Justifiable Group Choice[1]

Klaus Nehring

Department of Economics, University of California at Davis
Davis, CA 95616, U.S.A.
kdnehring@ucdavis.edu

and

Clemens Puppe

Department of Economics, University of Karlsruhe
D – 76128 Karlsruhe, Germany
puppe@wior.uni-karlsruhe.de

May 2007

[1]A preliminary version of this paper has been presented at the Cowles Foundation Workshop on "Aggregation of Opinions" at Yale University in September 2006 under the title "Reason-Based Group Choice under Partial Consensus." The second author wishes to thank the Institute of Advanced Study, Princeton, and the Economic Theory Center at the University of Melbourne where part of this research was conducted for their hospitality. Further acknowledgements to be added.

Abstract. *We study the judgment aggregation problem from the perspective of justifying a particular collective decision by a corresponding aggregation on the decision criteria. The first main result characterizes the logical relations between the decision and the decision criteria that enable justification of a majority decision through a proposition-wise aggregation rule with no veto power on the criteria. While the well-studied "doctrinal paradox" provides a negative example in which no such justification exists, we show that genuine possibility results emerge if there is a gap between the necessary and the sufficient conditions for the decision. A number of examples and applications are discussed.*

1 Introduction

Consider a group of individuals who have to take a collective decision and want to justify their decision based on reasons which reflect the opinions of the group members. As an example, consider a court of three judges who has to decide on the liability of a defendant (proposition d). Suppose that, by legal doctrine, the defendant is to be held liable if and only if (s)he did a particular action (proposition c_1) and no special exculpatory circumstances apply (proposition c_2). If the court members' judgments are as shown in Table 1, proposition-wise majority voting on both the decision and the "reasons" leads to a set of collective judgments that is inconsistent with the legal doctrine: the affirmation of both c_1 and c_2 but at the same time the rejection of d.

	action done (c_1)	no special circumstances (c_2)	liable (d)
Judge 1	true	true	true
Judge 2	true	false	false
Judge 3	false	true	false
Majority	true	true	false

Table 1: The doctrinal paradox / discursive dilemma

This is the well-known "doctrinal paradox" or "discursive dilemma" studied in the judgment aggregation literature, following Kornhauser and Sager (1986) and List and Pettit (2002). The literature has demonstrated the robustness of the discursive dilemma, both with respect to the class of admissible aggregation methods and with respect to the structure of the logical relation between the "decision" (d) and the reasons or "decision criteria" (c_1 and c_2).[2]

In Nehring and Puppe (2006b), we have shown that the discursive dilemma extends to all "truth-functional" contexts, i.e. to all situations in which a binary decision is completely determined by the judgment on a set of decision criteria. In such situations the only consistent proposition-wise aggregation methods are oligarchic and often even dictatorial. For instance, in the doctrinal paradox above the only anonymous proposition-wise aggregation method is the unanimity rule according to which the collective affirmation of each proposition requires unanimous consent.[3]

Forcing truth-functionality is, however, restrictive and arguably unnatural in the present case since the presence of "special circumstances" creates a scope of discretion. Specifically, assume that the logical interrelation between the decision and the criteria is as follows: (i) negating that the action has been done necessarily leads to the verdict "not liable," no matter whether or not special circumstances are granted, (ii) affirming both c_1 and c_2 (i.e. affirming that the action has been done but denying special circumstances) necessarily implies the verdict "liable," and (iii) affirming c_1 but negating c_2

[2]See, e.g., Pauly and van Hees (2006), Dietrich (2006), Dokow and Holzman (2005), Nehring and Puppe (2006b), Dietrich and List (2006). List and Puppe (2007) provide a survey of the recent literature on judgment aggregation.

[3]Whether there exist anonymous proposition-wise aggregation methods in truth-functional contexts depends on the precise logical relation between the decision and the decision criteria. In many cases, there are in fact no anonymous rules at all, see Dokow and Holzman (2005) and Nehring and Puppe (2006b).

(thus granting special circumstances) is consistent with either a positive and a negative verdict. Clause (iii) creates a gap between the necessary and the sufficient conditions for the decision, thereby introducing a "scope of discretion" that reflects the assessment of the special circumstances for the case at hand.

Relaxing the assumption of truth-functionality in this way allows one to resolve the doctrinal paradox. Specifically, a consistent proposition-wise aggregation method can be obtained in a natural way by requiring unanimous consent in order to affirm c_2 (i.e. to deny the presence of special circumstances), deciding all other propositions by majority vote as before. If the individual judgments are as above, this aggregation method results in the collective judgment according to which c_1 is affirmed, but special circumstances are granted and the verdict is "not liable" (see Table 2).

	action done (c_1)	no special circumstances (c_2)	liable (d)
Judge 1	true	true	true
Judge 2	true	false	false
Judge 3	false	true	false
Majority with unanimity on c_2	true	false	false

Table 2: The doctrinal paradox resolved

As is easily verified, the suggested aggregation method always yields a consistent collective judgment, no matter what the individual judgments are.[4] Thus, reaching a verdict as the result of a majority vote on the decision can be *justified* by an appropriate independent aggregation of the decision criteria. In this paper, we ask in which cases the gap between necessary and sufficient conditions for justifying an outcome decision opens interesting possibility results more generally.

Viewing the group decision problem as a problem of justification shall be taken to mean that (a) the group uses a *given* procedure which aggregates the individuals' views on the decision, e.g. majority voting as in the above example, and (b) there is a set of agreed upon constraints on how outcome decisions can be justified by judgments on the decision criteria, both at the individual and social level. While this viewpoint is consistent with some approaches in the literature, for instance with Pettit's (2004) notion of conversability, it has to be distinguished from an interpretation in terms of "reason-based" group choice that attempts to optimally use the information contained in the individuals' judgments on the decision criteria ("premises"). Note that under the latter interpretation, the so-called premise-based procedure appears to be an attractive way out of the discursive dilemma.[5] However, from the present perspective of justifying a *given* collective decision the premise-based procedure is simply not applicable.

The question posed by a group justification problem is to find aggregation procedures on the decision criteria that appropriately reflect the individuals' views on these

[4]Indeed, if c_2 is collectively rejected, collective consistency is guaranteed by individual consistency. On the other hand, c_2 can only be collectively affirmed if *all* individuals affirm it, in which case the decision d is uniquely determined by the judgment on c_1, again guaranteeing collective consistency under the considered aggregation method.

[5]In a truth-functional context, the *premise-based procedure* consists in aggregating the premises and deriving the decision by logical implication.

and at the same time respect the justification constraints at the aggregate level. A natural starting point is to require independence among the decision criteria, especially when these are logically independent as we shall assume throughout. Compared to a general, abstract judgment aggregation approach, the justification perspective motivates the distinction between the aggregation of the outcome decision versus the aggregation of the decision criteria, and in particular the imposition of stronger normative requirements on the former than on the latter.

