Justifiable Group Choice[1]

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Abstract. *We study the judgment aggregation problem from the perspective of justifying a particular collective decision by a corresponding aggregation on the criteria. In particular, we characterize the logical relations between the decision and the criteria that enable justification of a majority decision through a proposition-wise aggregation rule with no veto power on the criteria. While the well-studied "doctrinal paradox" provides a negative example in which no such justification exists, we show that genuine possibility results emerge if there is a gap between the necessary and the sufficient conditions for the decision. This happens, for instance, if there is only a partial consensus about the appropriate criteria for the decision, if only a subset of these criteria can be elicited, or if the judgment on criteria is based on probabilistic acceptance thresholds.*

1 Introduction

Consider a group of individuals who have to take a collective decision and want to justify their decision based on reasons which reflect the opinions of the group members. As an example, consider a court of three judges who has to decide on the liability of a defendant (proposition d). Suppose that, by legal doctrine, the defendant is to be held liable if and only if (s)he did a particular action (proposition c_1) and no special exculpatory circumstances apply (proposition c_2). If the court members' judgments are as shown in Table 1, proposition-wise majority voting on both the decision and the "reasons" leads to a set of collective judgments that is inconsistent with the legal doctrine: the affirmation of both c_1 and c_2 but at the same time the rejection of d.

	action done (c_1)	no special circumstances (c_2)	liable (d)
Judge 1	true	true	true
Judge 2	true	false	false
Judge 3	false	true	false
Majority	true	true	false

Table 1: The doctrinal paradox / discursive dilemma

This is the well-known "doctrinal paradox" or "discursive dilemma" studied in the judgment aggregation literature, following Kornhauser and Sager (1986) and List and Pettit (2002). The literature has demonstrated the robustness of the discursive dilemma, both with respect to the class of admissible aggregation methods and with respect to the structure of the logical relation between the "decision" (d) and the reasons or "criteria" (c_1 and c_2).[2]

In Nehring and Puppe (2008), we have shown that the discursive dilemma extends to all "truth-functional" contexts. These are contexts in which each judgment set forces *either* the acceptance *or* the rejection of the decision. In such situations the only consistent proposition-wise aggregation methods are oligarchic and often even dictatorial. For instance, in the doctrinal paradox above the only anonymous proposition-wise aggregation method is the unanimity rule according to which the collective affirmation of each proposition requires unanimous consent.[3]

Assuming truth-functionality is, however, restrictive and arguably unnatural in the present case since the presence of "special circumstances" creates a scope of discretion. Specifically, assume that the logical interrelation between the decision and the criteria is as follows: (i) negating that the action has been done necessarily leads to the verdict "not liable," no matter whether or not special circumstances are granted, (ii) affirming both c_1 and c_2 (i.e. affirming that the action has been done but denying special circumstances) necessarily implies the verdict "liable," and (iii) affirming c_1 but negating c_2 (thus granting special circumstances) is consistent with either a positive or a negative

[2]See, e.g., Pauly and van Hees (2006), Dietrich (2006), Dietrich and List (2007), Dokow and Holzman (2009, 2010), Nehring and Puppe (2008, 2010). List and Puppe (2009) provide a survey of the recent literature on judgment aggregation.

[3]Whether there exist anonymous proposition-wise aggregation methods in truth-functional contexts depends on the precise logical relation between the decision and the criteria. In many cases, there are in fact no anonymous rules at all, see Dokow and Holzman (2009) and Nehring and Puppe (2008).

verdict, depending on further details of the case. Clause (iii) creates a gap between the necessary and the sufficient conditions for the decision, thereby introducing a "scope of discretion" that reflects the assessment of the special circumstances for the case at hand.

Relaxing the assumption of truth-functionality in this way allows one to avoid the doctrinal paradox. Specifically, a consistent proposition-wise aggregation method can be obtained in a natural way by requiring unanimous consent in order to affirm c_2 (i.e. in order to deny the presence of special circumstances), deciding all other propositions by majority vote as before. If the individual judgments are as above, this aggregation method results in the collective judgment according to which c_1 is affirmed, but special circumstances are granted and the verdict is "not liable" (see Table 2).

	action done (c_1)	no special circumstances (c_2)	liable (d)
Judge 1	true	true	true
Judge 2	true	false	false
Judge 3	false	true	false
Majority with unanimity on c_2	true	false	false

Table 2: The doctrinal paradox avoided

As is easily verified, the suggested aggregation method always yields a consistent collective judgment, no matter what the individual judgments are. Indeed, by individual consistency, any voter for d must also vote for c_1, so there can never be a majority for d without there being one for c_1. And, under the unanimity rule, c_2 is collectively affirmed only if everyone votes for it, in which case (by individual consistency) every voter for c_1 must also vote for d. Thus whenever c_2 is collectively affirmed, there cannot be a majority for c_1 without there being one for d.[4] Thus, reaching a verdict as the result of a majority vote on the decision can be *justified* by an appropriate independent aggregation of the criteria. In this paper, we ask under which circumstances the gap between necessary and sufficient conditions for justifying an outcome decision opens interesting possibility results more generally.

There are various situations in which there may be a gap between the necessary and the sufficient conditions for a decision. For instance, the decision may depend only on the *number* of affirmed criteria with a threshold between the minimal number of affirmed criteria forcing a positive decision and the maximal number of affirmed criteria forcing a negative decision (see Example 1 in Subsection 2.3 below). Other examples arise if there is only a *partial consensus* about the appropriate (truth-functional) relationship between the criteria and the decision, or if only a subset of the relevant criteria can be *elicited* (see Subsection 2.3, Examples 2 and 3, respectively). Finally, non-truth-functional contexts arise by employing probabilistic thresholds, or Dietrich's (2010) notion of subjunctive implication (see Subsection 2.3, Examples 4 and 5, respectively).

[4]Requiring unanimity just for c_2 does not ensure collective consistency in the former truth-functional case since the individual judgment profile could have majorities for c_1 and for d without having unanimity on c_2.

Viewing the group decision problem as a problem of justification shall be taken to mean that (a) the group uses a *given* procedure which aggregates the individuals' views on the decision, e.g. majority voting as in the above example, and (b) there is a set of agreed upon constraints on how outcome decisions can be justified by judgments on the criteria, both at the individual and social level. This viewpoint is consistent with other approaches in the literature, for instance with Pettit's (2004) notion of conversability according to which legitimate collective discourse must be susceptible to reason and to the perception of what reasons demand.

On the other hand, our approach is not compatible with an interpretation in terms of "reason-based" group choice that attempts to optimally use the information contained in the individuals' judgments on the criteria ("premises"). Under the latter interpretation, the so-called premise-based procedure appears to be an attractive way out of the discursive dilemma. In a truth-functional context, the *premise-based procedure* consists in aggregating the premises and deriving the decision by logical implication (see Nehring (2005), Mongin (2008), Dietrich and Mongin (2010)). However, from the present perspective of justifying a *given* collective decision the premise-based procedure is simply not applicable. List (2006) classifies different responses to the discursive dilemma in terms of a spectrum that ranges from purely reason-driven approaches, such as the premise-based procedure, to purely outcome-oriented (conclusion-based) approaches. The justification perspective of the present paper occupies a middle ground on this spectrum in that the collective decision is not *derived* from an aggregation of the criteria but has nevertheless to be complemented ("justified") by an appropriate aggregation of these criteria.

The question posed by a group justification problem is to find aggregation procedures on the criteria that appropriately reflect the individuals' views on these and at the same time respect the justification constraints at the aggregate level. A natural starting point is to require independence among the criteria, especially when these are logically independent as we shall assume throughout. Compared to a general, abstract judgment aggregation approach, the justification perspective motivates the distinction between the aggregation of the decision versus the aggregation of the criteria, and in particular the imposition of stronger normative requirements on the former than on the latter.

