Karlsruhe Institute of Technology (KIT)

Condorcet Admissibility

Indeterminacy and Path-Dependence under Majority Voting on Interconnected Decisions

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Freudenstadt
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Agenda

1. Defining Majoritarianism: The Condorcet Admissible Set

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2. Path-Dependence and Indeterminacy

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- **Example (Preference Aggregation):** Strict orderings over alternatives a, b, c. Issue 1: "$a \succ b$?", issue 2: "$b \succ c$?", issue 3: "$c \succ a$?" Thus, $X^{\mathrm{pref}} = \{0,1\}^3 \setminus \{(0,0,0),(1,1,1)\}$.

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Definition

Given a profile $\mu \in X^I$ of feasible views, the **Condorcet admissible set** *$Cond(X, \mu) \subseteq X$ is the set of all $x \in X$ such that no feasible view coincides with the (issue-wise) majority view on a strictly larger set of issues than x.*

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Consider alternatives a, b, c, d and suppose that one third of the population endorses the preference orderings $a \succ_1 b \succ_1 c \succ_1 d$, $b \succ_2 c \succ_2 d \succ_2 a$ and $c \succ_3 d \succ_3 a \succ_3 b$, respectively.

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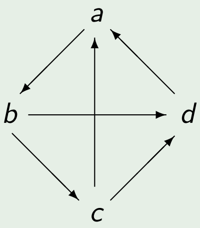

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Proposition

For all μ, $\mathit{Cond}(X^{\mathrm{alloc}}, \mu) = \mathit{Slater}(X^{\mathrm{alloc}}, \mu)$

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For all μ, $\mathit{Cond}(X^{\mathrm{alloc}},\mu) = \mathit{Slater}(X^{\mathrm{alloc}},\mu)$ and $\mathit{Cond}(X^{\mathrm{com}},\mu) = \mathit{Slater}(X^{\mathrm{com}},\mu)$

Agenda

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- As above, suppose that one third of the population endorse each of the preference orderings $a \succ_1 b \succ_1 c \succ_1 d$, $b \succ_2 c \succ_2 d \succ_2 a$, $c \succ_3 d \succ_3 a \succ_3 b$, respectively.

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- Then, sequential majority voting yields $d \succ a \succ b \succ c$, in particular $d \succ c$ by transitivity.
- But there is **unanimous** agreement that $c \succ d$!

Unanimity Violations

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- A profile μ is called *totally indeterminate* if $Cond(X, \mu) = X$.
- A space X is called *issue-wise indeterminate* if there exists an issue-wise indeterminate profile, and X is called *totally indeterminate* if there exists a totally indeterminate profile.

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Corollary

If X is McGarvey, then X is issue-wise indeterminate if and only if there exists some $x \in \{0,1\}^K$ that is critical for X.

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- How many individuals does an issue-wise indeterminate profile minimally involve? Often the minimal necessary number is only 3.

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- While few spaces are totally indeterminate, many are *asymptotically* totally indeterminate, i.e. the relative (logarithmic) size of the maximal Condorcet admissible set converges to 1. For instance, the relative size of the maximal Condorcet admissible set in the spaces X^{pref} converges to 1 when the number of alternatives grows without bound.

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- For instance, the preference aggregation problem has a particular combinatorial structure that makes it special in many respects.

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