We will ask specifically when justification is possible with majority voting on the decision. It is immediate from general results on judgment aggregation that majority voting on the decision is only in trivial cases consistent with majority voting on all decision criteria (see, e.g. List and Puppe (2007)). But interesting possibilities emerge if one combines majority voting on the outcome decision with no veto power on the criteria. Our first main result (Theorem 1) characterizes the class of all functional relations between the decision and the decision criteria, henceforth "justification constraints," that enable aggregation rules with majority voting on the decision and no veto power on the criteria.[6] This class is shown to coincide with the class of constraints that are "one-sided" and "monotone." A set of constraints is *one-sided* if either (i) no combination of affirmed criteria forces acceptance of the decision, or (ii) no combination of affirmed criteria forces rejection the decision. A simple example of one-sided justification constraints is Dietrich's (2007) subjunctive implication (see Section 2 for more discussion and further examples). A set of constraints is *monotone* if the labeling of the criteria vis-á-vis their negations can be chosen in such a way that (i) affirming additional criteria can never turn acceptance of the decision into non-acceptance, and (ii) negating additional criteria can never turn rejection of the decision into non-rejection.

There are two possible avenues to obtain further possibility results by weakening the aggregation requirements with respect to either the outcome decision or the decision criteria. By itself, the former is ineffective: requiring majority voting on the decision is without loss of generality in the monotone case under no veto power on the decision criteria (cf. Proposition 1 below). By contrast, dropping the no veto requirement on the aggregation of the criteria leads to new possibilities as illustrated in the introductory example (note that the example displays two-sided justification constraints). Our second set of results (Theorems 2 and 3) characterize the monotone justification constraints that enable justification of more general aggregation procedures on the decision. Specifically, we consider aggregation with no veto *on the decision* and non-dictatorial aggregation.

As a further illustration of the justification perspective developed in this paper, consider the following re-interpretation of our introductory example. An editor of an academic journal wants to justify the publication decision in case of a particular submitted work. The editor asks three referees to evaluate the paper according to the criteria of correctness of results (c_1) and originality of ideas (c_2), and to give a publication recommendation (d). In this context, one would probably consider correctness of the results to be a necessary condition for a positive publication recommendation, i.e. ($\neg c_1 \to \neg d$). Moreover, correctness and originality are arguably jointly sufficient for a positive decision, i.e. ($c_1 \wedge c_2 \to d$). On the other hand, *sometimes* one may wish to recommend publication of correct results even if the ideas are not considered

[6]Remember that in the truth-functional case all admissible aggregation rules are oligarchic; in particular, every admissible aggregation rule entails veto power on *all* propositions (decision and decision criteria).

to be original, i.e. the judgment $c_1 \wedge \neg c_2$ would be considered to be consistent with either a positive and a negative publication recommendation. The example is formally equivalent to the example underlying Table 2. Note in particular that the justification constraints are not one-sided since some evaluations of the criteria force a positive publication decision while others force a negative decision. By the results established here, an outcome decision based on majority voting on the publication recommendations can be justified via an independent aggregation rule on the criteria, but all such aggregation rules entail veto power on at least one criterion for at least one agent. As in Table 2 above, an admissible rule emerges by taking majority voting on both the decision d and the first criterion c_1 (correctness of results), and to affirm the second criterion c_2 (originality of ideas) if and only if it is unanimously accepted. It follows from the methods developed here that this is in fact the only anonymous aggregation rule in the present example.

The remainder of the paper is organized as follows. In the following section, we introduce our framework and show its applicability in a wide variety of contexts. Section 3 contains the first main characterization result. Section 4 analyzes monotone justification constraints in greater detail and demonstrates that weaker possibility results may obtain also in the two-sided case. Section 5 concludes; all proofs are collected in an appendix.

2 Framework and Further Examples

The set $C = \{c_1, ...c_m\}$ represents a collection of *criteria* for a binary decision d. The elements of $C \cup \{d\}$ and their negations are also referred to as *propositions*. A *judgment* is a subset $J \subseteq C \cup \{d\}$ with the interpretation that the elements of J are exactly the accepted propositions. By consequence, $(C \cup \{d\}) \setminus J$ is the set of propositions rejected by J.[7] In particular, $d \in J$ (resp. $d \notin J$) means that the judgment J entails the acceptance (resp. rejection) of d. Throughout, we assume that any combination of affirmed criteria is logically possible. Individual judgments are denoted by J_i with the subscripts referring to individuals.

2.1 Justification Constraints

There is unanimous agreement among individuals that some combinations of affirmed criteria force the acceptance of d; similarly, other combinations of affirmed criteria force the rejection of d. Formally, denote by $\mathcal{A} \subseteq 2^C$ the collection of subsets $A \subseteq C$ for which it is agreed upon that affirmation of exactly the criteria in A leads to the acceptance of the decision d. We call $\mathcal{A}$ the *acceptance region* of d. Similarly, $\mathcal{R} \subseteq 2^C$ is the collection of subsets $R \subseteq C$ for which it is agreed upon that affirmation of exactly the criteria in R leads to the rejection of the decision d. We call $\mathcal{R}$ the *rejection region* of d. A pair $(\mathcal{A}, \mathcal{R})$ is referred to as a set of *justification constraints*, or simply *constraints*. A judgment J is *consistent* with the constraints if $J \cap C \in \mathcal{A}$ implies $d \in J$ and $J \cap C \in \mathcal{R}$ implies $d \notin J$.

Throughout, we impose the following non-triviality requirements: $\mathcal{A} \cup \mathcal{R} \neq \emptyset$ (the decision is at least sometimes restricted), $\mathcal{A} \cap \mathcal{R} = \emptyset$ (any combination of affirmed

[7] We are always employing standard two-valued propositional logic and identify doubly negated propositions with the propositions themselves.

criteria must be consistent with some decision), $\mathcal{A} \neq 2^C$ (the decision does not always have to be accepted), and $\mathcal{R} \neq 2^C$ (the decision does not always have to be rejected). Moreover, we assume that the decision is not equivalent to any single criterion, formally, for no $c \in C$, $[A \in \mathcal{A} \Leftrightarrow c \in A$ and $R \in \mathcal{R} \Leftrightarrow c \notin R]$, and similarly, for no $c \in C$, $[A \in \mathcal{A} \Leftrightarrow c \notin A$ and $R \in \mathcal{R} \Leftrightarrow c \in R]$.

For the following, it is important that there may be a gap between the acceptance and rejection regions, i.e. there may be combinations of affirmed criteria for which it is neither agreed that they lead to acceptance, nor to rejection of the decision. The special case in which any combination of affirmed criteria either leads to the acceptance or to the rejection of the decision, i.e. the case $\mathcal{A} \cup \mathcal{R} = 2^C$ is referred to as the *truth-functional* case and has already been studied before (see Nehring and Puppe (2006b)). In the truth-functional case when there is no gap between the necessary and the sufficient conditions for the outcome decision, we will refer to the pair $(\mathcal{A}, \mathcal{R})$ also as a *decision rule*. Below, we provide a number of examples that motivate to consider the non-truth-functional case.