We will ask specifically when justification is possible with majority voting on the decision. The case of majority voting on the decision is a particularly natural benchmark case, since in our context majority voting is the only anonymous aggregation method that treats acceptance and rejection symmetrically. It is immediate from general results on judgment aggregation that majority voting on the decision is only in trivial cases consistent with majority voting on all criteria (see, e.g. Nehring and Puppe (2010, Theorem 4)). But interesting possibilities emerge if one combines majority voting on the decision with weaker requirements on the aggregation of the criteria. As an important example we consider the condition of "no veto power," i.e. the requirement that a single individual can neither force a proposition to be collectively accepted nor force a proposition to be rejected. Our first main result (Theorem 1) characterizes the class of all functional relations between the decision and the criteria, henceforth "justification constraints," that enable aggregation rules with majority voting on the decision and no veto power on the criteria.[5]

[5] Remember that in the truth-functional case all admissible aggregation rules are oligarchic; in particular, every admissible aggregation rule entails veto power on *all* propositions (decision and

The key to this result are the notions of "one-sided" and "monotone" justification constraints, respectively. A set of justification constraints (i.e. the logical relation between the criteria and the decision) is said to be *one-sided* if either (i) no combination of affirmed criteria ever forces the acceptance of the decision, or (ii) no combination of affirmed criteria ever forces the rejection of the decision. The justification constraints described in our example above are not one-sided since, by assumption, affirming no criteria (in particular, denying that the action has been done) necessarily leads to the rejection of the decision, while affirming both criteria forces the acceptance of the decision. Examples of one-sided justification constraints are discussed below.

A set of justification constraints is *monotone* if the criteria can be labeled in such a way that the affirmation of a criterion always has a positive effect on the decision, no matter what other criteria are affirmed. For instance, accepting a decision if and only if *at least* two out of three criteria are affirmed corresponds to monotone justification constraints; by contrast, accepting a decision if and only if *exactly* two out of three criteria are affirmed does not correspond to monotone justification constraints. Equipped with these notions, we can now state our first theorem more precisely: There exist consistent proposition-wise aggregation rules with majority voting on the decision and no veto power on the criteria if and only if the underlying set of justification constraints is one-sided and monotone.

There are two possible avenues to obtain further possibility results by weakening the aggregation requirements with respect to either the decision or the criteria. Under monotonicity, dropping the restriction to majority rule for the decision does not help if we still require no veto on the criteria: it is still necessary and sufficient that the justification constraints be one-sided (cf. Proposition 1 below). By contrast, dropping the no veto requirement on the aggregation of the criteria leads to new possibilities as illustrated in the introductory example.[6] Our second result (Theorem 2) characterizes the monotone justification constraints that, allowing vetoes on the criteria, enable justification of various aggregation procedures on the decision. Specifically, we consider aggregation on the decision without veto and without dictators.

As a further illustration of the justification perspective developed in this paper, consider the following re-interpretation of our introductory example. An editor of an academic journal wants to justify the publication decision in case of a particular submitted work. The editor asks three referees to evaluate the paper according to the criteria of correctness of results (c_1) and originality of ideas (c_2), and to give a publication recommendation (d). In this context, one would probably consider correctness of the results to be a necessary condition for a positive publication recommendation, i.e. ($\neg c_1 \to \neg d$). Moreover, correctness and originality are arguably jointly sufficient for a positive decision, i.e. ($c_1 \wedge c_2 \to d$). On the other hand, *sometimes* one may wish to recommend publication of correct results even if the ideas are not considered to be original, i.e. the judgment $c_1 \wedge \neg c_2$ would be considered to be consistent with either a positive and a negative publication recommendation. The example is formally equivalent to the example underlying Table 2. Note in particular that the justification constraints are not one-sided since some evaluations of the criteria force a positive

criteria). We also note that the existence of aggregation rules with no veto power often requires a sufficiently large number of individuals.

[6] Remember that the example does not display one-sided justification constraints, and note that the aggregation rule described above entails a veto for every voter over c_2 since that proposition is accepted only under unanimous consent.

publication decision while others force a negative decision. By the results established here, an outcome decision based on majority voting on the publication recommendations can be justified via an independent aggregation rule on the criteria, but all such aggregation rules entail veto power on at least one criterion for at least one agent. As in Table 2 above, an admissible rule emerges by taking majority voting on both the decision d and the first criterion c_1 (correctness of results), and to affirm the second criterion c_2 (originality of ideas) if and only if it is unanimously accepted. It follows from the methods developed here that this is in fact the only anonymous aggregation rule with majority voting on the decision in the present example.[7] Stronger possibility results would arise in a variant of the example involving, say, a top economics journal for which *no* combination of affirmed criteria is sufficient to force acceptance of a submitted paper, while some combinations of affirmed criteria are sufficient to force its rejection. In this case, the justification constraints are one-sided.

The remainder of the paper is organized as follows. In the following section, we introduce our framework and show its applicability in a wide variety of contexts. Section 3 contains the first main characterization result. Section 4 analyzes monotone justification constraints in greater detail and demonstrates that weaker possibility results may obtain also in the two-sided case. Section 5 concludes; all proofs are collected in an appendix.

2 Framework and Further Examples

The set $C = \{c_1, ...c_m\}$ represents a collection of *criteria* for a binary decision d. The elements of $C \cup \{d\}$ and their negations are also referred to as *propositions*. A *judgment set* is a subset $J \subseteq C \cup \{d\}$ with the interpretation that the accepted propositions are exactly the elements of J plus the negations of the elements of $[C \cup \{d\}] \setminus J$, and that all other propositions are rejected.[8] Throughout, we assume that any combination of affirmed criteria is logically possible. Individual judgment sets are denoted by J_i with the subscripts referring to individuals.

2.1 Justification Constraints

There is unanimous agreement among individuals that some combinations of affirmed criteria force the acceptance of d; similarly, other combinations of affirmed criteria force the rejection of d. Formally, denote by $\mathcal{A} \subseteq 2^C$ the collection of subsets $A \subseteq C$ for which it is agreed upon that affirmation of exactly the criteria in A forces the acceptance of the decision d. We call $\mathcal{A}$ the *acceptance region* of d. Similarly, $\mathcal{R} \subseteq 2^C$ is the collection of subsets $R \subseteq C$ for which it is agreed upon that affirmation of exactly the criteria in R forces the rejection of the decision d. We call $\mathcal{R}$ the *rejection region* of d. A pair $(\mathcal{A}, \mathcal{R})$ is referred to as a set of *justification constraints*, or simply *constraints*. A judgment set J is *consistent* with the constraints if $J \cap C \in \mathcal{A}$ implies $d \in J$ and $J \cap C \in \mathcal{R}$ implies $d \notin J$.

[7] Note that majority voting on the decision is not consistent with a rule according to which *both* criteria are collectively affirmed only under unanimous consent, since this could lead to the aggregate conclusion that the paper is incorrect but can still be published.

[8] Unlike in a number of other contributions to the literature, the completeness requirement on judgement sets is thus built into our framework.

Throughout, we impose the following non-triviality requirements: $\mathcal{A} \cup \mathcal{R} \neq \emptyset$ (the decision is at least sometimes restricted), $\mathcal{A} \cap \mathcal{R} = \emptyset$ (any combination of affirmed criteria must be consistent with some decision), $\mathcal{A} \neq 2^C$ (the decision does not always have to be accepted), and $\mathcal{R} \neq 2^C$ (the decision does not always have to be rejected). Moreover, we assume that the decision is not equivalent to any single criterion, formally, for no $c \in C$, $[A \in \mathcal{A} \Leftrightarrow c \in A$ and $R \in \mathcal{R} \Leftrightarrow c \notin R]$, and similarly, for no $c \in C$, $[A \in \mathcal{A} \Leftrightarrow c \notin A$ and $R \in \mathcal{R} \Leftrightarrow c \in R]$.

For the following, it is important that there may be a gap between the acceptance and rejection regions, i.e. there may be combinations of affirmed criteria for which it is neither agreed that they force acceptance, nor that they force rejection of the decision. The special case in which any combination of affirmed criteria either forces the acceptance or the rejection of the decision is referred to as the "truth-functional" case and has been studied e.g. in Nehring and Puppe (2008) and Dokow and Holzman (2009).

Definition (Truth-Functionality). A set of justification constraints $(\mathcal{A}, \mathcal{R})$ is called *truth-functional* if $\mathcal{A} \cup \mathcal{R} = 2^C$.

Below, we provide a number of examples that motivate to consider non-truth-functional contexts.

2.2 Aggregation Rules

Denote by $N = \{1, ..., n\}$ the set of individuals. An *aggregation rule* is a mapping F that assigns a consistent collective judgment set J to each profile $(J_1, ..., J_n)$ of consistent individual judgment sets. Throughout, we require the following properties. First, F is defined for all logically possible combinations of consistent individual judgment sets ("unrestricted domain"). Second, any consistent judgment set J is the collective result of F for some suitable profile of individual judgment sets ("voter sovereignty").[9] Finally, we require the following condition of "monotone independence." Consider $(J_1, ..., J_n)$ and $(J'_1, ..., J'_n)$ such that, for some p and all i, $p \in J_i \Rightarrow p \in J'_i$. Then, $p \in F(J_1, ..., J_n) \Rightarrow p \in F(J'_1, ..., J'_n)$; similarly, if for some p and all i, $p \notin J_i \Rightarrow p \notin J'_i$, then $p \notin F(J_1, ..., J_n) \Rightarrow p \notin F(J'_1, ..., J'_n)$. Monotone Independence thus asserts that if the collective judgment set entails the affirmation (resp. negation) of p, and if the individual support for p increases (resp. decreases), then p must remain affirmed (resp. negated) in the collective judgment set. From the perspective of the present paper, the independence requirement is strong but arguably very natural: independence of the aggregation among the criteria is justified by their logical independence, and independence of the aggregation of the decision is implied by the task to justify a collective decision resulting from a *given* aggregation of individual decisions.

