if and only if he judges that x is preferable to y, i.e., accepts the proposition that x is preferable to y. This clarifies the claim that one concept encompasses the other, but how does it translate into the respective formalisms?

We answer this question by following Dietrich and List's (2007a) footsteps. They derive a version of Arrow's impossibility theorem in which the individuals and the group express only *strict* preferences on the set of alternatives Z, and these preferences are assumed to be not only transitive but also *complete*. Although this is a restrictive case from the viewpoint of social choice theory, it deserves being studied, because it elegantly clarifies the general connection of that theory with the new one. The first step is to turn the G mappings defined on the domain of preferences into particular cases of F. To do so, one takes a first-order language $\mathcal{L}$ whose elementary formulas xPy express "x is strictly preferable to y", for all $x, y \in Z$, and one defines a logic for $\mathcal{L}$ by enriching the inference relation $\vdash$ of first-order logics with the axioms expressing the asymmetry, transitivity and completeness of P. The conditions for general logic hold. Now, if one takes the agenda X to be the set of elementary formulas of $\mathcal{L}$ and defines the set of judgment sets D from this choice, it is possible to associate with each given G, an $F : D^n \to D$ having the same informal content. The next step is to make good the results of judgment aggregation theory. Dietrich and List show that X satisfies the agenda conditions of the theorem of last section (second form). To finish the proof that Arrow's axiomatic conditions on G are incompatible, it is enough to check that they translate into the corresponding ones on F, and apply the the theorem (in its sufficiency part).

A more roundabout construction takes care of Arrow's general case, in which the individuals and the group can express indifferences (see Dokow and Holzman, 2010b). However, this is a technicality, and Dietrich and List's derivation sets a standard for the ensuing work. Generally speaking, each theorem in judgment aggregation theory can deliver a theorem in social choice theory by a suitable choice of a preference language and of a preference logic, followed a suitable translation of the axioms. The work done along this line has hardly begun, but one can expect that it will produce novel theorems, beside those, like Arrow's, which it simply recovers. In this way, judgment aggregation theory will enrich the substance of social choice theory, over and beyond the more refined analysis of voting rules that it has already provided.

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