2.2 Aggregation Rules

Denote by $N = \{1, ..., n\}$ the set of individuals. An *aggregation rule* is a mapping F that assigns a consistent collective judgment J to each profile $(J_1, ..., J_n)$ of consistent individual judgments. Throughout, we require the following properties. First, F is defined for all logically possible combinations of consistent individual judgments ("unrestricted domain"). Second, any consistent judgment J is the collective result of F for some suitable profile of individual judgments ("voter sovereignty").[8] Finally, we require the following condition of "monotone independence." Consider $(J_1, ..., J_n)$ and $(J'_1, ..., J'_n)$ such that, for some p and all i, $p \in J_i \Rightarrow p \in J'_i$. Then, $p \in F(J_1, ..., J_n) \Rightarrow p \in F(J'_1, ..., J'_n)$. Similarly, if for some p and all i, $p \notin J_i \Rightarrow p \notin J'_i$, then $p \notin F(J_1, ..., J_n) \Rightarrow p \notin F(J'_1, ..., J'_n)$. Monotone Independence thus asserts that if the collective judgment entails the affirmation (resp. negation) of p, and if the individual support for p increases (resp. decreases), then p must remain affirmed (resp. negated) in the collective judgment. From the perspective of the present paper, the independence requirement is strong but arguably very natural: independence of the aggregation among the decision criteria is justified by their logical independence, and independence of the aggregation of the decision is implied by the task to justify a collective decision resulting from a *given* aggregation of individual outcome decisions.

2.3 Examples

The following examples are meant to illustrate the great variety of situations of the above kind, i.e. those in which there may exist a gap between the necessary and the sufficient condition for a particular decision.

1. Substitutable decision criteria. A simple example of a set of justification constraints is given by $\mathcal{A} = \{A \subseteq C : \#A \geq k\}$ and $\mathcal{R} = \{R \subseteq C : \#R \leq l\}$, where $k > l$. For instance, a job candidate will be hired if she fares well on a sufficiently high number of criteria, and is not hired if she fares well only on a low number of criteria. In intermediate case, either a positive and a negative hiring decision is justifiable. Note

[8] A slightly stronger condition ("unanimity") would require that J be the collective judgment whenever all individuals agree on J.

that the decision is truth-functionally determined by the criteria if and only if $k = l+1$.

2. Partial consensus. Suppose that there is a set of different truth-functional decision rules $\{(\mathcal{A}_k, \mathcal{R}_k)\}_{k\in K}$ each of which is considered to reflect a reasonable "point of view." Taking $\mathcal{A} := \cap_k \mathcal{A}_k$ and $\mathcal{R} := \cap_k \mathcal{R}_k$ in such a situation creates a gap between the necessary and the sufficient conditions for the outcome decision and reflects a weak notion of justifiability. A particular decision is justifiable *tout court* if and only if it is justifiable from some reasonable point of view. As a concrete example one may take again the hiring procedure, say at an academic institution, in which the constitutional rules allow each prospective member of a committee to use any of the decision rules classified ex-ante as reasonable. Note that the domain of the admissible aggregation rules is thus still the Cartesian product of a common domain of individual judgments although different *actual* members of a particular committee may operate with different decision rules. For a "non-Cartesian" model in this context, in which different individuals face different domain restrictions corresponding to different logical constraints on the decision, see Miller (2007).

3. Partial elicitation. Suppose now that each individual uses the *same* truth-functional decision rule, but only a subset of criteria are elicited. Say, for instance, that there are 10 criteria and the common (truth-functional) decision rule prescribes a positive hiring decision if and only if the candidate fares well on at least 5 criteria. If e.g. 3 of the criteria are not elicited, a positive assessment on at least 2 and no more than 4 of the remaining criteria is consistent with either a positive or a negative decision. On the other hand, a positive assessment in only one of the elicited criteria would not be consistent with a positive hiring decision.

4. Acceptance Thresholds. A large class of examples emerges by interpreting the affirmation of a proposition as "sufficient confidence" in its truth in an uncertain environment. For instance, suppose that the criteria c_1 and c_2 are affirmed if and only if the probability of a positive evaluation exceeds some common threshold q, where $0 < q < 1$. On the other hand, assume that the decision d is accepted if and only if the product of the individual probabilities for the two criteria exceeds r. Most interesting and quite natural is the case of $r = q^2$, which results in non-truth-functional justification constraints since affirming exactly one criterion is consistent with either a positive and a negative decision, depending on the precise probabilities.[9]

5. Subjunctive Implication. A special class of non-truth-functional decision contexts arise by considering "subjunctive implications" as introduced by Dietrich (2007). Specifically, consider a decision d and the two decision criteria a and $a \hookrightarrow d$, where the latter is interpreted as a *subjunctive implication.* By definition, this means that the decision d is restricted only if both a and $a \hookrightarrow d$ are affirmed, in which case d has to be accepted. Thus, $\mathcal{A} = \{a, a \hookrightarrow d\}$ and $\mathcal{R} = \emptyset$ in this case.[10] More generally, any decision problem with decision d and set of criteria $C = \{a_1, a_1 \hookrightarrow a_2, a_2 \hookrightarrow a_3, ..., a_k \hookrightarrow d\}$ gives rise to constraints of the form $\mathcal{A} = \{C\}$ and $\mathcal{R} = \emptyset$, provided that all implications

[9] This example suggests to study the judgment aggregation problem in a probabilistic framework, see Nehring (2007) for such a model. For a related account of the discursive dilemma as a form of the well-known "lottery paradox," see Levi (2004).

[10] See Dietrich (2007) for a detailed analysis and interpretation of judgment aggregation problems with subjunctive implications.

are interpreted in the subjunctive sense.[11]

2.4 One-Sidedness and Monotonicity

The following two properties of justification constraints will play a central role in the our analysis.

A set of constraints is called *one-sided* if either $\mathcal{A}$ or $\mathcal{R}$ is empty, i.e. if either the rejection of the decision is always consistent, or its acceptance is always consistent. One-sidedness is clearly restrictive but arguably quite natural in a number of applications. Natural examples arise under the partial consensus interpretation above. Specifically, consider again a hiring decision at an academic department. The members of the hiring committee may agree on certain *necessary* criteria that a prospective candidate would have to satisfy, say teaching experience and decent research. This would lead to a non-empty rejection region. At the same time, the hiring committee may find it difficult to agree on sets of criteria that are jointly *sufficient* for a positive hiring decision. For instance, the committee members may come from different areas with different performance standards, etc. In this case, the acceptance region, i.e. the intersection of the individual acceptance regions, could well be empty.

In order to define the second concept, a monotonicity property, we need some auxiliary concepts. For any two subsets $A \subseteq C$ and $\chi \subseteq C$, denote by $A^\chi := [A \setminus \chi] \cup [(C \setminus A) \cap \chi]$. The intended interpretation is as follows. Suppose that for the criteria in χ the meaning of affirmation versus negation is exchanged. Then, affirming exactly the criteria in A (and thus negating the criteria in $C \setminus A$) before the swap means the same as affirming the criteria in A^χ after the swap. Two justification constraints $(\mathcal{A}, \mathcal{R})$ and $(\mathcal{A}', \mathcal{R}')$, defined on the same set C of criteria, are said to be *equivalent* if there exists a subset $\chi \subseteq C$ such that $A \in \mathcal{A} \Leftrightarrow A^\chi \in \mathcal{A}'$ and $R \in \mathcal{R} \Leftrightarrow R^\chi \in \mathcal{R}'$. For instance, the constraints $(\mathcal{A}, \mathcal{R})$ and $(\mathcal{A}', \mathcal{R}')$ defined on $C = \{c_1, c_2\}$ with $\mathcal{A} = \{\{c_2\}\}$, $\mathcal{R} = \{\{c_1\}, \{c_1, c_2\}\}$, $\mathcal{A}' = \{\{c_1, c_2\}\}$ and $\mathcal{R}' = \{\emptyset, \{c_2\}\}$ are equivalent, as is easily verified by taking $\chi = \{c_1\}$.