An aggregation rule is called *anonymous* if it is invariant with respect to permutations of the individuals. An individual i is said to have a *veto* on proposition p if i can force p to be collectively rejected, or force p to be collectively affirmed. Formally, i has a veto on p under the aggregation rule F if, for all J_l, $l \neq i$, there exists J_i such that $p \notin F(J_1, ..., J_i, ..., J_n)$, or for all J_l, $l \neq i$, there exists J_i such that $p \in F(J_1, ..., J_i, ..., J_n)$. An aggregation has *veto power* if some individual has a veto on some proposition. An individual i is said to be a *dictator on* p if i can force p to be

[9] A slightly stronger condition ("unanimity") would require that J be the collective judgment set whenever all individuals agree on J.

collectively rejected *and* force p to be collectively affirmed under the aggregation rule F, i.e. for some i and all J_l, $l \neq i$, there exists J_i such that $p \notin F(J_1, ..., J_i, ..., J_n)$, and $\hat{J}_i$ such that $p \in F(J_1, ..., \hat{J}_i, ..., J_n)$. An aggregation rule is called *dictatorial on p* if some individual is a dictator on p. Finally, an aggregation rule is called *dictatorial* if there exists an individual i who is a dictator on all propositions.

2.3 Examples

The following examples are meant to illustrate the great variety of situations in which there may exist a gap between the necessary and the sufficient condition for a particular decision.

1. Substitutable criteria. A simple example of a set of justification constraints is given by $\mathcal{A} = \{A \subseteq C : \#A \geq k\}$ and $\mathcal{R} = \{R \subseteq C : \#R \leq l\}$, where $k > l$. For instance, a job candidate will be hired if she fares well on a sufficiently large number of criteria, and is not hired if she fares well only on a small number of criteria. In intermediate case, either a positive and a negative hiring decision is justifiable. Note that the decision is truth-functionally determined by the criteria if and only if $k = l+1$.

2. Partial consensus. Suppose that there is a set of different truth-functional justification constraints $\{(\mathcal{A}_k, \mathcal{R}_k)\}_{k \in K}$ each of which is considered to reflect a reasonable "point of view." Taking $\mathcal{A} := \cap_k \mathcal{A}_k$ and $\mathcal{R} := \cap_k \mathcal{R}_k$ in such a situation creates a gap between the necessary and the sufficient conditions for the outcome decision and reflects a weak notion of justifiability. Indeed, employing the set of justification constraints $(\mathcal{A}, \mathcal{R})$ means to consider a decision to be justifiable if and only if it is justifiable from *some* reasonable point of view. As a concrete example one may take again the hiring procedure, say at an academic institution, in which the constitutional rules allow each prospective member of a committee to use any of the truth-functional justification constraints classified ex-ante as reasonable. Note that the domain of the admissible aggregation rules is thus still the Cartesian product of a common domain of individual judgment sets although different *actual* members of a particular committee may operate with different truth-functional justification constraints. For a "non-Cartesian" model in this context, in which different individuals face different domain restrictions corresponding to different logical constraints on the decision, see Miller (2008).

3. Partial elicitation. Suppose now that each individual uses the *same* truth-functional justification constraints, but only a subset of criteria are elicited. Say, for instance, that there are 10 criteria and the common (truth-functional) justification constraint prescribes a positive hiring decision if and only if the candidate fares well on at least 5 criteria. If e.g. only 7 out of the 10 criteria are elicited, then a positive assessment of between 2 and 4 elicited criteria is consistent with either a positive or a negative decision. On the other hand, a positive assessment in only one of the elicited criteria would not be consistent with a positive hiring decision.

4. Acceptance Thresholds. A large class of examples emerges by interpreting the affirmation of a proposition as "sufficient confidence" in its truth in an uncertain environment. For instance, suppose that the two criteria c_1 and c_2 are affirmed if and only if the probability of a positive evaluation exceeds some common threshold q, where $0 < q < 1$. On the other hand, assume that the decision d is accepted if and only if the product of the individual probabilities for the two criteria exceeds r. Most interesting and quite natural is the case of $r = q^2$, which results in the non-truth-functional set of justification constraints $\mathcal{A} = \{\{c_1, c_2\}\}$ and $\mathcal{R} = \{\emptyset\}$. Indeed, if both criteria are affirmed, we obtain $prob(c_i) > q$ for $i = 1, 2$, and thus $prob(c_1) \cdot prob(c_2) > q^2 = r$, i.e. d is accepted; similarly, negation of both criteria forces rejection of d. On the other hand, affirming exactly one criterion is clearly consistent with either a positive and a negative decision, depending on the precise probabilities $prob(c_1)$ and $prob(c_2)$.[10]

5. Subjunctive Implication. A special class of non-truth-functional decision contexts arises by considering "subjunctive implications" as introduced by Dietrich (2010). Specifically, consider a decision d and the two criteria a and $a \hookrightarrow d$, where the latter is interpreted as a *subjunctive implication*. By definition, this means that the decision d is restricted only if both a and $a \hookrightarrow d$ are affirmed, in which case d has to be accepted. Thus, $\mathcal{A} = \{a, a \hookrightarrow d\}$ and $\mathcal{R} = \emptyset$ in this case. As a concrete example, let a stand for "carbon dioxide emissions will increase further" and d for "climate will change." In everyday language, the implication "*if* carbon dioxide emissions will increase further *then* climate will change" is usually interpreted not in its material but in its subjunctive form. In particular, negating this implication is considered to be consistent with either the acceptance or the rejection of d no matter whether a is affirmed or not.[11] Thus, the subjunctive interpretation of the implication introduces a gap between the necessary and sufficient conditions for the decision. More generally, any decision problem with decision d and set of criteria $C = \{a_1, a_1 \hookrightarrow a_2, a_2 \hookrightarrow a_3, ..., a_k \hookrightarrow d\}$ gives rise to constraints of the form $\mathcal{A} = \{C\}$ and $\mathcal{R} = \emptyset$, provided that all implications are interpreted in the subjunctive sense.[12]

2.4 One-Sidedness and Monotonicity

The following two properties of justification constraints will play a central role in our analysis.

Definition (One-Sidedness). A set of justification constraints $(\mathcal{A}, \mathcal{R})$ is called *one-sided* if either $\mathcal{A}$ or $\mathcal{R}$ is empty, i.e. if either the rejection of the decision is always consistent, or its acceptance is always consistent.

One-sidedness is clearly restrictive but applicable in a number of contexts. Natural examples arise under the partial consensus interpretation above. Specifically, consider again a hiring decision at an academic department. The members of the hiring committee may agree on certain *necessary* criteria that a prospective candidate would have to satisfy, say teaching experience and decent research. This would lead to a non-empty rejection region. At the same time, the hiring committee may find it difficult to agree on sets of criteria that are jointly *sufficient* for a positive hiring decision. For instance, the committee members may come from different areas with different performance standards, etc. In this case, the acceptance region, i.e. the intersection of the individual acceptance regions, could well be empty. Another example of one-sided justification constraints is the case of the subjunctive implication. A set of justification constraints

[10] This example suggests to study the judgment aggregation problem in a probabilistic framework, see Nehring (2007) for such a model. For a related account of the discursive dilemma as a form of the well-known "lottery paradox," see Levi (2004).

[11] By contrast, negating the implication "$a \to d$" but affirming a forces one to reject d under the standard material interpretation of "$a \to d$."

[12] Including some of the propositions $a_2, ..., a_k$ as criteria would destroy the logical independence of the criteria as assumed here. For an analysis of the case with logically interdependent premises, see Dietrich (2010).

is called *two-sided* if it is not one-sided.