A set of constraints $(\mathcal{A}, \mathcal{R})$ is called *monotone* if there exists an equivalent set of constraints $(\mathcal{A}', \mathcal{R}')$ such that $\mathcal{A}'$ is closed under taking supersets and $\mathcal{R}'$ is closed under taking subsets.[12] Monotone justification constraints are arguably the most relevant ones. They correspond to contexts in which the criteria/negation pairs can be labeled in such a way that the affirmation of a criterion always has a positive effect on the decision, no matter what other criteria are affirmed. For instance, in the hiring decision example one can arguably often label the criteria in such a way that affirming additional criteria is never harmful for a positive hiring decision. On the other hand, monotonicity does rule out, say, standards of "mediocrity," according to which job candidates are deemed eligible if they satisfy a certain number of desirable criteria but not too many of them. Specifically, the set of constraints $(\mathcal{A}, \mathcal{R})$ with $\mathcal{A} = \{A \subseteq C : k \leq \#A \leq l\}$ is not monotone whenever $0 < k \leq l < m$. Note that if $m = 2$, $k = l = 1$ and $\mathcal{R} = 2^C \setminus \mathcal{A}$ the resulting decision rule is equivalent to a rule according to which the decision is accepted if and only if the two criteria are either both affirmed or both negated.

[11] Including some of the propositions $a_2, ..., a_k$ would destroy the logical independence of the decision criteria as assumed here. For an analysis of the case with logically interdependent premises, see Dietrich (2007).

[12] As can be seen from the simple example just given, closedness under taking supersets and subsets is in general not preserved by replacing one set of constraints by an equivalent one.

3 A General Possibility Result

The following is our first main result. A voter i is said to have a *veto* on proposition p if i can either force p to be affirmed in the social judgment, or force p to be negated in the social judgment.

Theorem 1 (majority voting on decision, no veto on criteria) *A set of justification constraints admits monotonically independent aggregation rules with no veto power and majority voting on the decision if and only if it is one-sided and monotone.*

The proof of this result, provided in the appendix, in fact shows that monotonicity and one-sidedness jointly guarantee the existence of an *anonymous* aggregation rule with the stated properties. In the introduction above, we provided an example of an independent aggregation rule with veto power and majority voting on the decision in the two-sided case (cf. Table 2); this shows that the no veto condition cannot be dropped in Theorem 1. The following example shows that, similarly, in the one-sided but non-monotone case independent aggregation rules with veto power on some criteria and majority voting on the decision may exist. Specifically, assume that there are three criteria c_1, c_2 and c_3, and that the justification constraints can be described by the two implications $(c_1 \wedge c_2 \to d)$ and $(\neg c_1 \wedge c_3 \to d)$. As can be verified, these constraints are one-sided but not monotone.[13] The aggregation rule according to which the outcome and the first criterion are decided by majority voting while the other two criteria are affirmed only under unanimous consent is easily seen to be consistent. Note that, by Theorem 1, the entailed veto on some of the criteria cannot be avoided.

The proof of Theorem 1 relies on several auxiliary results, some of which are of independent interest. To formulate them, we need the following additional definitions. Given a set of constraints $(\mathcal{A}, \mathcal{R})$, say that a criterion c is *positively relevant for the acceptance of* d if affirming c can turn a rejection of d into a necessary acceptance. Formally, the set $C^+_{\mathcal{A}}$ of all criteria that are positively relevant for acceptance is defined by

$$C^+_{\mathcal{A}} := \{c \in C : \text{ there exists } A \in \mathcal{A} \text{ with } c \in A \text{ and } A \setminus \{c\} \notin \mathcal{A}\}.$$

Analogously, a criterion c is *negatively relevant for the acceptance of* d if negating c can turn a a rejection of d into a necessary acceptance. The corresponding set $C^-_{\mathcal{A}}$ is formally defined by

$$C^-_{\mathcal{A}} := \{c \in C : \text{ there exists } A \in \mathcal{A} \text{ with } c \notin A \text{ and } A \cup \{c\} \notin \mathcal{A}\}.$$

Similarly, the sets of the criteria that are *positively (respectively, negatively) relevant for the rejection of* d are the sets of those criteria the affirmation (respectively, negation) of which can turn a necessary rejection of d into a possible acceptance. Formally, these sets are defined by

$$C^+_{\mathcal{R}} := \{c \in C : \text{ there exists } R \in \mathcal{R} \text{ with } c \notin R \text{ and } R \cup \{c\} \notin \mathcal{R}\}$$

and

$$C^-_{\mathcal{R}} := \{c \in C : \text{ there exists } R \in \mathcal{R} \text{ with } c \in R \text{ and } R \setminus \{c\} \notin \mathcal{R}\},$$

[13] The corresponding rejection and acceptance regions are given by $\mathcal{R} = \emptyset$ and $\mathcal{A} = \{\{c_1, c_2\}, \{c_3\}, \{c_2, c_3\}, \{c_1, c_2, c_3\}\}$, respectively. The non-monotonicity of these justification constraints follows from Lemma 1 below.

respectively.

Lemma 1 *A set of constraints is monotone if and only if no criterion is both positively and negatively relevant, i.e. if and only if*

$$\left(C^+_{\mathcal{A}} \cup C^+_{\mathcal{R}}\right) \cap \left(C^-_{\mathcal{A}} \cup C^-_{\mathcal{R}}\right) = \emptyset.$$

Lemma 2 *A set of constraints is one-sided if and only if no criterion is simultaneously relevant for acceptance and rejection (positively, or negatively), i.e. if and only if*

$$\left(C^+_{\mathcal{A}} \cup C^-_{\mathcal{A}}\right) \cap \left(C^+_{\mathcal{R}} \cup C^-_{\mathcal{R}}\right) = \emptyset.$$

The proof of the necessity part of our main result provided in the appendix proceeds as follows. First, we show that the existence of an monotonically independent aggregation rule with no veto power and majority voting on the decision implies that the pairwise intersections of the four sets $C^+_{\mathcal{A}}$, $C^-_{\mathcal{A}}$, $C^+_{\mathcal{R}}$ and $C^-_{\mathcal{R}}$ have to be empty. By Lemmas 1 and 2 this implies that the decision rule has to be monotone and one-sided.

The sufficiency part of the above theorem is provided by explicitly constructing, for any monotone and one-sided decision rule, an anonymous aggregation method with majority voting on the decision and appropriate super-majority quotas on the criteria.

4 Monotone Justification Constraints

We now want to determine to which extent weakening the aggregation requirements on either the outcome decision and the criteria creates a scope for further possibilities. To obtain sharper results and facilitate the exposition, we will assume throughout monotonicity of the justification constraints.

First, we note that requiring *majority voting* on the decision is without loss of generality under monotonicity (and no veto power on the decision criteria). Specifically, we have the following result.