The second concept is a monotonicity property. A set of justification constraints will be called "monotone" if the criteria negation pairs can be labeled in such a way that the affirmation of a criterion always has a positive effect on the decision, no matter what other criteria are affirmed. To provide a formal definition, we first need some notation to cover the possible need to relabel the criteria and their negations. For any two subsets $A \subseteq C$ and $\chi \subseteq C$, denote by $A^{\chi} := [A \setminus \chi] \cup [(C \setminus A) \cap \chi]$. The intended interpretation is as follows. Suppose that for the criteria in χ the meaning of affirmation versus negation is exchanged. Then, affirming exactly the criteria in A (and thus negating the criteria in $C \setminus A$) before the swap means the same as affirming the criteria in A^{χ} after the swap. Two justification constraints $(\mathcal{A}, \mathcal{R})$ and $(\mathcal{A}', \mathcal{R}')$, defined on the same set C of criteria, are said to be *equivalent* if there exists a subset $\chi \subseteq C$ such that $A \in \mathcal{A} \Leftrightarrow A^{\chi} \in \mathcal{A}'$ and $R \in \mathcal{R} \Leftrightarrow R^{\chi} \in \mathcal{R}'$. For instance, the constraints $(\mathcal{A}, \mathcal{R})$ and $(\mathcal{A}', \mathcal{R}')$ defined on $C = \{c_1, c_2\}$ with $\mathcal{A} = \{\{c_2\}\}$, $\mathcal{R} = \{\{c_1\}, \{c_1, c_2\}\}$, $\mathcal{A}' = \{\{c_1, c_2\}\}$ and $\mathcal{R}' = \{\emptyset, \{c_2\}\}$ are equivalent, as is easily verified by taking $\chi = \{c_1\}$.

Definition (Monotonicity). A set of justification constraints $(\mathcal{A}, \mathcal{R})$ is called *monotone* if there exists an equivalent set of constraints $(\mathcal{A}', \mathcal{R}')$ such that $\mathcal{A}'$ is closed under taking supersets and $\mathcal{R}'$ is closed under taking subsets.[13]

Monotone justification constraints are arguably the most relevant ones. For instance, in the hiring decision example one can often label the criteria in such a way that affirming additional criteria is never harmful for a positive hiring decision. On the other hand, monotonicity does rule out, say, standards of "mediocrity," according to which job candidates are deemed eligible if they satisfy a certain number of desirable criteria but not too many of them. Specifically, the set of constraints $(\mathcal{A}, \mathcal{R})$ with $\mathcal{A} = \{A \subseteq C : k \leq \#A \leq l\}$ is not monotone whenever $0 < k \leq l < m$.

3 A General Possibility Result

The following is our first main result. Say that a set of justification constraints *admits* aggregation rules with particular properties if there exists a number n of voters and aggregation rules with the specified properties that map all n-profiles $(J_1, ..., J_n)$ of consistent individual judgement sets to consistent collective judgement sets. Note that consistency of majority voting requires an odd number of individuals. Also note that the existence of aggregation rules without veto power that are different from majority voting requires more than three voters.

Theorem 1 (majority voting on decision, no veto on criteria) *A set of justification constraints admits monotonically independent aggregation rules with no veto power on the criteria and majority voting on the decision if and only if it is one-sided and monotone.*

The proof of this result, provided in the appendix, in fact shows that monotonicity and one-sidedness jointly guarantee the existence of an aggregation rule with majority voting on the decision that has no veto power and is in addition anonymous on the criteria, i.e. a quota rule with no veto power on the criteria. The proof also yields an

[13] As can be seen from the simple example just given, closedness under taking supersets and subsets is in general not preserved by replacing a set of justification constraints by an equivalent one.

upper bound on the minimal number of voters needed to ensure the no veto power condition. While the precise number depends on the specific structure of the justification constraints, a crude condition that is always sufficient is $n \geq 2m$, where m is the number of criteria.

In the introduction above, we provided an example of an independent aggregation rule with veto power and majority voting on the decision in the two-sided case (cf. Table 2); this shows that the no veto condition cannot be dropped in Theorem 1. The following example shows that, similarly, in the one-sided but non-monotone case independent aggregation rules with veto power on some criteria and majority voting on the decision may exist. Specifically, assume that there are three criteria c_1, c_2 and c_3, and that the justification constraints can be described by the two implications $(c_1 \wedge c_2 \rightarrow d)$ and $(\neg c_1 \wedge c_3 \rightarrow d)$. These constraints are one-sided but not monotone.[14] The aggregation rule according to which the outcome and the first criterion are decided by majority voting while the other two criteria are affirmed only under unanimous consent is easily seen to be consistent.

Remark Theorem 1 relies on our assumption that the criteria are logically independent, i.e. that any combination of affirmed criteria is consistent. Otherwise, there may exist consistent aggregation rules with no veto power on the criteria and majority voting on the decision also in the non-monotone and two-sided case. As an example consider the following situation. There are three criteria, c_1, c_2 and c_3, any affirmation/negation combination of which is consistent except the affirmation of all three criteria. The justification constraints are given by $\mathcal{A} = \{\{c_1, c_3\}\}$ and $\mathcal{R} = \{\{c_2, c_3\}\}$. These justification constraints are easily seen to be neither monotone nor one-sided. Nevertheless, the following aggregation rule is consistent: the decision is determined by majority voting while each criterion is collectively affirmed if and only if more than 3/4 of the voters vote for it. This rule entails no veto power if there are at least 5 voters.[15] The example may seem surprising, since one might expect additional restrictions in form of logical interdependencies between the criteria to *decrease* the scope for possibility results, as in the truth-functional case (cf. Nehring and Puppe (2008)). The problem of logically interdependent criteria in the non-truth-functional case poses interesting open questions for further research.

The proof of Theorem 1 relies on several auxiliary results, some of which are of independent interest. To formulate them, we need the following additional definitions. Given a set of constraints $(\mathcal{A}, \mathcal{R})$, say that a criterion c is *positively pivotal for acceptance* of d if affirming c can turn a possible rejection of d into a forced acceptance. Formally, the set $C^+_{\mathcal{A}}$ of all criteria that are positively pivotal for acceptance is defined by

$$C^+_{\mathcal{A}} := \{c \in C : \text{ there exists } A \in \mathcal{A} \text{ with } c \in A \text{ and } A \setminus \{c\} \notin \mathcal{A}\}.$$

[14]The corresponding rejection and acceptance regions are given by $\mathcal{R} = \emptyset$ and $\mathcal{A} = \{\{c_1, c_2\}, \{c_3\}, \{c_2, c_3\}, \{c_1, c_2, c_3\}\}$, respectively. The non-monotonicity of these justification constraints follows from Lemma 1 below.

[15]To verify the consistency of the aggregation rule if the number of voters is odd, note that the decision is only restricted if c_3 and exactly one of the other two criteria is affirmed. If, for instance, both c_3 and c_1 are collectively affirmed, a supermajority of more than 3/4 of the voters must have voted for each, in which case more than half of all voters must have in fact voted for both c_3 *and* c_1. By individual consistency, these voters must vote for the negation of c_2, and therefore also for d, hence c_2 is collectively negated and d is collectively accepted. A similar argument shows that c_1 and d are collectively rejected if c_3 and c_2 are collectively affirmed, and that c_3 must be collectively negated if c_1 and c_2 are collectively affirmed.

Similarly, a criterion c is *negatively pivotal for acceptance* of d if negating c can turn a possible rejection of d into a forced acceptance. The corresponding set $C_{\mathcal{A}}^{-}$ is formally defined by

$$C_{\mathcal{A}}^{-} := \{c \in C : \text{ there exists } A \in \mathcal{A} \text{ with } c \notin A \text{ and } A \cup \{c\} \notin \mathcal{A}\}.$$

Analogously, a criterion c is *positively pivotal for rejection* of d if affirming c can turn a possible acceptance of d into a forced rejection, and a criterion c is *negatively pivotal for rejection* of d if negating c can turn a possible acceptance of d into a forced rejection. Formally, the two corresponding sets are defined by

$$C_{\mathcal{R}}^{+} := \{c \in C : \text{ there exists } R \in \mathcal{R} \text{ with } c \in R \text{ and } R \setminus \{c\} \notin \mathcal{R}\}$$

and

$$C_{\mathcal{R}}^{-} := \{c \in C : \text{ there exists } R \in \mathcal{R} \text{ with } c \notin R \text{ and } R \cup \{c\} \notin \mathcal{R}\},$$

respectively.

Lemma 1 *A set of constraints is monotone if and only if*

$$\left(C_{\mathcal{A}}^{+} \cup C_{\mathcal{R}}^{-}\right) \cap \left(C_{\mathcal{A}}^{-} \cup C_{\mathcal{R}}^{+}\right) = \emptyset.$$

Lemma 2 *A set of constraints is one-sided if and only if*

$$\left(C_{\mathcal{A}}^{+} \cup C_{\mathcal{A}}^{-}\right) \cap \left(C_{\mathcal{R}}^{+} \cup C_{\mathcal{R}}^{-}\right) = \emptyset.$$

The proof of the necessity part of our main result provided in the appendix proceeds as follows. First, we show that the existence of an monotonically independent aggregation rule with no veto power and majority voting on the decision implies that the pairwise intersections of the four sets $C_{\mathcal{A}}^{+}$, $C_{\mathcal{A}}^{-}$, $C_{\mathcal{R}}^{+}$ and $C_{\mathcal{R}}^{-}$ have to be empty. By Lemmas 1 and 2 this implies that the underlying set of justification constraints has to be monotone and one-sided.

The sufficiency part of the above theorem is proved by explicitly constructing, for any monotone and one-sided set of justification constraints, an anonymous aggregation method with majority voting on the decision and appropriate super-majority quotas on the criteria.

4 Monotone Justification Constraints

We now want to determine to which extent weakening the aggregation requirements on either the outcome decision and the criteria creates a scope for further possibilities. To obtain sharper results and facilitate the exposition, we will assume throughout monotonicity of the justification constraints.

First, we note that under monotonicity the existence of an aggregation rule with *majority voting* on the decision and no veto power on the criteria is implied by the existence of an aggregation rule without veto power on the criteria alone. Specifically, we have the following result.

Proposition 1 *Consider a set of monotone justification constraints* $(\mathcal{A}, \mathcal{R})$. *The following statements are equivalent.*

(i) $(\mathcal{A}, \mathcal{R})$ *admits monotonically independent aggregation rules with no veto power on the criteria.*
(ii) $(\mathcal{A}, \mathcal{R})$ *admits monotonically independent aggregation rules with majority voting on the decision and no veto power on the criteria.*
(iii) $(\mathcal{A}, \mathcal{R})$ *is one-sided.*

Let $(\mathcal{A}, \mathcal{R})$ be monotone. By appropriately relabeling the criteria, we may assume without loss of generality for the remainder of this section that $C^-_{\mathcal{A}}$ and $C^+_{\mathcal{R}}$ are empty, i.e. that $\mathcal{A}$ is closed under taking supersets and that $\mathcal{R}$ is closed under taking subsets. A decision criterion c is called *doubly pivotal* if it is simultaneously relevant for acceptance and rejection, i.e. if $c \in C^+_{\mathcal{A}} \cap C^-_{\mathcal{R}}$. The following result gives the necessary and sufficient conditions under which different types of aggregation procedures on the decision are justifiable via an independent aggregation of the criteria.

Theorem 2 (a) (no veto power / majority voting on decision) *Suppose that the justification constraints* $(\mathcal{A}, \mathcal{R})$ *are monotone. Then* $(\mathcal{A}, \mathcal{R})$ *admits monotonically independent aggregation rules with no veto power on the decision if and only if all minimal elements of* $\mathcal{A}$ *contain at most one doubly pivotal criterion* and *all maximal elements of* $\mathcal{R}$ *omit at most one doubly pivotal criterion. Moreover, in this case the aggregation on the decision can be taken to be majority voting.*
(b) (no-dictatorship / anonymity on decision) *Suppose that the justification constraints* $(\mathcal{A}, \mathcal{R})$ *are monotone. Then* $(\mathcal{A}, \mathcal{R})$ *admits monotonically independent aggregation rules that are non-dictatorial on the decision if and only if all minimal elements of* $\mathcal{A}$ *contain at most one doubly pivotal criterion* or *all maximal elements of* $\mathcal{R}$ *omit at most one doubly pivotal criterion. Moreover, in this case the aggregation on the decision can be taken to be anonymous.*

Our introductory example (cf. Table 2) illustrates Theorem 2(a). Recall that in this example we have $\mathcal{A} = \{\{c_1, c_2\}\}$ and $\mathcal{R} = \{\emptyset, \{c_2\}\}$. By consequence, we obtain $C^+_{\mathcal{A}} = \{c_1, c_2\}$ (both criteria are positively pivotal for acceptance), $C^-_{\mathcal{R}} = \{c_1\}$ (only the first criterion is negatively pivotal for rejection), and $C^-_{\mathcal{A}} = C^+_{\mathcal{R}} = \emptyset$. Thus, only the criterion c_1 is doubly pivotal, corresponding to the fact that, if c_2 is affirmed ("no special circumstances"), the affirmation of c_1 ("action done") forces acceptance of d ("liable") while the negation of c_1 ("action not done") forces rejection of d ("not liable"). By Theorem 2(a), there exist proposition-wise aggregation rules with majority voting on the decision, but by Proposition 1, all such rules necessarily entail a veto of some voter on some criterion. One example is the rule described in the introduction which requires unanimous consent for the collective affirmation of c_2.

An example illustrating Theorem 2(b) is the (truth-functional) case of a decision which is only accepted if *all* criteria are affirmed and rejected in all other cases, i.e. $\mathcal{A} = \{C\}$ and $\mathcal{R} = 2^C \setminus \{C\}$. In this case, all criteria are doubly pivotal and the maximal elements of $\mathcal{R}$ omit exactly one doubly pivotal criterion. In accordance with Theorem 2(b) there exist aggregation rules that are anonymous on the decision. An example is the rule that requires unanimous consent for the affirmation of each criterion and for the acceptance of the decision (by Proposition 2 below, this is in fact the only anonymous rule in this case).

In the truth-functional case, *all* criteria are doubly pivotal.[16] But this holds much more generally. For instance, also in the case of a candidate who is accepted whenever

[16] Assuming, without loss of generality, that no criterion is entirely irrelevant in the sense that it never makes a difference for the decision.

he or she fulfills at least k out of m criteria ($1 \leq k \leq m$), and who is rejected whenever he/she fulfills at most l of the m criteria ($0 \leq l \leq m-1$) with $k > l$ (cf. Example 1 in Section 2.3 above), all criteria are doubly pivotal. For these cases, we obtain the following very crisp characterization. An aggregation rule is called *oligarchic* if there exists a group $M \subseteq N$ of individuals (the "oligarchs") and a consistent default judgment set J_0 such that, for each proposition, a departure from J_0 in the collective judgment set requires unanimous consent among the members of M. Note that, in particular, every oligarch has a veto on all criteria and on the decision.

Proposition 2 *Let $(\mathcal{A}, \mathcal{R})$ be monotone, and assume that all criteria are doubly pivotal. Then, all aggregation rules are oligarchic (in particular, any aggregation method entails a veto on the decision). There exist anonymous aggregation rules if and only if either $\mathcal{A} = 2^C \setminus \{\emptyset\}$, or $\mathcal{R} = 2^C \setminus \{C\}$, i.e. if and only if the decision is equivalent to either the disjunction, or the conjunction of all criteria. In all other cases, all aggregation methods are dictatorial.*

Note that this result implies in particular that if all criteria are doubly pivotal, anonymous rules can only exist in the truth-functional case.

5 Conclusion

In this paper, we have focused on the perspective of the discursive dilemma as a problem of *justifying* a particular collective decision by complementing it with an appropriate aggregation of the criteria. As is well-known, there is no consistent proposition-wise aggregation rule in the standard discursive dilemma that takes the form of majority voting on the decision. Here, we have shown that relaxing the assumption of truth-functionality, i.e. allowing for a gap between the necessary and the sufficient conditions for the decision, weakens the dilemma and opens the possibility of justifying a majority decision by means of a supporting (consistent) aggregation of the criteria. The strongest possibility results (majority voting on the decision, no veto power on the criteria) emerge when the justification constraints are one-sided and monotone (Theorem 1, Proposition 1). Weakening the desiderata on the aggregation rule enlarges the scope of the possibility result but only moderately (Theorem 2). Read in a more negative vein, under monotonicity Theorem 2 yields also a substantial generalization (Proposition 2) of the oligarchic and dictatorial impossibilities that are unavoidable in the standard truth-functional case in which there is no gap between the necessary and the sufficient conditions for the justification of a decision.