Proposition 1 *Consider a set of monotone justification constraints* $(\mathcal{A}, \mathcal{R})$. *The following statements are equivalent.*
(i) $(\mathcal{A}, \mathcal{R})$ *admits monotonically independent aggregation rules with majority voting on the decision and no veto power on the decision criteria.*
(ii) $(\mathcal{A}, \mathcal{R})$ *admits monotonically independent aggregation rules with no veto power on the decision criteria.*
(iii) $(\mathcal{A}, \mathcal{R})$ *is one-sided.*

Let $(\mathcal{A}, \mathcal{R})$ be monotone. By Lemma 1, we may assume without loss of generality for the remainder of this section that $C^-_{\mathcal{A}}$ and $C^-_{\mathcal{R}}$ are empty, i.e. that $\mathcal{A}$ is closed under taking supersets and that $\mathcal{R}$ is closed under taking subsets. A decision criterion is called *two-sided* if it is simultaneously relevant for acceptance and rejection. The constraints $(\mathcal{A}, \mathcal{R})$ will be called $\mathcal{A}$-*simple* if all minimal elements of $\mathcal{A}$ contain at most one two-sided decision criterion. Similarly, $(\mathcal{A}, \mathcal{R})$ will be called $\mathcal{R}$-*simple* if all maximal elements of $\mathcal{R}$ do not contain at most one two-sided decision criterion. The following results give the necessary and sufficient conditions under which different types of aggregation procedures on the decision are justifiable via an independent aggregation of the decision criteria.

Theorem 2 (majority voting / no veto power on decision) *Let* $(\mathcal{A}, \mathcal{R})$ *be monotone. The following statements are equivalent.*
(i) $(\mathcal{A}, \mathcal{R})$ *admits monotonically independent aggregation rules with majority voting on the decision.*
(ii) $(\mathcal{A}, \mathcal{R})$ *admits monotonically independent aggregation rules with no veto power on the decision.*
(iii) $(\mathcal{A}, \mathcal{R})$ *is both* $\mathcal{A}$*-simple and* $\mathcal{R}$*-simple.*

Theorem 3 (no-dictatorship on decision) *Let* $(\mathcal{A}, \mathcal{R})$ *be monotone. The following statements are equivalent.*
(i) $(\mathcal{A}, \mathcal{R})$ *admits monotonically independent aggregation rules that are non-dictatorial on the decision.*
(ii) $(\mathcal{A}, \mathcal{R})$ *is* $\mathcal{A}$*-simple or* $\mathcal{R}$*-simple.*

Frequently, *all* decision criteria are two-sided; this is true in particular in the truth-functional case, but substantially more generally. Examples are justification constraints according to which a candidate is accepted whenever he or she fulfills at least k out of m criteria ($1 \leq k \leq m$), and rejected whenever he/she fulfills at most l of the m criteria ($0 \leq l \leq m-1$) with $k > l$ (cf. Example 1 in Section 2.3 above). For these cases, we obtain the following very crisp characterization. An aggregation rule is called *oligarchic* if there exists a group $M \subseteq N$ of individuals (the "oligarchs") and a consistent default judgment J_0 such that a departure from J_0 in the collective judgment requires unanimous consent among the members of M (proposition-by-proposition). Note that, in particular, every oligarch has a veto on all criteria and on the decision.

Proposition 2 *Let* $(\mathcal{A}, \mathcal{R})$ *be monotone, and assume that all decision criteria are two-sided. Then, all aggregation rules are oligarchic (in particular, any aggregation method entails a veto on the decision). There exist anonymous aggregation rules if and only if either* $\mathcal{A} = 2^C \setminus \{\emptyset\}$*, or* $\mathcal{R} = 2^C \setminus \{C\}$*, i.e. if and only if the decision is equivalent to either the disjunction, or the conjunction of all decision criteria. In all other cases, all aggregation methods are dictatorial.*

Note that this result implies in particular that in the two-sided case, anonymous rules can only exist in the truth-functional case. Proposition 2 can be derived from existing general characterizations of the possibility of non-oligarchic (Nehring (2006)) and non-dictatorial proposition-wise aggregation (Nehring and Puppe (2005)). By contrast, our other results are not special cases of these general characterizations.

Appendix A: Proofs

For the proofs of the above results we invoke our general characterization of monotonically independent aggregation methods in terms of the Intersection Property (see Nehring and Puppe (2006a)), as follows. A family of *winning coalitions* is a non-empty family $\mathcal{W}$ of subsets of the set N of all individuals satisfying $[W \in \mathcal{W}$ and $W' \supseteq W] \Rightarrow W' \in \mathcal{W}$. Denote by Z the set of all propositions, i.e. $Z = C \cup \{d\}$, and by Z^* the *negation closure* of Z, i.e. $Z^* := Z \cup \{\neg p : p \in Z\}$ where $\neg p$ denotes the negation of p. A *structure of winning coalitions* on Z^* is a mapping $p \mapsto \mathcal{W}_p$ that assigns a family of winning coalitions to each proposition $p \in Z^*$ satisfying the following condition,

$$W \in \mathcal{W}_p \Leftrightarrow (N \setminus W) \notin \mathcal{W}_{\neg p}. \tag{A.1}$$

In words, a coalition is winning for p if and only if its complement is not winning for the negation of p. An aggregation rule F is called *voting by issues*, or, in our context simply *proposition-wise voting*, if for some structure of winning coalitions and all $p \in Z$,

$$p \in F(J_1, ..., J_n) \Leftrightarrow \{i : p \in J_i\} \in \mathcal{W}_p.$$

Observe that, so far, nothing guarantees that the outcome judgment $F(J_1, ..., J_n)$ of proposition-wise voting is consistent. The necessary and sufficient condition for consistency can be described as follows.

A *critical family* is a minimal subset $Q \subseteq Z^*$ of propositions that is logically inconsistent. The sets $\{p, \neg p\}$ are called *trivial* critical families. A structure of winning coalitions satisfies the *Intersection Property* if for any critical family $\{p_1, ..., p_l\} \subseteq Z^*$, and any selection $W_j \in \mathcal{W}_{p_j}$,

$$\bigcap_{j=1}^{l} W_j \neq \emptyset.$$

In Nehring and Puppe (2006a, Theorem 3), we have shown that an aggregation rule satisfies unrestricted domain, voter sovereignty and monotone independence if and only if it is proposition-wise voting satisfying the Intersection Property.

Using (A.1) and the fact that families of winning coalitions are closed under taking supersets, we obtain

$$\mathcal{W}_{\neg p} = \{W \subseteq N : W \cap W' \neq \emptyset \text{ for all } W' \in \mathcal{W}_p\}. \tag{A.2}$$

The following *conditional entailment relation* plays a central role. For all $p, q \in Z^*$,

$$p \geq^0 q \Leftrightarrow [p \neq \neg q \text{ and there exists a critical family containing } p \text{ and } \neg q]. \tag{A.3}$$

By $\geq$ we denote the transitive closure of $\geq^0$, and by $\equiv$ the symmetric part of $\geq$. Note that $\geq$ is "negation adapted" in the sense that $p \geq q \Leftrightarrow \neg q \geq \neg p$.

The following two lemmas are proved in Nehring and Puppe (2005).

Lemma A.1 *Suppose that a structure of winning coalitions satisfies the Intersection Property. Then, $p \geq q \Rightarrow \mathcal{W}_p \subseteq \mathcal{W}_q$.*

Lemma A.2 (Veto Lemma) *Suppose that a structure of winning coalitions satisfies the Intersection Property, and assume that p, q, r are contained in some critical family. If $\mathcal{W}_{\neg p} \subseteq \mathcal{W}_q$, then $\{i\} \in \mathcal{W}_{\neg r}$, for some $i \in N$.*

The following lemma characterizes the conditional entailment relation between the decision criteria and the decision for any set of justification constraints.