Appendix A: Proofs

For the proofs of the above results we invoke our general characterization of monotonically independent aggregation methods in terms of the Intersection Property (see Nehring and Puppe (2007)), as follows. A family of *winning coalitions* is a non-empty family $\mathcal{W}$ of subsets of the set N of all individuals satisfying $[W \in \mathcal{W}$ and $W' \supseteq W] \Rightarrow W' \in \mathcal{W}$. Denote by Z the set of all propositions, i.e. $Z = C \cup \{d\}$, and by Z^* the *negation closure* of Z, i.e. $Z^* := Z \cup \{\neg p : p \in Z\}$ where $\neg p$ denotes the negation of p and doubly negated propositions are identified with the propositions themselves. A *structure of winning coalitions* on Z^* is a mapping $p \mapsto \mathcal{W}_p$ that assigns a family of winning coalitions to each proposition $p \in Z^*$ satisfying the following condition,

$$W \in \mathcal{W}_p \Leftrightarrow (N \setminus W) \notin \mathcal{W}_{\neg p}. \tag{A.1}$$

In words, a coalition is winning for p if and only if its complement is not winning for the negation of p. An aggregation rule F is called *voting by issues*, or, in our context simply *proposition-wise voting*, if for some structure of winning coalitions and all $p \in Z$,

$$p \in F(J_1, ..., J_n) \Leftrightarrow \{i : p \in J_i\} \in \mathcal{W}_p.$$

Observe that an arbitrarily given structure of winning coalitions does, in general, not generate an aggregation rule in our sense, since nothing guarantees that the collective propositions determined by proposition-wise voting form a consistent judgement set. The necessary and sufficient condition for consistency can be described as follows.

A *critical family* is a minimal subset $Q \subseteq Z^*$ of propositions that is logically inconsistent. The sets $\{p, \neg p\}$ are called *trivial* critical families. A structure of winning coalitions satisfies the *Intersection Property* if for any critical family $\{p_1, ..., p_l\} \subseteq Z^*$, and any selection $W_j \in \mathcal{W}_{p_j}$,

$$\bigcap_{j=1}^{l} W_j \neq \emptyset.$$

In Nehring and Puppe (2007, Theorem 3), we have shown that an aggregation rule satisfies unrestricted domain, voter sovereignty and monotone independence if and only if it is proposition-wise voting satisfying the Intersection Property.

Using (A.1) and the fact that the aggregation rule is monotone and hence that families of winning coalitions are closed under taking supersets, we obtain

$$\mathcal{W}_{\neg p} = \{W \subseteq N : W \cap W' \neq \emptyset \text{ for all } W' \in \mathcal{W}_p\}. \tag{A.2}$$

The following *conditional entailment relation* plays a central role. For all $p, q \in Z^*$,

$$p \geq^0 q \Leftrightarrow [p \neq \neg q \text{ and there exists a critical family containing } p \text{ and } \neg q]. \tag{A.3}$$

By $\geq$ we denote the transitive closure of $\geq^0$, and by $\equiv$ the symmetric part of $\geq$. Note that $\geq$ is *negation adapted* in the sense that $p \geq q \Leftrightarrow \neg q \geq \neg p$.

The following two lemmas are proved in Nehring and Puppe (2010).

Lemma A.1 (Contagion Lemma) *Suppose that a structure of winning coalitions satisfies the Intersection Property. Then,* $p \geq q \Rightarrow \mathcal{W}_p \subseteq \mathcal{W}_q$.

Lemma A.2 (Veto Lemma) *Suppose that a structure of winning coalitions satisfies the Intersection Property, and assume that p, q, r are jointly contained in some critical family. If $\mathcal{W}_{\neg p} \subseteq \mathcal{W}_q$, then $\{i\} \in \mathcal{W}_{\neg r}$, for some $i \in N$, i.e. voter i has a veto on r.*

The following lemma characterizes the conditional entailment relation between the criteria and the decision for any set of justification constraints.

Lemma A.3 *Let $(\mathcal{A}, \mathcal{R})$ be a set of justification constraints, then*
(i) $c \in C^+_{\mathcal{A}} \Leftrightarrow c \geq d$,
(ii) $c \in C^-_{\mathcal{A}} \Leftrightarrow \neg d \geq c$,
(iii) $c \in C^+_{\mathcal{R}} \Leftrightarrow c \geq \neg d$, *and*
(iv) $c \in C^-_{\mathcal{R}} \Leftrightarrow d \geq c$.

Proof of Lemma A.3 (i) Let $c \in C^+_{\mathcal{A}}$ and consider any subset $A \in \mathcal{A}$ with $c \in A$ such that $A \setminus \{c\} \notin \mathcal{A}$. Since $A \in \mathcal{A}$, the set $A \cup \{\neg p : p \in C \setminus A\} \cup \{\neg d\}$ forms an inconsistent family of propositions. Let P be a minimally inconsistent subset of this family. By assumption, since $A \setminus \{c\} \notin \mathcal{A}$, the set P contains c; moreover, since any combination of affirmed criteria is consistent, P also contains $\neg d$. This shows that $c \geq d$.

Now suppose conversely that $c \geq d$, i.e. that there exists a critical family $P \subseteq Z^*$ containing c and $\neg d$. Since P is inconsistent, we have $P \cap C \in \mathcal{A}$; thus, by criticality of P, we obtain that c is positively pivotal for the acceptance of d.

(ii) Let $c \in C^-_{\mathcal{A}}$ and consider any subset $A \in \mathcal{A}$ with $c \notin A$ such that $A \cup \{c\} \notin \mathcal{A}$. Since $A \in \mathcal{A}$, the set $A \cup \{\neg p : p \in C \setminus A\} \cup \{\neg d\}$ forms an inconsistent family of propositions. Let P be a minimally inconsistent subset of this family. By assumption, since $A \cup \{c\} \notin \mathcal{A}$, the set P contains $\neg c$; moreover, since any combination of affirmed criteria is consistent, P also contains $\neg d$. This shows that $\neg d \geq c$.

Now suppose conversely that $\neg d \geq c$, i.e. that there exists a critical family $P \subseteq Z^*$ containing $\neg c$ and $\neg d$. Since P is inconsistent, we have $P \cap C \in \mathcal{A}$; thus, by criticality of P, we obtain that c is negatively pivotal for the acceptance of d.

The proof of part (iii) is analogous to the proof of part (i) with d replaced by $\neg d$; similarly, the proof of part (iv) is analogous to the proof of part (ii).

Proof of Lemma 1 Evidently, we have $C^-_{\mathcal{A}} \cup C^+_{\mathcal{R}} = \emptyset$ if and only if $\mathcal{A}$ is closed under taking supersets and $\mathcal{R}$ is closed under taking subsets. Moreover, if $(\mathcal{A}', \mathcal{R}')$ arises from $(\mathcal{A}, \mathcal{R})$ by swapping the meaning of affirmation versus negation for the criteria in $\chi \subseteq C$, then

$$C^+_{\mathcal{A}'} \cup C^-_{\mathcal{R}'} = ((C^+_{\mathcal{A}} \cup C^-_{\mathcal{R}}) \setminus \chi) \cup ((C^-_{\mathcal{A}} \cup C^+_{\mathcal{R}}) \cap \chi), \tag{A.4}$$
$$C^-_{\mathcal{A}'} \cup C^+_{\mathcal{R}'} = ((C^-_{\mathcal{A}} \cup C^+_{\mathcal{R}}) \setminus \chi) \cup ((C^+_{\mathcal{A}} \cup C^-_{\mathcal{R}}) \cap \chi). \tag{A.5}$$

Now, suppose that for the set of justification constraints $(\mathcal{A}, \mathcal{R})$ we have $(C^+_{\mathcal{A}} \cup C^-_{\mathcal{R}}) \cap (C^-_{\mathcal{A}} \cup C^+_{\mathcal{R}}) = \emptyset$. Then, by taking $\chi = (C^-_{\mathcal{A}} \cup C^+_{\mathcal{R}})$, we can transform $(\mathcal{A}, \mathcal{R})$ into an equivalent set of justification constraints $(\mathcal{A}', \mathcal{R}')$ with $(C^-_{\mathcal{A}'} \cup C^+_{\mathcal{R}'}) = \emptyset$, as is immediate from (A.5). Thus, $(\mathcal{A}, \mathcal{R})$ is monotone.

Conversely, suppose that $(C^+_{\mathcal{A}} \cup C^-_{\mathcal{R}}) \cap (C^-_{\mathcal{A}} \cup C^+_{\mathcal{R}}) \neq \emptyset$. It is immediate from (A.4) and (A.5) that the criteria contained in this set are invariant under a χ-transformation. In particular, we have $C^-_{\mathcal{A}'} \cup C^+_{\mathcal{R}'} \neq \emptyset$ for all sets of justification constraints $(\mathcal{A}', \mathcal{R}')$ that are equivalent to $(\mathcal{A}, \mathcal{R})$. Thus, $(\mathcal{A}, \mathcal{R})$ is not monotone.

Proof of Lemma 2 Evidently, if $(\mathcal{A},\mathcal{R})$ is one-sided, then either $C^+_\mathcal{A}\cup C^-_\mathcal{A}=\emptyset$ or $C^+_\mathcal{R}\cup C^-_\mathcal{R}=\emptyset$, thus in particular, $(C^+_\mathcal{A}\cup C^-_\mathcal{A})\cap(C^+_\mathcal{R}\cup C^-_\mathcal{R})=\emptyset$.

Now suppose, conversely, $(C^+_\mathcal{A}\cup C^-_\mathcal{A})\cap(C^+_\mathcal{R}\cup C^-_\mathcal{R})=\emptyset$, and assume, by way of contradiction that both $\mathcal{A}$ and $\mathcal{R}$ are non-empty. Let A^m and R^m be maximal elements (with respect to set inclusion) of $\mathcal{A}$ and $\mathcal{R}$, respectively. Moreover, denote $C^0 := C\setminus(C^+_\mathcal{A}\cup C^-_\mathcal{A}\cup C^+_\mathcal{R}\cup C^-_\mathcal{R})$ (which may be empty). We have $A^m\supseteq C^+_\mathcal{R}\cup C^-_\mathcal{R}\cup C^0$; indeed, if $c\notin A^m$, then $A^m\cup\{c\}\notin\mathcal{A}$ by maximality of A^m, and therefore $c\in C^+_\mathcal{A}\cup C^-_\mathcal{A}$. Thus, we can write $A^m=\tilde{A}\cup C^+_\mathcal{R}\cup C^-_\mathcal{R}\cup C^0$ for some $\tilde{A}\subseteq C^+_\mathcal{A}\cup C^-_\mathcal{A}$. Note that $\tilde{A}$ may be empty. We have,

$$(\tilde{A}\cup R)\in\mathcal{A}\text{ for all }R\subseteq C^+_\mathcal{R}\cup C^-_\mathcal{R}\cup C^0,\tag{A.6}$$

since otherwise one would obtain $c\in C^+_\mathcal{A}\cup C^-_\mathcal{A}$ for some $c\in C^+_\mathcal{R}\cup C^-_\mathcal{R}\cup C^0$.

By a completely symmetric argument, we obtain $R^m=\tilde{R}\cup C^+_\mathcal{A}\cup C^-_\mathcal{A}\cup C^0$ for some $\tilde{R}\subseteq C^+_\mathcal{R}\cup C^-_\mathcal{R}$, and

$$(\tilde{R}\cup A)\in\mathcal{R}\text{ for all }A\subseteq C^+_\mathcal{A}\cup C^-_\mathcal{A}\cup C^0.\tag{A.7}$$

But (A.6) and (A.7) together imply $\tilde{A}\cup\tilde{R}\in\mathcal{A}\cap\mathcal{R}$, a contradiction.

Proof of Theorem 1 Suppose that $(\mathcal{A},\mathcal{R})$ is monotone and one-sided. Without loss of generality, suppose that $\mathcal{R}=\emptyset$ and $C^-_\mathcal{A}=\emptyset$ (the argument is completely analogous if $\mathcal{A}=\emptyset$ and $C^+_\mathcal{R}=\emptyset$). By Lemma A.3, any critical family is of the form $A\cup\{\neg d\}$ for some $A\subseteq C^+_\mathcal{A}$. Let $k\geq 2$ be the largest cardinality of such a critical family, and consider the following structure of winning coalitions: for all $c\in C^+_\mathcal{A}$, the family of winning coalitions is given by

$$\mathcal{W}_c=\mathcal{W}_{\frac{2k-3}{2k-2}}:=\{W\subseteq N:\#W>\frac{2k-3}{2k-2}\cdot n\},$$

while all other criteria (if any) and the decision d are determined by majority voting, thus in particular,

$$\mathcal{W}_d=\mathcal{W}_{\frac{1}{2}}:=\{W\subseteq N:\#W>\frac{1}{2}\cdot n\}.$$

It is easily verified that, for any collection of $k-1$ sets $W_j\in\mathcal{W}_{\frac{2k-3}{2k-2}}$, $j=1,...,k-1$ and any set $W\in\mathcal{W}_{\frac{1}{2}}$, one has

$$\left(\bigcap_{j=1}^{k-1}W_j\right)\cap W\neq\emptyset.$$

Thus, the given structure of winning coalitions satisfies the Intersection Property and hence gives rise to a (consistent) aggregation rule if n is odd. Note that the aggregation rule is evidently anonymous. It entails no veto power if $n\geq 2(k-1)$.

To prove that monotonicity of the justification constraints is necessary, suppose to the contrary that $(C^+_\mathcal{A}\cup C^-_\mathcal{R})\cap(C^-_\mathcal{A}\cup C^+_\mathcal{R})\neq\emptyset$. There are four cases to consider.
(i) Suppose that there exists $c\in C^+_\mathcal{A}\cap C^-_\mathcal{A}$. Then, by Lemma A.3, $c\geq d$ and $\neg d\geq c$, i.e. there exists a critical family P containing c and $\neg d$, and a critical family P' containing $\neg c$ and $\neg d$. Majority voting on d implies $\mathcal{W}_d=\mathcal{W}_{\neg d}$ and thus, by Lemma A.1, $\mathcal{W}_c=\mathcal{W}_{\neg c}=\mathcal{W}_d=\mathcal{W}_{\neg d}$ (recall that $\geq$ is negation adapted). One of the critical

families P or P' must contain at least three elements since otherwise d would always have to be accepted. Without loss of generality suppose that $c' \in P$ for $c' \neq c$. By Lemma A.2, for some i, $\{i\} \in \mathcal{W}_{\neg c'}$, i.e. i has a veto.

(ii) Suppose that there exists $c \in C^+_{\mathcal{A}} \cap C^+_{\mathcal{R}}$. Then, by Lemma A.3, $c \geq d$ and $c \geq \neg d$, i.e. there exists a critical family P containing c and $\neg d$, and a critical family P' containing c and d. Majority voting on d implies $\mathcal{W}_d = \mathcal{W}_{\neg d}$. Now observe that the set $(P \cup P') \setminus \{d, \neg d\}$ is inconsistent, since otherwise it would have to be consistent with either d or $\neg d$. This implies by the assumed logical independence of the criteria that, for some $c' \neq c$, $[c' \in P$ and $\neg c' \in P']$ or $[c' \in P'$ and $\neg c' \in P]$. In either case, this implies $\mathcal{W}_{c'} = \mathcal{W}_{\neg c'} = \mathcal{W}_d = \mathcal{W}_{\neg d}$ as in case (i) (applied to c'). By Lemma A.2, for some i, $\{i\} \in \mathcal{W}_{\neg c}$, i.e. i has a veto on c.

(iii) Suppose that there exists $c \in C^-_{\mathcal{R}} \cap C^-_{\mathcal{A}}$. Then, by Lemma A.3, $d \geq c$ and $\neg d \geq c$, i.e. there exists a critical family P containing $\neg c$ and d, and a critical family P' containing $\neg c$ and $\neg d$. By the argument of case (ii) with c replaced by $\neg c$ one obtains a veto on $\neg c$.

(iv) Finally, suppose that there exists $c \in C^-_{\mathcal{R}} \cap C^+_{\mathcal{R}}$. Then, by Lemma A.3, $d \geq c$ and $c \geq \neg d$, i.e. there exists a critical family P containing $\neg c$ and d, and a critical family P' containing c and d. Majority voting on d implies $\mathcal{W}_c = \mathcal{W}_{\neg c} = \mathcal{W}_d = \mathcal{W}_{\neg d}$. One of the critical families P or P' must contain at least three elements since otherwise d would always have to be rejected. As in case (i), this implies a veto using Lemma A.2.

Since in all four cases a veto results, we conclude that the existence of an aggregation rule with majority voting on the decision and no veto power implies that $(C^+_{\mathcal{A}} \cup C^-_{\mathcal{R}}) \cap (C^-_{\mathcal{A}} \cup C^+_{\mathcal{R}}) = \emptyset$. By Lemma 1, justification constraints admitting such aggregation rules must thus be monotone.

To prove necessity of one-sidedness, suppose to the contrary that $(C^+_{\mathcal{A}} \cup C^-_{\mathcal{A}}) \cap (C^+_{\mathcal{R}} \cup C^-_{\mathcal{R}}) \neq \emptyset$. Again, there are four cases to consider, two of which have already been covered above.