Lemma A.3 *Let $(\mathcal{A}, \mathcal{R})$ be a set of justification constraints, then*
(i) $c \in C^+_{\mathcal{A}} \Leftrightarrow c \geq d$,
(ii) $c \in C^-_{\mathcal{A}} \Leftrightarrow \neg d \geq c$,
(iii) $c \in C^+_{\mathcal{R}} \Leftrightarrow d \geq c$, *and*
(iv) $c \in C^-_{\mathcal{R}} \Leftrightarrow c \geq \neg d$.

Proof of Lemma A.3 (i) Let $c \in C^+_{\mathcal{A}}$ and consider any subset $A \in \mathcal{A}$ with $c \in A$ such that $A \setminus \{c\} \notin \mathcal{A}$. Since $A \in \mathcal{A}$, the set $A \cup \{\neg p : p \in C \setminus A\} \cup \{\neg d\}$ forms an inconsistent family of propositions. Let P a minimally inconsistent subset of this family. By assumption, since $A \setminus \{c\} \notin \mathcal{A}$, the set P contains c; moreover, since any combination of affirmed criteria is consistent, P also contains $\neg d$. This shows that $c \geq d$.

Now suppose conversely that $c \geq d$, i.e. that there exists a critical family $P \subseteq Z^*$ containing c and $\neg d$. Since P is inconsistent, we have $P \cap C \in \mathcal{A}$; thus, by criticality of P, we obtain that c is positively relevant for the acceptance of d.
(ii) Let $c \in C^-_{\mathcal{A}}$ and consider any subset $A \in \mathcal{A}$ with $c \notin A$ such that $A \cup \{c\} \notin \mathcal{A}$. Since $A \in \mathcal{A}$, the set $A \cup \{\neg p : p \in C \setminus A\} \cup \{\neg d\}$ forms an inconsistent family of propositions. Let P a minimally inconsistent subset of this family. By assumption, since $A \cup \{c\} \notin \mathcal{A}$, the set P contains $\neg c$; moreover, since any combination of affirmed criteria is consistent, P also contains $\neg d$. This shows that $\neg d \geq c$.

Now suppose conversely that $\neg d \geq c$, i.e. that there exists a critical family $P \subseteq Z^*$ containing $\neg c$ and $\neg d$. Since P is inconsistent, we have $P \cap C \in \mathcal{A}$; thus, by criticality of P, we obtain that c is negatively relevant for the acceptance of d.
The proof of part (iii) is analogous to the proof of part (ii) with d replaced by $\neg d$; similarly, the proof of part (iv) is analogous to the proof of part (i).

For the remaining proofs, we introduce the following notation:

$$\begin{aligned} C^+_{(\mathcal{A},\mathcal{R})} &:= C^+_{\mathcal{A}} \cup C^+_{\mathcal{R}}, \\ C^-_{(\mathcal{A},\mathcal{R})} &:= C^-_{\mathcal{A}} \cup C^-_{\mathcal{R}}, \\ C_{\mathcal{A}} &:= C^+_{\mathcal{A}} \cup C^-_{\mathcal{A}}, \\ C_{\mathcal{R}} &:= C^+_{\mathcal{R}} \cup C^-_{\mathcal{R}}. \end{aligned}$$

Proof of Lemma 1 Evidently, we have $C^-_{(\mathcal{A},\mathcal{R})} = \emptyset$ if and only if $\mathcal{A}$ is closed under taking supersets and $\mathcal{R}$ is closed under taking subsets. Moreover, if $(\mathcal{A}', \mathcal{R}')$ arises from $(\mathcal{A}, \mathcal{R})$ by swapping the meaning of affirmation versus negation for the criteria in $\chi \subseteq C$, then

$$C^+_{(\mathcal{A}',\mathcal{R}')} = (C^+_{(\mathcal{A},\mathcal{R})} \setminus \chi) \cup (C^-_{(\mathcal{A},\mathcal{R})} \cap \chi), \tag{A.4}$$

$$C^-_{(\mathcal{A}',\mathcal{R}')} = (C^-_{(\mathcal{A},\mathcal{R})} \setminus \chi) \cup (C^+_{(\mathcal{A},\mathcal{R})} \cap \chi). \tag{A.5}$$

Now, suppose that for the decision rule $(\mathcal{A}, \mathcal{R})$ we have $C^+_{(\mathcal{A},\mathcal{R})} \cap C^-_{(\mathcal{A},\mathcal{R})} = \emptyset$. Then, by taking $\chi = C^-_{(\mathcal{A},\mathcal{R})}$, we can transform $(\mathcal{A}, \mathcal{R})$ into an equivalent decision rule $(\mathcal{A}', \mathcal{R}')$ with $C^-_{(\mathcal{A}',\mathcal{R}')} = \emptyset$, as is immediate from (A.5). Thus, $(\mathcal{A}, \mathcal{R})$ is monotone.

Conversely, suppose that $C^+_{(\mathcal{A},\mathcal{R})} \cap C^-_{(\mathcal{A},\mathcal{R})} \neq \emptyset$. It is immediate from (A.4) and (A.5) that the set of criteria that are both positively and negatively relevant is invariant under a χ-transformation. In particular, we have $C^-_{(\mathcal{A}',\mathcal{R}')} \neq \emptyset$ for all decision rules that are equivalent to $(\mathcal{A},\mathcal{R})$. Thus, $(\mathcal{A},\mathcal{R})$ is not monotone.

Proof of Lemma 2 Evidently, if $(\mathcal{A},\mathcal{R})$ is one-sided, then either $C_{\mathcal{A}} = \emptyset$ or $C_{\mathcal{R}} = \emptyset$, thus in particular, $C_{\mathcal{A}} \cap C_{\mathcal{R}} = \emptyset$.

Now suppose, conversely, $C_{\mathcal{A}} \cap C_{\mathcal{R}} = \emptyset$, and assume, by way of contradiction that both $\mathcal{A}$ and $\mathcal{R}$ are non-empty. Let A^m and R^m be maximal elements (with respect to set inclusion) of $\mathcal{A}$ and $\mathcal{R}$, respectively. Moreover, denote $C^0 := C \setminus (C_{\mathcal{A}} \cup C_{\mathcal{R}})$ (which may be empty). We have $A^m \supseteq C_{\mathcal{R}} \cup C^0$; indeed, if $c \notin A^m$, then $A^m \cup \{c\} \notin \mathcal{A}$ by maximality of A^m, and therefore $c \in C_{\mathcal{A}}$. Thus, we can write $A^m = \tilde{A} \cup C_{\mathcal{R}} \cup C^0$ for some $\tilde{A} \subseteq C_{\mathcal{A}}$. Note that $\tilde{A}$ may be empty. We have,

$$(\tilde{A} \cup R) \in \mathcal{A} \text{ for all } R \subseteq C_{\mathcal{R}} \cup C^0, \tag{A.6}$$

since otherwise one would again have $c \in C_{\mathcal{A}}$ for some $c \in C_{\mathcal{R}} \cup C^0$.

By a completely symmetric argument, we obtain $R^m = \tilde{R} \cup C_{\mathcal{A}} \cup C^0$ for some $\tilde{R} \subseteq C_{\mathcal{R}}$, and

$$(\tilde{R} \cup A) \in \mathcal{R} \text{ for all } A \subseteq C_{\mathcal{A}} \cup C^0. \tag{A.7}$$

But (A.6) and (A.7) together imply $\tilde{A} \cup \tilde{R} \in \mathcal{A} \cap \mathcal{R}$, a contradiction.