(v) Suppose that there exists $c \in C^+_{\mathcal{A}} \cap C^-_{\mathcal{R}}$. Then, by Lemma A.3, $c \geq d$ and $d \geq c$, i.e. there exists a critical family P containing c and $\neg d$, and a critical family P' containing $\neg c$ and d. By Lemma A.1, we obtain that $\mathcal{W}_c = \mathcal{W}_d$ and $\mathcal{W}_{\neg c} = \mathcal{W}_{\neg d}$. One of the critical families P or P' must contain at least three elements since otherwise c and d would be logically equivalent. Without loss of generality suppose that $c' \in P$ for $c' \neq c$. By Lemma A.2, for some i, $\{i\} \in \mathcal{W}_{\neg c'}$, i.e. i has a veto. For later reference, we note that the derivation of the veto in the present case did not use the assumption that the aggregation takes the form of majority voting on d.

(vi) Suppose that there exists $c \in C^-_{\mathcal{A}} \cap C^+_{\mathcal{R}}$. Then, by Lemma A.3, $\neg d \geq c$ and $c \geq \neg d$, i.e. there exists a critical family P containing $\neg c$ and $\neg d$, and a critical family P' containing c and d. As in case (v), we obtain $\mathcal{W}_{\neg c} = \mathcal{W}_d$ and $\mathcal{W}_c = \mathcal{W}_{\neg d}$ using Lemma A.1. One of the critical families P or P' must contain at least three elements since otherwise c and $\neg d$ would be logically equivalent. As above, this implies a veto using Lemma A.2.

The other two cases have already been treated above as cases (ii) and (iii). Again in all four cases a veto results, and we conclude that the existence of an aggregation rule with majority voting on the decision and no veto power implies that $(C^+_{\mathcal{A}} \cup C^-_{\mathcal{A}}) \cap (C^+_{\mathcal{R}} \cup C^-_{\mathcal{R}}) = \emptyset$. By Lemma 2, justification constraints admitting such aggregation rules must thus be one-sided. This concludes the proof of Theorem 1.

The remaining proofs concern the monotone case, and we may therefore assume without loss of generality that $C_{\mathcal{A}}^{-} \cup C_{\mathcal{R}}^{+} = \emptyset$ in all what follows.

Proof of Proposition 1 By Theorem 1, conditions (ii) and (iii) are equivalent; moreover, (ii) obviously implies (i). The implication "(i) $\Rightarrow$ (iii)" follows as in case (v) of the proof of Theorem 1 above. Indeed, an inspection of the argument given there shows that in the monotone but two-sided case one can deduce the existence of a veto without further assumption.

The key to the proof of Theorem 2 is the observation that in the monotone case, all critical families are either of the form $\{c_1, ..., c_k\} \cup \{\neg d\}$ where $\{c_1, ..., c_k\}$ is a minimal element in $\mathcal{A}$, or of the form $\{\neg c_1, ..., \neg c_l\} \cup \{d\}$ where $C \setminus \{c_1, ..., c_l\}$ is a maximal element in $\mathcal{R}$.

Proof of Theorem 2 (a) Suppose first that some minimal element of $\mathcal{A}$ contains more than one doubly pivotal criterion. By the preceding observation, this implies that there exists a critical family containing $\neg d$ and at least two doubly pivotal criteria, say c and c'. By Lemma A.3, we have $c \geq d$ and $d \geq c$ as well as $c' \geq d$ and $d \geq c'$, thus by Lemma A.1, $\mathcal{W}_c = \mathcal{W}_{c'} = \mathcal{W}_d$. By (A.2), this also implies $\mathcal{W}_{\neg c} = \mathcal{W}_{\neg c'} = \mathcal{W}_{\neg d}$. By Lemma A.2, we have $\{i\} \in \mathcal{W}_{\neg c'}$ and hence also $\{i\} \in \mathcal{W}_{\neg d}$ for some i, i.e. individual i has a veto on the decision. A completely symmetric argument shows that a veto on the decision also results if some maximal element of $\mathcal{R}$ omits more than one doubly pivotal criterion. This proves the necessity of the stated condition for the existence of a monotonically independent aggregation rule with no veto power on the decision.

To prove its sufficiency, consider the following proposition-wise aggregation rule: the outcome decision d and all doubly pivotal criteria are decided by majority voting; moreover, any criterion $c \in C_{\mathcal{A}}^{+} \setminus C_{\mathcal{R}}^{-}$ is collectively affirmed only under unanimous consent, and any criterion $c \in C_{\mathcal{R}}^{-} \setminus C_{\mathcal{A}}^{+}$ is collectively negated only under unanimous consent. Using the Intersection Property and the special structure of critical families in the monotone case, this aggregation rule is easily seen to be consistent. Indeed, consider e.g. a critical family of the form $A \cup \{\neg d\}$ for some minimal $A \in \mathcal{A}$ with $\#A = l$. By minimality, $A \subseteq C_{\mathcal{A}}^{+}$, and by assumption A contains at most one doubly pivotal criterion. Thus, the only winning coalition for at least $l - 1$ elements of A is the grand coalition N. This immediately implies that the given structure of winning coalitions satisfies the Intersection Property. A similar argument applies to critical families of the form $R \cup \{d\}$ for some maximal $R \in \mathcal{R}$. Note also that the given rule is anonymous and takes the form of majority voting on the decision.

(b) By the arguments given in the proof of part (a), if some minimal element of $\mathcal{A}$ contains more than one doubly pivotal criterion, we can find an individual i such that $\{i\} \in \mathcal{W}_{\neg d}$. Similarly, if some maximal element of $\mathcal{R}$ omits more than one doubly pivotal criterion, we can find i' such that $\{i'\} \in \mathcal{W}_d$. But by (A.2), we must in fact have $i = i'$, i.e. the aggregation rule is dictatorial on the decision (and on all doubly pivotal criteria).

Now suppose, conversely, that all maximal elements of $\mathcal{R}$ omit at most one doubly pivotal criterion. Then, the rule according to which d and any doubly pivotal criterion are collectively affirmed only under unanimous consent, while all other criteria are decided dictatorially according to the judgment set of some fixed individual is consistent and entails no-dictatorship on the decision. Moreover, it is even anonymous on the decision. A symmetric argument applies if all minimal elements of $\mathcal{A}$ contain at most one doubly pivotal criterion.

Proof of Proposition 2 By assumption, $C^+_{\mathcal{A}} = C^-_{\mathcal{R}} = C$, hence by Lemmas A.1 and A.3, $\mathcal{W}_c = \mathcal{W}_d$ and $\mathcal{W}_{\neg c} = \mathcal{W}_{\neg d}$ for all $c \in C$. Let P_c be a critical family containing c and $\neg d$ and let $P_{\neg c}$ be a critical family containing $\neg c$ and d (these must exist since c is doubly pivotal). It is not possible that both P_c and $P_{\neg c}$ have only two elements, since otherwise c would be logically equivalent to d. Suppose that P_c has at least three elements, say $P_c \supseteq \{c, c', \neg d\}$. Then, by Lemma A.2, $\{i\} \in \mathcal{W}_{\neg c'}$ for some i, and therefore $\{i\} \in \mathcal{W}_{\neg c}$ for all c and $\{i\} \in \mathcal{W}_{\neg d}$. Similarly, if $P_{\neg c}$ has at least three elements, we obtain $\{i'\} \in \mathcal{W}_c$ for all c and $\{i'\} \in \mathcal{W}_d$ for some i'. Now if, for some c, both P_c and $P_{\neg c}$ contain at least three elements, we must have $i = i'$ by (A.2), and the only consistent aggregation rules are dictatorial.

If all critical families of the form P_c have two elements, then the affirmation of any single criterion already forces the acceptance of d, i.e. $\mathcal{A} = 2^C \setminus \{\emptyset\}$ and d corresponds to the logical disjunction of the criteria. In this case, let $M := \{h \in N : \{h\} \in \mathcal{W}_d\}$. Using the Intersection Property and (A.2), it is easily verified that consistency forces the aggregation method to be oligarchic with oligarchy M. If $M = N$, we obtain the anonymous (and consistent) aggregation rule according to which each criterion and the decision are collectively rejected if and only if every voter rejects them.

Similarly, if all critical families of the form $P_{\neg c}$ have two elements, then the negation of any single criterion forces the rejection of d, i.e. $\mathcal{R} = 2^C \setminus \{C\}$ and d corresponds to the logical conjunction of the criteria. In this case, let $M := \{h \in N : \{h\} \in \mathcal{W}_{\neg d}\}$. Again it is easily verified that consistency forces the aggregation method to be oligarchic with oligarchy M. If $M = N$, we obtain the anonymous (and consistent) aggregation rule according to which each criterion and the decision are collectively affirmed if and only if every voter affirms them.

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