Proof of Theorem 1 Suppose that $(\mathcal{A},\mathcal{R})$ is monotone and one-sided. Without loss of generality, suppose that $\mathcal{R} = \emptyset$ and $C^-_{\mathcal{A}} = \emptyset$ (the argument is completely analogous if $\mathcal{A} = \emptyset$ and $C^-_{\mathcal{R}} = \emptyset$). By Lemma A.3, any critical family is of the form $A \cup \{\neg d\}$ for some $A \subseteq C^+_{\mathcal{A}}$. Let $m_{\mathcal{A}}$ be the largest cardinality of such a critical family. Using the (anonymous version) of the Intersection Property (see Nehring and Puppe (2006a, Fact 3.4)), it is easily verified that a quota rule that assigns the quota $\frac{2m_{\mathcal{A}}-3}{2m_{\mathcal{A}}-2}$ to all $c \in C^+_{\mathcal{A}}$, and the quota 1/2 to d (and all other criteria, if any) is consistent. If there are sufficiently many individuals, no single voter has a veto.

To prove the converse statement, suppose first that $C^+_{(\mathcal{A},\mathcal{R})} \cap C^-_{(\mathcal{A},\mathcal{R})} \neq \emptyset$. There are four cases to consider. The arguments are similar in each case.
(i) Suppose that there exists $c \in C^+_{\mathcal{A}} \cap C^-_{\mathcal{A}}$. Then, by Lemma A.3, $c \geq d$ and $\neg d \geq c$, i.e. there exists a critical family P containing c and $\neg d$, and a critical family P' containing $\neg c$ and $\neg d$. Majority voting on d implies $\mathcal{W}_d = \mathcal{W}_{\neg d}$ and thus, by Lemma A.1, $\mathcal{W}_c = \mathcal{W}_{\neg c} = \mathcal{W}_d = \mathcal{W}_{\neg d}$. One of the critical families P or P' must contain at least three elements since otherwise d would always have to be accepted. Without loss of generality suppose that $c' \in P$ for $c' \neq c$. By Lemma A.2, for some i, $\{i\} \in \mathcal{W}_{\neg c'}$, i.e. i has a veto.
(ii) Suppose that there exists $c \in C^+_{\mathcal{A}} \cap C^-_{\mathcal{R}}$. Then, by Lemma A.3, $c \geq d$ and $c \geq \neg d$, i.e. there exists a critical family P containing c and $\neg d$, and a critical family P' containing c and d. As in case (i), majority voting on d implies $\mathcal{W}_c = \mathcal{W}_{\neg c} = \mathcal{W}_d = \mathcal{W}_{\neg d}$. One of the critical families P or P' must contain at least three elements since otherwise affirming c would be inconsistent with either decision. As above, this implies a veto using Lemma A.2.
(iii) Suppose that there exists $c \in C^+_{\mathcal{R}} \cap C^-_{\mathcal{A}}$. Then, by Lemma A.3, $d \geq c$ and $\neg d \geq c$, i.e. there exists a critical family P containing $\neg c$ and d, and a critical family P' containing $\neg c$ and $\neg d$. As above, majority voting on d implies $\mathcal{W}_c = \mathcal{W}_{\neg c} = \mathcal{W}_d = \mathcal{W}_{\neg d}$. One of the critical families P or P' must contain at least three elements since otherwise

negating c would be inconsistent with either decision. As above, this implies a veto using Lemma A.2.
(iv) Finally, suppose that there exists $c \in C^+_{\mathcal{R}} \cap C^-_{\mathcal{R}}$. Then, by Lemma A.3, $d \geq c$ and $c \geq \neg d$, i.e. there exists a critical family P containing $\neg c$ and d, and a critical family P' containing c and d. Again, majority voting on d implies $\mathcal{W}_c = \mathcal{W}_{\neg c} = \mathcal{W}_d = \mathcal{W}_{\neg d}$. One of the critical families P or P' must contain at least three elements since otherwise d would always have to be rejected. As above, this implies a veto using Lemma A.2.
Since in all four cases a veto results, we conclude that the existence of an aggregation rule with majority voting on the decision and no veto power implies that $C^+_{(\mathcal{A},\mathcal{R})} \cap C^-_{(\mathcal{A},\mathcal{R})} = \emptyset$. By Lemma 1, justification constraints admitting such aggregation rules must thus be monotone.

Now suppose that $C_{\mathcal{A}} \cap C_{\mathcal{R}} \neq \emptyset$. Again, there are four cases to consider, two of which have already been covered above.
(v) Suppose that there exists $c \in C^+_{\mathcal{A}} \cap C^+_{\mathcal{R}}$. Then, by Lemma A.3, $c \geq d$ and $d \geq c$, i.e. there exists a critical family P containing c and $\neg d$, and a critical family P' containing $\neg c$ and d. By Lemma A.1, we obtain that $\mathcal{W}_c = \mathcal{W}_d$ and $\mathcal{W}_{\neg c} = \mathcal{W}_{\neg d}$. One of the critical families P or P' must contain at least three elements since otherwise c and d would be logically equivalent. Without loss of generality suppose that $c' \in P$ for $c' \neq c$. By Lemma A.2, for some i, $\{i\} \in \mathcal{W}_{\neg c'}$, i.e. i has a veto.
(vi) Suppose that there exists $c \in C^-_{\mathcal{A}} \cap C^-_{\mathcal{R}}$. Then, by Lemma A.3, $\neg d \geq c$ and $c \geq \neg d$, i.e. there exists a critical family P containing $\neg c$ and $\neg d$, and a critical family P' containing c and d. As in case (v), we obtain $\mathcal{W}_{\neg c} = \mathcal{W}_d$ and $\mathcal{W}_c = \mathcal{W}_{\neg d}$ using Lemma A.1. One of the critical families P or P' must contain at least three elements since otherwise c and $\neg d$ would be logically equivalent. As above, this implies a veto using Lemma A.2.
The other two cases have already been treated above as cases (ii) and (iii). Again in all four cases a veto results, and we conclude that the existence of an aggregation rule with majority voting on the decision and no veto power implies that $C_{\mathcal{A}} \cap C_{\mathcal{R}} = \emptyset$. By Lemma 2, justification constraints admitting such aggregation rules must thus be one-sided. This concludes the proof of Theorem 1.

The remaining proofs concern the monotone case, and we may therefore assume without loss of generality that $C^-_{(\mathcal{A},\mathcal{R})} = \emptyset$ in all what follows.

Proof of Proposition 1 By Theorem 1, conditions (i) and (iii) are equivalent; moreover, (i) obviously implies (ii). The implication "(ii) $\Rightarrow$ (iii)" follows as in case (v) of the proof of Theorem 1 above. Indeed, an inspection of the argument given there shows that in the monotone case one can deduce the existence of a veto without further assumption.

The key to the proofs of Theorems 2 and 3 is the observation that in the monotone case, all critical families are either of the form $\{c_1, ..., c_k\} \cup \{\neg d\}$ where $\{c_1, ..., c_k\}$ is a minimal element in $\mathcal{A}$, or of the form $\{\neg c_1, ..., \neg c_l\} \cup \{d\}$ where $C \setminus \{c_1, ..., c_l\}$ is a maximal element in $\mathcal{R}$.

Proof of Theorem 2 Evidently, (i) implies (ii). We show that (ii) implies (iii) by contraposition. Suppose first that $(\mathcal{A}, \mathcal{R})$ is not $\mathcal{A}$-simple. By the preceding observation, this implies that there exists a critical family containing $\neg d$ and at least two two-sided criteria, say c and c'. By Lemmas A.1 and A.3, we have $\mathcal{W}_c = \mathcal{W}_{c'} = \mathcal{W}_d$ and $\mathcal{W}_{\neg c} = \mathcal{W}_{\neg c'} = \mathcal{W}_{\neg d}$. By Lemma A.2, we have $\{i\} \in \mathcal{W}_{\neg c'}$ and hence also $\{i\} \in \mathcal{W}_{\neg d}$

for some i, i.e. individual i has a veto on the decision. A completely symmetric argument shows that a veto on the decision also results if $(\mathcal{A}, \mathcal{R})$ is not $\mathcal{R}$-simple. This shows that no veto power on the decision implies both $\mathcal{A}$- and $\mathcal{R}$-simplicity, i.e. "(ii) $\Rightarrow$ (iii)."

Finally, to show that (iii) implies (i), let $(\mathcal{A}, \mathcal{R})$ be both $\mathcal{A}$- and $\mathcal{R}$-simple. Using the Intersection Property and given the structure of critical families in the monotone case, the following proposition-wise aggregation rule is easily seen to be consistent: the outcome decision d and all two-sided criteria are decided by majority voting; moreover, any criterion $c \in C_{\mathcal{A}}^{+} \setminus C_{\mathcal{R}}^{+}$ is collectively affirmed only under unanimous consent, and any criterion $c \in C_{\mathcal{R}}^{+} \setminus C_{\mathcal{A}}^{+}$ is collectively negated only under unanimous consent.

Proof of Theorem 3 By the arguments given in the proof of Theorem 2, if $(\mathcal{A}, \mathcal{R})$ is not $\mathcal{A}$-simple, we can find an individual i such that $\{i\} \in \mathcal{W}_{\neg d}$. Similarly, if $(\mathcal{A}, \mathcal{R})$ is also not $\mathcal{R}$-simple, we can find i' such that $\{i'\} \in \mathcal{W}_{d}$. But by (A.2), we must in fact have $i = i'$, i.e. the aggregation rule is dictatorial on the decision (and on all two-sided criteria).

Now suppose, conversely, that $(\mathcal{A}, \mathcal{R})$ is $\mathcal{R}$-simple. Then, the rule according to which d and any two-sided criterion is collectively affirmed only under unanimous consent, while all other criteria are decided dictatorially according to the judgment of some fixed individual is consistent and entails no-dictatorship on the outcome decision. A symmetric argument applies when $(\mathcal{A}, \mathcal{R})$ is $\mathcal{A}$-simple.

Proof of Proposition 2 By the argument in part (v) of the proof of Theorem 1, we have $\{i\} \in \mathcal{W}_{\neg c'}$ for some i and some $c' \in C_{\mathcal{A}}^{+}$ if there exists a critical family containing $\neg d$ and at least three elements. By assumption, $C_{\mathcal{A}}^{+} = C_{\mathcal{R}}^{+} = C$, hence by Lemmas A.1 and A.3, $\{i\} \in \mathcal{W}_{\neg c} = \mathcal{W}_{\neg d}$ for all $c \in C$. Similarly, we obtain $\{i'\} \in \mathcal{W}_{c''}$ for some i' and some $c'' \in C_{\mathcal{R}}^{+}$, and thus for all $c'' \in C$, if there exists a critical family containing d and at least three elements. Moreover, there is at least one critical family containing at least three elements. Without loss of generality, suppose that $\{i\} \in \mathcal{W}_{\neg c} = \mathcal{W}_{\neg d}$. Define $M := \{h \in N : \{h\} \in \mathcal{W}_{\neg d}\}$. Using the Intersection Property and (A.2), it can be verified that consistency forces the aggregation method to be oligarchic with oligarchy M.

Using the Intersection Property once again, it is easily seen from the preceding argument that non-dictatorial rules can only exist if either all critical families containing $\neg d$ have only two elements, or all critical families containing d have only two elements. The first case corresponds to the case $\mathcal{A} = 2^C \setminus \{\emptyset\}$, the second to $\mathcal{R} = 2^C \setminus \{C\}$. In either case, the oligarchic rule can be chosen to be anonymous, i.e. a "unanimity rule."

References

[1] DIETRICH, F. (2006), “Judgment Aggregation: (Im)Possibility Theorems,” *Journal of Economic Theory* 126, 286-298.

[2] DIETRICH, F. (2007), “On the Possibility of Judgment Aggregation with Subjunctive Implications,” *mimeographed.*

[3] DIETRICH, F. and LIST, C. (2006), “Arrow’s Theorem in Judgment Aggregation,” *Social Choice and Welfare*, forthcoming.

[4] DOKOW, E. and HOLZMAN, R. (2005), “Aggregation of Binary Evaluations,” *mimeographed.*

[5] KORNHAUSER, L. and SAGER, L. (1986), “Unpacking the Court,” *Yale Law Journal* 96, 82-117.

[6] LEVI, I. (2004), “List and Pettit,” *Synthese* 140, 237-242.

[7] LIST, C. and PETTIT, P. (2002), “Aggregating Sets of Judgments: An Impossibility Result,” *Economics and Philosophy* 18, 89-110.

[8] LIST, C. and PUPPE, C. (2007), “Judgment Aggregation: A Survey,” *mimeographed.*

[9] MILLER, M.K. (2007), “Judgement Aggregation with Differing Agendas: An Extension to Democratic Theory,” *mimeographed.*

[10] MONGIN, P. (2005), “Factoring out the Impossibility of Logical Aggregation,” *mimeographed.*

[11] NEHRING, K. (2005), “The Impossibility of a Paretian Rational ,” *mimeographed.*

[12] NEHRING, K. (2006), “Oligarchies in Judgment Aggregation,” *mimeographed.*

[13] NEHRING, K. (2007), “The Impossibility of a Paretian Rational: A Bayesian Perspective,” *Economics Letters*, forthcoming.

[14] NEHRING, K. and PUPPE, C. (2005), “Strategy-Proof Social Choice without Dictators,” *mimeographed.*

[15] NEHRING, K. and PUPPE, C. (2006a), “The Structure of Strategy-Proof Social Choice. Part I: General Characterization and Possibility Results on Median Spaces,” *Journal of Economic Theory*, forthcoming.

[16] NEHRING, K. and PUPPE, C. (2006b), “Consistent Judgement Aggregation: The Thruth-Functional Case,” *mimeographed*

[17] PAULY, M. and VAN HEES, M. (2006), “Logical Constraints on Judgment Aggregation,” *Journal of Philosophical Logic* 35, 569-585.

[18] PETTIT, P. (2001), “Deliberative Democracy and the Discursive Dilemma,” *Philosophical Issues* 11, 268-299.

[19] PETTIT, P. (2004), “Groups with Minds of their Own,” in: F. Schmitt (Ed.) *Socializing Metaphysics*, New York, Rowman and Littlefield.